

# Theoretical and Experimental Problems of the Problem of Rotation of a Heat-Conducting Solid Relative to a Fixed Point

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**Abstract.** The article investigates a variant of a non-relativistic model of the motion of a rigid body in a uniform magnetic field described by a system of differential equations in a deterministic problem of the rotation of an isotropic body relative to the center of inertia fixed in the geometric center and offers experimental characteristics of the studies, the results of tests of the inductive orientation system of a moving homogeneous body. Thus, the article covers both theoretical and practical issues of the dynamics of rigid bodies under the influence of a magnetic field, offering methods and approaches to controlling their orientation and motion.

## 1 Introduction

The problem of the motion of a rigid body under the influence of external forces and moments of force is presented as follows. Since a rigid body generally possesses six degrees of freedom, the general system of equations of motion must contain six differential equations. They are represented as time derivatives of two vectors – the momentum of the body  $P$  and the kinetic moment  $L$  [1].

$$\frac{dP}{dt} = F ; \quad \frac{dL}{dt} = M , \tag{1}$$

where  $F$  and  $M$  are the resultant force and momentum.

If the mass  $m$  and the velocity of motion of the center of mass  $v$ , then the momentum

$$P = mv . \tag{2}$$

Expression for Kinetic Moment

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$$L = \int_V \rho \{ r^2 \Omega - r(r\Omega) \} dV, \tag{3}$$

where  $\rho$  is the density of the body;  
 $V$  - volume;  
 $\Omega$  - instantaneous angular velocity vector;  
 $r$  - radius – vector.

By choosing the directions of the coordinate axes appropriately, the inertia tensor

$$I_{i\mathfrak{R}} = \int_V \rho \{ x_i^2 \delta_{i\mathfrak{R}} - x_i x_{\mathfrak{R}} \} dV, \quad \delta_{i\mathfrak{R}} = \begin{cases} 1, \mathfrak{R} = i \\ 0, \mathfrak{R} \neq i \end{cases}$$

can be converted to a diagonal appearance. These directions are called the principal axes of inertia, and the corresponding values of the tensor components are called the principal moments of inertia.

After reducing to the principal axes for the components of the kinetic moment, the following is obtained:

$$L_1 = I_1 \Omega_1, \quad L_2 = I_2 \Omega_2, \quad L_3 = I_3 \Omega_3, \tag{4}$$

where  $I_1, I_2, I_3$  are the main moments of inertia.

To describe the motion of a rigid body, two coordinate systems are introduced: a fixed system  $\xi\eta\zeta$  and a mobile system  $xyz$ , the origin of which coincides with the center of mass of the rigid body and the axes are directed along the main axes of inertia.

The equations of solid-state dynamics consist of three equations of motion of the center of mass:

$$m \frac{d^2 \xi}{dt^2} = F_\xi; \quad m \frac{d^2 \eta}{dt^2} = F_\eta; \quad m \frac{d^2 \zeta}{dt^2} = F_\zeta, \tag{5}$$

where  $F_\xi, F_\eta, F_\zeta$  are the coordinate projections of the resultant force applied to the body and the equations of motion relative to the center of mass, recorded in a movable coordinate system:

$$\begin{aligned} \frac{dI_1 \Omega_1}{dt} + \Omega_2 I_3 \Omega_3 - \Omega_3 I_2 \Omega_2 &= M_x, \\ \frac{dI_2 \Omega_2}{dt} - \Omega_1 I_3 \Omega_3 + \Omega_3 I_1 \Omega_1 &= M_y, \\ \frac{dI_3 \Omega_3}{dt} + \Omega_1 I_2 \Omega_2 - \Omega_2 I_1 \Omega_1 &= M_z, \end{aligned} \tag{6}$$

where  $M_x, M_y, M_z$  are the coordinate projections of the resultant moment of forces.

Moving on to the study of the dynamics of solids in an electromagnetic field, it is necessary to take into account electromagnetic forces and moments of forces. If no external

forces, except for electromagnetic forces, act, then the components  $F_x, F_y, F_z, M_x, M_y, M_z$  are directly components of electromagnetic forces and moments of forces.

The most common form of notation of electromagnetic forces for a rigid body [2]:

$$F = \epsilon_0 \oint_S \left\{ E(nE) - \frac{1}{2} E^2 n \right\} dS + \frac{1}{\mu_0} \oint_S \left\{ B(nB) - \frac{1}{2} B^2 n \right\} dS, \tag{7}$$

where  $E$  is the strength of the electric field;

$B$  - Magnetic field induction

$\epsilon_0$  - dielectric constant of vacuum;

$\mu_0$  -magnetic inductivity;

$n$  is a normal vector to the surface of a solid.

Integration is carried out on the surface covering the body under consideration. The only condition regarding the choice of this surface is that it should not contain sources that create an electromagnetic field inside [3].

In the above expression of electromagnetic forces, it would also be necessary to take into account the force arising from a change in the momentum of the electromagnetic field, the density of which

$$f = \frac{1}{\mu c^2} \frac{\partial}{\partial t} [E, B] \tag{8}$$

and the magnitude of this force is considered negligible.

The written expression (7) is also applied to a simple special case, namely, when considering the electromagnetic pressure acting on the surface of a hemisphere.

Let us choose the coordinate system in such a way that the surface of the half-space coincides with the plane  $(x, z)$  and the positive direction of the normal coincides with the positive direction of the  $y$ -axis, which within a half-space decays at infinity [4]. In this case, the normal electric and magnetic pressures are determined by the expressions

$$P_E = -\frac{\epsilon_0}{2} E_x^2; \quad P_M = -\frac{1}{2\mu_0} B_x^2. \tag{9}$$

If the external uniform fields are directed along the  $y$ -axis with a similar attenuation at infinity within a half-space, then the corresponding pressures:

$$P_E = \frac{\epsilon_0}{2} E_y^2; \quad P_M = \frac{1}{2\mu_0} B_y^2. \tag{10}$$

pressures are created in the direction of the normal and the surface of the half-space is drawn into the field.

The nature of electromagnetic forces and moments of forces acting on a rigid body depends on the distribution of the field, the finding of which is associated with the integration of Maxwell's equations in the medium:

$$\begin{aligned} \operatorname{rot} E &= -\frac{\partial B}{\partial t}; & \operatorname{rot} H &= j + \frac{\partial D}{\partial t}; \\ \operatorname{div} D &= \rho_e; & \operatorname{div} B &= 0, \end{aligned} \tag{11}$$

where  $\rho_e$  is the density of electric charges.

The main factors that determine the different nature of electromagnetic forces and moments of forces are:

- spatial dependence of the external field;
- Time dependence of the external field;
- geometric shape of the body;
- Body material: dielectric, ferromagnetic, conductor.

In many practical problems, alternating fields are used to generate electromagnetic forces. If the frequency is high enough, then when calculating forces and moments of forces, averaging over time is performed [5]. When the fields change according to the sinusoidal law, the electromagnetic pressure will have the following dependence on time:

$$P = P_0 \sin^2 \omega t$$

Suppose that the pressure is exerted on the surface of the wafer with an area  $S$  and a thickness  $d$  (the density of the wafer material  $\rho$ ).

The  $x$  coordinate is chosen, which characterizes the position of the plate surface in the direction of the pressure. Then, taking into account the expression  $\frac{dP}{dt} = F$ , where  $F$  is the resultant force,

$$\rho d \frac{d^2 x}{dt^2} = P_0 \sin^2 \omega t \tag{12}$$

As a result of integrating this equation under zero initial conditions

$$x|_{t=0} = 0, \quad \frac{dx}{dt}|_{t=0} = 0$$

Obtained for speed  $V$  and movement  $x$  [6]:

$$v = \frac{P_0 t}{2\rho d} \left( 1 - \frac{\sin 2\omega t}{2\omega t} \right), \quad x = \frac{P_0 t^2}{4\rho d} \left[ 1 - \frac{\sin^2 \omega t}{(\omega t)^2} \right]. \tag{13}$$

The first terms of the right-hand parts of the obtained expressions characterize the motion of the plate under the influence of constant pressure  $\frac{1}{2}P_0$ , which can be determined as a result of averaging the pressure  $p = p_0 \sin^2 \omega t$  over the period. The second terms

characterize the motion caused by field oscillations. For large values of time, they  $\omega t \gg 1$  disappear [7] and uniformly accelerated motion becomes decisive.

Taking into account the fact that with uniformly accelerated motion, time  $t = \frac{2\rho v d}{P_0}$  or  $t = 2\sqrt{\frac{\rho x d}{P_0}}$  the following criteria are obtained to determine the applicability of time averaging:

$$\frac{2\rho v \omega d}{P_0} \gg 1 \quad \text{or} \quad \frac{4\rho x \omega^2 d}{P_0} \gg 1 \tag{14}$$

Of scientific interest are the possible results of integrating the equation (12) discussed here under zero initial conditions:

$$v = \frac{P_0 t}{2\rho d} \left( 1 - \frac{\sin 2\omega t}{2\omega t} \right) \quad x = \frac{P_0 t^2}{4\rho d} \left[ 1 + \frac{\cos \omega t \cos \omega t}{(\omega t)^2} \right] \tag{15}$$

The author of this paper proposes a modern approach to solving the desired equation in a deterministic class of nonstationary functions, to study the motions caused by field fluctuations for small values of time, and to consider equation (12) here as follows:

$ms^{-1} \frac{d^2 x}{dt^2} = P_0 \sin^2 \omega t$  under various non-zero deterministic initial conditions, and with the representation performed

$$\rho d \frac{du}{dt} = P_0 \sin^2 \omega t, \quad u(t_0 = 0) = u_0, \quad u_0 \neq 0 \tag{16}$$

Let's write down [8]:

$$u = u_0 + \int_0^t \rho^{-1} d^{-1} P_0 \sin^2 \omega t dt \tag{17}$$

Herewith

$$u \int_0^t \rho^{-1} d^{-1} P_0 \sin^2 \omega t dt - uu_0 + u_0^2 = \int_0^t \rho^{-1} d^{-1} P_0 \sin^2 \omega t dt^2 \tag{18}$$

From where

$$\int_0^t \rho^{-1} d^{-1} P_0 \sin^2 \omega t dt = 0.5u \pm (0.25u^2 - uu_0 + u_0 u_0)^{\frac{1}{2}} \tag{19}$$

On the other hand:

$$u_{0,1,2} = 0.5u \pm \left( 0.25u^2 - u \int_0^t \rho^{-1} d^{-1} P_0 \sin^2 \omega t dt + \left( \int_0^t \rho^{-1} d^{-1} P_0 \sin^2 \omega t dt \right)^2 \right)^{\frac{1}{2}} \quad (20)$$

The most important problems of the force effect of the electromagnetic field are reduced to the problems of linear electrodynamics, which makes it possible to take into account linear relations:

$$D = \varepsilon E, \quad B = \mu H, \quad j = \sigma E \quad (21)$$

Neglecting the insignificant influence of displacement currents, a system of initial equations is obtained:

$$\begin{aligned} \operatorname{rot} E &= -\frac{\partial B}{\partial t}, \quad \operatorname{rot} H = \sigma E, \\ \operatorname{div} D &= \rho, \quad \operatorname{div} B = 0 \end{aligned}$$

and distinguish three main tasks of electrodynamics:

- dielectric or conductive bodies in a stationary or alternating electric field;
- ferromagnetic bodies in a stationary or alternating magnetic field;
- conductive bodies in an alternating magnetic field.

## 2 Materials and methods

A noticeable effect of the natural magnetic field is used in orientation on the terrain with the help of a magnetic compass.

The presence of other magnetic sources, conductive areas distorts the actual readings. The author of this paper proposes to take into account the results of experimental tests of gyroscopic elements of inertial systems in the design recommendations of advanced devices and devices in a wide range of electromagnetic disturbances.

In an alternating magnetic field, the magnetic compass is idle.

A short-term alternating magnetic field has less disturbing effect on the functioning of a magnetically shielded sensing element compared to a uniform fixed field of measurable strength.

The displacement of a rigid body in a homogeneous magnetic, electric field is described by a system of differential equations, the right-hand parts of which contain electromagnetic moments of forces. In the most general case, they are functions of the angles of the vector of the external magnetic field and the axes of the observed coordinate system, as well as functions of the vector of angular velocity  $\Omega$  solid. Sometimes it is possible to regard these moments as depending only on the angles that determine the orientation of the rigid body with respect to the external field. Let us consider here the dependence of the electromagnetic moment of the forces  $M$  on the guide cosines  $\gamma, \gamma', \gamma''$  between the vector of the external field  $B$  and the coordinate axes  $x, y, z$  of the movable coupled coordinate system. It is assumed that the vector of the external field is directed along the axis  $\zeta$  of the oriented initial coordinate system. Then the components of the field along the coordinate axes of the movable reference system are determined by the relations [9,10]:

$$B_x = B\gamma ; B_y = B\gamma' ; B_z = B\gamma'' . \tag{22}$$

Equations of rotation of a rigid body with respect to a fixed point in a homogeneous deterministic field:

$$\begin{aligned} \frac{dI_x\Omega_x}{dt} + \Omega_y I_z \Omega_z - \Omega_z I_y \Omega_y &= 2PV \cdot \\ &\left[ (\alpha_{yy} - \alpha_{zz})\gamma'\gamma'' + \alpha_{yz}(\gamma'^2 - \gamma''^2) + (\alpha_{xy}\gamma'' - \alpha_{xz}\gamma')\gamma \right] \\ \frac{dI_y\Omega_y}{dt} - \Omega_x I_z \Omega_z + \Omega_z I_x \Omega_x &= 2PV \cdot \\ &\left[ (\alpha_{zz} - \alpha_{xx})\gamma\gamma'' + \alpha_{zx}(\gamma^2 - \gamma''^2) + (\alpha_{yz}\gamma - \alpha_{yx}\gamma'')\gamma' \right], \\ \frac{dI_z\Omega_z}{dt} + \Omega_x I_y \Omega_y - \Omega_y I_x \Omega_x &= 2PV \cdot \\ &\left[ (\alpha_{xx} - \alpha_{yy})\gamma\gamma' + \alpha_{xy}(\gamma^2 - \gamma'^2) + (\alpha_{zx}\gamma' - \alpha_{zy}\gamma)\gamma'' \right] \end{aligned} \tag{23}$$

where  $P$  is the magnetic pressure

$$\begin{aligned} \dot{\gamma} &= \Omega_z\gamma' - \Omega_y\gamma'' ; \\ \dot{\gamma}' &= \Omega_x\gamma'' - \Omega_z\gamma ; \\ \dot{\gamma}'' &= \Omega_y\gamma - \Omega_x\gamma' . \end{aligned} \tag{24}$$

The coefficients  $\alpha_{iR}$  depend on the shape and magnetic permeability of the body.

If guide cosines are known  $\gamma, \gamma', \gamma''$ , then their connection with Euler angles is established using the known formulas:

$$\begin{aligned} \varphi &= \arctg\gamma' / \gamma' ; \nu = \arccos\gamma'' ; \\ \frac{d\Psi}{dt} &= \frac{1}{\gamma''} \left( \Omega_z - \frac{\gamma'\dot{\gamma} - \gamma\dot{\gamma}'}{\gamma^2 + \gamma'^2} \right) . \end{aligned} \tag{25}$$

System (23) – (25) is not generally integrated.

The most important special case is a rigid body that has the shape of an ellipsoid, since in this case the components of the electromagnetic moment of forces are expressed by explicit formulas:

$$\begin{aligned} M_x &= pSl \frac{4(\mathfrak{R}_x - 1)^2 (N_z - N_y)\gamma'\gamma''}{3\lambda_1 [1 + (\mathfrak{R}_x - 1)N_y] [1 + (\mathfrak{R}_x - 1)N_z]} ; M_y = pSl \frac{4(\mathfrak{R}_y - 1)^2 (N_x - N_z)\gamma\gamma''}{3\lambda_1 [1 + (\mathfrak{R}_y - 1)N_x] [1 + (\mathfrak{R}_y - 1)N_z]} \\ M_z &= pSl \frac{4(\mathfrak{R}_z - 1)^2 (N_y - N_x)\gamma\gamma'}{3\lambda_1 [1 + (\mathfrak{R}_z - 1)N_x] [1 + (\mathfrak{R}_z - 1)N_y]} , \end{aligned} \tag{26}$$

where the odds are

$$N_x = \frac{\lambda_1 \lambda_2}{2} \int_0^\infty \frac{dS}{(S + \lambda_1^2)R_S}; \quad N_y = \frac{\lambda_1 \lambda_2}{2} \int_0^\infty \frac{dS}{(S + 1)R_S}; \quad N_z = \frac{\lambda_1 \lambda_2}{2} \int_0^\infty \frac{dS}{(S + \lambda_2^2)R_S};$$

$$R_S = \sqrt{(S + 1)(S + \lambda_1^2)(S + \lambda_2^2)}; \quad \lambda_1 = \frac{a}{\epsilon}; \quad \lambda_2 = \frac{c}{\epsilon};$$

$S$  - cross-sectional area of the ellipsoid;  $l = 2a$  - the length of the ellipsoid.

Lemma. A Proposed Solution to the System of Real Equations (2.2), (2.5) of the Problem of Rotation of an Ellipsoid in a Homogeneous Magnetic Field with Respect to an Observed Fixed Point at

$$-\Omega_y I_z \Omega_z + \Omega_z I_y \Omega_y +$$

$$pSl \frac{4(\Re_x - 1)^2 (N_z - N_y) \gamma' \gamma''}{3\lambda_1 [1 + (\Re_x - 1)N_y] [1 + (\Re_x - 1)N_z]} \neq 0;$$

$$\Omega_x I_z \Omega_z - \Omega_z I_x \Omega_x +$$

$$pSl \frac{4(\Re_y - 1)^2 (N_x - N_z) \gamma' \gamma''}{3\lambda_1 [1 + (\Re_y - 1)N_x] [1 + (\Re_y - 1)N_z]} \neq 0;$$

$$-\Omega_x I_y \Omega_y + \Omega_y I_x \Omega_x +$$

$$pSl \frac{4(\Re_z - 1)^2 (N_y - N_x) \gamma' \gamma''}{3\lambda_1 [1 + (\Re_z - 1)N_x] [1 + (\Re_z - 1)N_y]} \neq 0$$

Designed as follows:

$$\frac{dI_x \Omega_x}{dt} + \Omega_y I_z \Omega_z - \Omega_z I_y \Omega_y +$$

$$\frac{dI_y \Omega_y}{dt} - \Omega_x I_z \Omega_z + \Omega_z I_x \Omega_x + \frac{dI_z \Omega_z}{dt} +$$

$$\Omega_x I_y \Omega_y = \Omega_y I_x \Omega_x + pSl \cdot$$

$$\left\{ \begin{aligned} & \frac{4(\Re_x - 1)^2 (N_z - N_y) \gamma' \gamma''}{3\lambda_1 [1 + (\Re_x - 1)N_y] [1 + (\Re_x - 1)N_z]} + \\ & \frac{4(\Re_y - 1)^2 (N_x - N_z) \gamma' \gamma''}{3\lambda_1 [1 + (\Re_y - 1)N_x] [1 + (\Re_y - 1)N_z]} + \\ & \frac{4(\Re_z - 1)^2 (N_y - N_x) \gamma' \gamma''}{3\lambda_1 [1 + (\Re_z - 1)N_x] [1 + (\Re_z - 1)N_y]} \end{aligned} \right\}$$

Let us represent a variant of the non-relativistic problem of the rotation of an asymmetric homogeneous rigid body with respect to a fixed point in a dissipative medium with reversible properties by a system of the following real equations:

$$dI_x \omega_x / dt + \omega_y I_z \omega_z - \omega_z I_y \omega_y = M_1 \omega_x,$$

$$\begin{aligned}
 dI_y \omega_y / dt - \omega_x I_z \omega_z + \omega_z I_x \omega_x &= M_2 \omega_y, \\
 dI_z \omega_z / dt + \omega_x I_y \omega_y - \omega_y I_x \omega_x &= M_3 \omega_z
 \end{aligned}
 \tag{27}$$

**Definition:** A real characteristic polynomial for diagnosing the rest of a system of coupled differential equations of a deterministic problem of the rotation of an asymmetric rigid body with respect to a fixed point is developed as follows [11,12]:

$$\begin{aligned}
 &I_x^{-1} M_1 I_y^{-1} M_2 I_z^{-1} M_3 - I_x^{-1} M_1 I_y^{-1} M_2 \alpha - \\
 &- I_x^{-1} M_1 \alpha I_z^{-1} M_3 + I_x^{-1} M_1 \alpha^2 - \alpha I_y^{-1} M_2 I_z^{-1} M_3 + \\
 &+ \alpha I_y^{-1} M_2 \alpha + \alpha^2 I_z^{-1} M_3 - \alpha^3
 \end{aligned}
 \tag{28}$$

**Assertion.** If the developed inequality is observed, theoretical conditions for the stability of equilibrium are developed:

$$\begin{aligned}
 &- I_x^{-1} M_1 I_x^{-1} M_1 I_y^{-1} M_2 - I_x^{-1} M_1 I_x^{-1} M_1 I_z^{-1} M_3 - \\
 &I_x^{-1} M_1 I_y^{-1} M_2 I_z^{-1} M_3 - I_y^{-1} M_2 I_x^{-1} M_1 I_y^{-1} M_2 + \\
 &(-1) I_y^{-1} M_2 I_x^{-1} \cdot M_1 I_z^{-1} M_3 - I_y^{-1} M_2 I_y^{-1} M_2 I_z^{-1} M_3 - \\
 &I_z^{-1} M_3 I_x^{-1} M_1 I_y^{-1} M_2 - I_z^{-1} M_3 I_x^{-1} M_1 I_z^{-1} M_3 - \\
 &I_z^{-1} M_3 I_y^{-1} M_2 I_x^{-1} M_1 + \\
 &+ I_x^{-1} M_1 I_y^{-1} M_2 I_z^{-1} M_3 > 0
 \end{aligned}
 \tag{29}$$

The initial version of the equations of the problem of rotation of an asymmetric heat-conducting solid with respect to a fixed point by inertia under rotary symmetry, represented by the following coordinate equations in the case of Euler, deserves attention:

$$\begin{aligned}
 dI\Omega_1 / dt + \Omega_2 I_3 \Omega_3 &= \Omega_3 I \Omega_2, \\
 dI\Omega_2 / dt - \Omega_1 I_3 \Omega_3 + \Omega_3 I \Omega_1 &= 0, \\
 dI_3 \Omega_3 / dt &= 0.
 \end{aligned}
 \tag{30}$$

Where does the possible variant of the initial equivalent equation of the system (30) come from:

$$-C^{-1} \frac{d^2 I \Omega_2}{dt^2} + C^{-1} \frac{dZ \Omega_1}{dt} + \Omega_2 Z = C I \Omega_2, \quad Z - const
 \tag{31}$$

Hence, when  $C^{-1} \frac{dZ \Omega_1}{dt} = -\Omega_2 Z$  we write the equivalent equation under consideration, as follows:

$$C^{-1} \frac{d^2 I \Omega_2}{dt^2} + C I \Omega_2 = 0, \quad C - \Omega_3 = 0,
 \tag{32}$$

Moreover, such an equation describes a harmonic oscillator with a known solution:

$$C_1 \cos \mathfrak{R}t + C_2 \sin \mathfrak{R}t$$

On the other hand, the initial equivalent equation is as follows:

$$\frac{d^2 I \Omega_2}{dt^2} + C^2 I \Omega_2 = \frac{dZ \Omega_1}{dt} + C \Omega_2 Z \tag{33}$$

The desired solution of which is developed as follows:

$$\begin{aligned} \mathfrak{R} - I_3^{-1} Z &= 0, \\ -\frac{1}{\mathfrak{R}} \int \frac{dZ \Omega_1}{dt} \sin \mathfrak{R}t + C \Omega_2 Z \sin \mathfrak{R}t dt \cdot \cos \mathfrak{R}t + \\ \frac{1}{\mathfrak{R}} \int \cos \mathfrak{R}t \frac{dZ \Omega_1}{dt} + \cos \mathfrak{R}t C \Omega_2 Z dt \cdot \sin \mathfrak{R}t + \\ C_1 \cos \mathfrak{R}t + C_2 \sin \mathfrak{R}t \end{aligned}$$

### 3 Research and results

Studies of gyroscopic sensitive elements and devices, which preceded laboratory tests of the physical model of the spherical inertial platform, revealed possible disturbances in the functioning of gyroblocs in deterministic external magnetic fields of measurable strength.

A variant of the experimental model of a spherical miniature platform in a load-bearing suspension different from the well-known gimbal contains electrically conductive outer and inner spheres, in the space of which an inertial sensing element, a source of a reference magnetic field of a fixed frequency are located, a magnetic field receiver and a device for recording the movement of the inner sphere are located outside the outer sphere [13].

When the inner sphere rotates in the electromagnetic field, electromagnetic fields will be induced in the wall, the interaction of which with the external field will lead to the appearance of counteracting disturbance moments. The possibility of the occurrence of an induced electromagnetic field in the conductive material of a spherical platform is noted by the depth of penetration of the external field into the material of the structure, determined according to [14-16]:

$$\delta = \sqrt{\frac{2}{\sigma \mu \omega}},$$

where  $\sigma$  is the specific electrical conductivity of the material;

$\mu$  - Magnetic permeability of the material;

$\omega$  - frequency of the electromagnetic field.

Experimental modeling consisted in evaluating the functioning of the developed version of the magnetometric system for determining the angular coordinates of the physical model

of a spherical miniature platform, taking into account the inductive method of measuring alternating magnetic fields [18, 19].

The source of the reference magnetic field includes an inductor located stationary in the geometric center of the inner sphere with a radius of 13.7 cm and a wall thickness of 0.1 cm, in the cavity of which a gyroscopic sensing element is installed [20].

The magnetic field receiver includes inductors enclosing the outer sphere with a circumference radius of 14 cm, wall thickness, and the radii of the receiver coils are 17.4 and , the windings of which are made of copper wire with 82 turns with an active resistance of 9.6 and 17 Ohm and an inductance of 8.2 mH and 10 mH, respectively. 0.1 cm 18 cm

## 4 Conclusion

The inductor of the magnetic field source has a winding of copper wire with a circumference radius of 6.3 cm with an active impedance of 9 ohms. Measurements of the parameters of the inductors were carried out using a universal digital voltmeter B7-16 and a high-frequency meter L and C E7-9.

A low-frequency signal generator F3-112/1 with an output voltage of 10 V was connected to the source inductor, the main frequency error of which does not exceed  $\pm(0.01F+0.5)$  Hz in the frequency range of 200 Hz - 20 kHz, where F is the frequency measured on the generator scale. With the help of the optical device of the experiment, consisting of a light source, a reflection plane and a screen, the directive angular deviation of the inner sphere was monitored, with the help of a reference field receiver connected to an electrical measuring device, the output signal was measured with the corresponding angular deviation of the inner sphere relative to the fixed outer one. Statistical processing of the test results showed that the standard error of the arithmetic mean value of the measurement result is 12 angles. min.

By installing a system of orthogonal inductors, it is possible to form a matrix of guide cosines of the autonomous method of spatial orientation of an isotropic rigid body when rotating relative to a fixed pole.

The proposed orientation system of a movable non-deformable body is based on the well-known law of electromagnetic induction, written by differential expression  $e = -\frac{dBS}{dt}$ , the research of which makes it possible to develop the substantive aspects of electromotive force induction and to substantiate the constructive basis of all existence.

## References

1. Antonina Deniskina, Georgy Kravchenko, Elena Ageeva, Nickolay Linkov, BIO Web Conf. **93**, 04022 (2024) <https://doi.org/10.1051/bioconf/20249304022>.
2. Rustem Tazetdinov, Gennady Tibrin, Yuri Deniskin, Svetlana Lapteva, E3S Web of Conf. **376**, 01099 (2023) <https://doi.org/10.1051/e3sconf/202337601099>.
3. G. Deniskina, & Y. Deniskin, & Y. Bityukov, About Biortogonal Wavelets, Created on the Basis of Scheme of Increasing of Lazy Wavelets. (2021) [https://doi.org/10.1007/978-3-030-71119-1\\_18](https://doi.org/10.1007/978-3-030-71119-1_18).
4. Dmitry Golovin, Andrey Smolyaninov, Dmitriy Degtev, Alexander Matusevich, E3S Web Conf. **363**, 04001 (2022) <https://doi.org/10.1051/e3sconf/202236304001>.
5. G.N. Kravchenko, Y.I. Popov, A.I. Kolosov, A.V. Smolyaninov, I.V. Pobeбneva, *Method of Design Calculation for Strength of Structures Made of Metal and Polymer Composite*. In: Radionov, A.A., Gasiyarov, V.R. (eds) Proceedings of the 8th International Conference on Industrial Engineering. ICIE 2022. Lecture Notes in

- Mechanical Engineering. Springer, Cham. (2023). [https://doi.org/10.1007/978-3-031-14125-6\\_54](https://doi.org/10.1007/978-3-031-14125-6_54).
6. Yuri Bityukov, Yuri Deniskin, Galina Deniskina, Irina Pocebneva, E3S Web Conf. **244**, 05004 (2021) <https://doi.org/10.1051/e3sconf/202124405004>.
  7. Physics and Mechanics of New Materials and Their Applications, 2021 - 2022. – Hauppauge: Nova Science Publishers, Inc., 2023. – 1000 p. – (Materials Science and Technologies). – ISBN 979-8—88697542-0. <https://doi.org/10.52305/QLWW2709>. – EDN JAJIMJ.
  8. M. Ghassan Younis, Babylonian Journal of Mathematics, 1-6 (2023) <https://doi.org/10.58496/bjm/2023/001>. – EDN PZNFJU.
  9. T.S. Tricco, Frontiers in Astronomy and Space Sciences **10**, (2023) <https://doi.org/10.3389/fspas.2023.1288219>. – EDN GLOFIE.
  10. N.A. Beissen, G.B. Serikakhmetova, M.E. Abishev, NNC RK Bulletin **2**, 99-103 (2024) <https://doi.org/10.52676/1729-7885-2024-2-99-103>. – EDN TUYQCR.
  11. K. A. Buist, T. M. J. Nijssen, Chemical Engineering Science **265**, 118212 (2023) <https://doi.org/10.1016/j.ces.2022.118212>. – EDN BJNNDJ.
  12. A.M. Abd-Alla, D.M. Salah, Journal of Strain Analysis for Engineering Design **59(1)**, 56-66 (2024) <https://doi.org/10.1177/03093247231188762>. – EDN PIDOTK.
  13. J. Wang, Ya. Wang, Journal of Physics: Conference Series **2512(1)**, 012005 (2023) <https://doi.org/10.1088/1742-6596/2512/1/012005>. – EDN JNRHOX.
  14. A. D. Libera, G. Giacomuzzo, R. Carli [et al.], IFAC-PapersOnLine **56(2)**, 519-524 (2023) <https://doi.org/10.1016/j.ifacol.2023.10.1620>. – EDN JZATMK.
  15. P. M. Atkinson, S. Nlend, Physics Essays **36(2)**, 129-139 (2023) <https://doi.org/10.4006/0836-1398-36.2.129>. – EDN TDZEJY.
  16. F. Magri, Physica D: Nonlinear Phenomena **454**, 133850 (2023) <https://doi.org/10.1016/j.physd.2023.133850>. – EDN MYWYRJ.
  17. C. Arran, P. Bradford, A. Dearling [et al.], Physical Review Letters **131(1)**, 015101 (2023) <https://doi.org/10.1103/physrevlett.131.015101>. – EDN EEJDWZ.
  18. Ch. Wollur, P. Shivananda, S. Harinath, M. Z. Kangda, Innovative Infrastructure Solutions **8(1)**, 27 (2023) <https://doi.org/10.1007/s41062-022-00965-y>. – EDN MKKSTM.
  19. Y. Nishiwaki, M. Takashige, European Journal of Physics **44(5)**, 055007 (2023) <https://doi.org/10.1088/1361-6404/ace294>. – EDN JQXUSR.
  20. S.N. Chukanov, I.S. Chukanov, Aerospace instrumentation **9**, 18-23 (2023) <https://doi.org/10.25791/aviakosmos.9.2023.1360>. – EDN JZNDUH.