

# Algorithm for determining the gas flow rate by the half-division method for pressure measurements in hydrate formation in a well

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**Abstract.** Previously, the algorithm for the inverse problem for determining the dynamics of mass flow rate changes in the systems of natural gas production and transportation by measuring the outlet pressure was generalized to the case of hydrate formation in the main gas pipeline and in the well. In this research the new algorithm based on the method of half division (bisection, dichotomy), which has unconditional convergence, is presented. In the process of solving the problem of formation and deposition of gas hydrates on the pipe walls, the dynamics of gas pressure and temperature changes, as well as the flowing section along the length of the well are calculated. Comparison of the calculation results showed that at constant bottomhole pressure the realization of this algorithm, unlike the previously proposed one, leads to an increase in the duration of the process of complete plugging of the well with hydrates and, accordingly, to an increase in the total amount of extracted gas.

## 1 Introduction

Nowadays, natural gas hydrates are of considerable interest not only as a prospective alternative source of hydrocarbon raw materials, but also as an object of research for their possible application for technological purposes (during transportation and storage of gas in hydrate state, separation of gas mixtures, water desalination, etc.) [1–3]. The problem of hydrate control is becoming more and more urgent, i.e. the tasks of forecasting and prevention of man-made accidents due to hydrate formation in gas production, transportation and processing systems are relevant. Prevention of gas hydrate formation is much easier and cheaper than the elimination of hydrates that have already been formed and accumulated.

In the northern regions due to low climatic temperatures and the presence of a thick layer of permafrost during the operation of wells at gas and gas condensate fields, hydrate formation can lead to a decrease in well productivity, or to the stoppage of gas production with complete blockage of pipes. In the monograph [4] the current situation with methods of combating anthropogenic hydrate formation in wells, gathering systems and field treatment of natural gases is analyzed. The main methods of hydrate control are [5–8]:

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maintenance of gas flow pressure below the phase equilibrium pressure of the system "gas + water – hydrate"; maintenance of temperature above the point of hydrate formation; dehydration of gas flow from moisture; and the most common and effective method of introducing inhibitors into the gas-water flow, leading to a change in the equilibrium conditions of hydrate formation. The application of one or another method or their combination is determined on the basis of a comprehensive technical and economic analysis in each specific case. In paper [9], four exploitation methods which are depressurization, heating, simultaneous depressurization combined with electrical heating, huff and puff, were considered to study the influences of different production methods on natural gas recovery through a vertical well.

When it is very difficult or impossible to study the physical processes under consideration in laboratory and field conditions, it is possible to obtain sufficiently reliable data using computational experiment methods based on their adequate mathematical model and its numerical realization. In this research work, based on the method of half division [10], the algorithm for solving the problem of determining the dynamics of gas mass flow rate change in the well at a specified wellhead pressure under the conditions of formation and deposition of gas hydrates on the inner wall of the pipe is presented. The chosen method of half division (bisection or dichotomy method) for solving nonlinear differential equations with respect to pressure, temperature and pipe cross-section has unconditional convergence, but its disadvantage is slow rate of convergence. The solution of the problem posed by this method is conditionally called the calculation of the "direct" problem. Comparison with the results of solving the inverse problem by the algorithm for determining the parameters of systems of ordinary differential equations by measurements of pressure at the outlet of natural gas production and transportation systems, which was generalized to the case [11, 12], when the pipe cross-section varies along its length and in time due to the formation of hydrate layer. In the paper [11] an example of hydrate formation identification is presented, corresponding to a section of the main gas pipeline in permafrost, and in the paper [12] there is a case of hydrate formation in a well.

## 2 Problem Statement and Algorithm for Computational Implementation

The process of formation and deposition of gas hydrates in a well is described by a quasi-stationary mathematical model presented in [11–16]. The full mathematical formulation of the problem will not be given here, but only its brief description. In this model, the motion of real gas in the well is described in the framework of pipe hydraulics, and the dynamics of hydrate formation is defined in the framework of the generalized Stefan problem, in which the equilibrium temperature of hydrate formation depends on the pressure of the gas flow  $T_h = a \ln(p) + b$ , where  $a$  and  $b$  are the coefficients of approximation of the equilibrium curve of gas hydrate formation. The equations of continuity, motion and energy for the gas flow are reduced to a system of two ordinary differential equations with respect to pressure  $p$  and temperature  $T$ . The non-linearity of these equations is determined by the fact that the density, throttling and compressibility coefficients of the gas are related to its pressure and temperature. The dimensionless equation describing the change of the borehole cross-sectional area  $S$  with time  $t$  is generalized to the case of dependence of the heat transfer coefficient between gas and hydrate layer on the time-varying area of the pipe cross-section. In this case, the coordinate along the borehole axis  $x$  enters this equation as a parameter. In the borehole sections where hydrate layer formation occurs and dimensionless value  $S < 1$ , the heat transfer coefficient is determined from the semiempirical formula for turbulent gas flow in the pipe and the value of rock temperature is replaced by the equilibrium temperature of hydrate formation. For the conjugate problem

of thermal interaction, the temperature distribution in the rocks surrounding the well is determined from the solution of the differential heat conduction equation, taking into account the phase transition "ice – water" in the area of permafrost.

In the article [17] it is noted that during gas production from underwater reservoirs the temperature and pressure in the well bores constantly change when natural gas, including water, flows upward. The hydrate formation and decomposition mainly depends on temperature and pressure. It has been established that a decrease in pressure in pipes leads to less hydrate formation in the well bore cells, respectively, to a decrease in the risk of hydrate plug formation. In our case, the gas temperature in the well bore is influenced by the surrounding permafrost instead of low-temperature seawater. The difference in the mathematical formulation is that in the work [17] the dynamics of the change in the molar amount of hydrate is calculated during its formation/decomposition in the well bore, and in this work the dynamics of the change in the well cross-section is determined. In addition to this difference, in mathematical models, changes in temperature and pressure along the length of the well are determined using various differential equations.

The mode of gas extraction with the given wellhead pressure  $p_{wh}^0$  is analyzed at constant bottomhole pressure. In this case, the algorithm for determining the gas flow rate dynamics  $M(t)$  at the conjugate problem of gas extraction, hydrate formation and heat exchange with minerals can be organized as follows:

1. The initial parameters of the model problem (geometric, physical, initial) are set.
2. The gas mass flow rate  $M$  is determined at each time step with a given accuracy by the half-division method, which provides wellhead pressure  $p_{wh}^0$ , i.e. the equation  $p_{wh}(M) = p_{wh}^0$  is solved. To localize and specify the sought value of  $M$  it is taken into account that  $p_{wh}(M)$  is a decreasing function, it is assumed a priori that  $p_{wh}(M = 0) > p_{wh}^0$ . In this case, to determine the wellhead pressure  $p_{wh}(M)$  depending on the specified  $M$  it is necessary at a fixed area of the borehole cross-section  $S$  by Runge-Kutta method of the 4th order to calculate gas pressure and temperature along the whole wellbore [14–16], including wellhead pressure  $p_{wh}(M)$ .
3. At the obtained value  $M$  from the equations describing the dynamics of the borehole cross-sectional area, a new cross-sectional value  $S$  is determined in the current time layer.
4. The temperature distribution in the rock massif is determined, i.e. the heat conduction problem is solved (in the area of permafrost the Stefan problem is solved).

At each time step, items 2-4 are repeated until  $M > 0$  and  $S > 0$ .

In point 2, the right boundary of the segment is determined  $M^{right}$ , containing the required value  $M$ , as well as the length of this segment  $h_M$ , the left boundary of which is not explicitly selected. The current segment in each iteration is divided in half and the one that contains the desired value  $M$  is selected. The advantage of the method of half division is that it always converges regardless of the function, its derivative, etc. The main point is that the function must be continuous and have a root at the considered section, i.e. the line  $y = p_{wh}(M)$  intersected the horizontal line  $y = p_{wh}^0$ .

### 3 Results of Computational Experiment

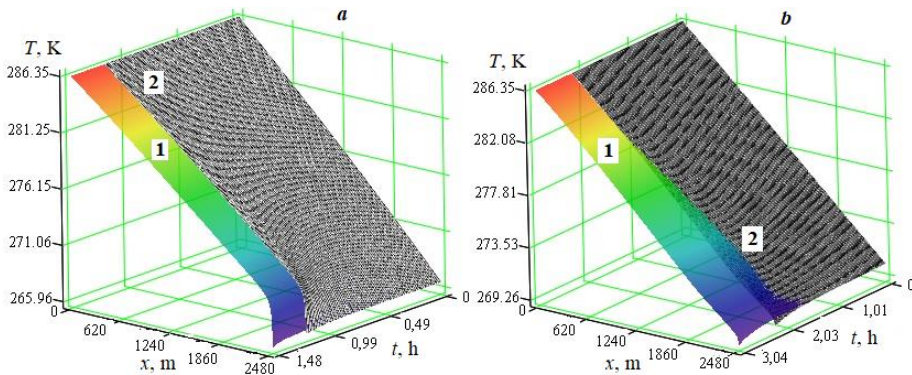
The calculations were performed for the well No. 314-2 of the Otradninskoye gas condensate field in the Republic of Sakha (Yakutia). This field is located on the northern side of the Nyusko-Djerbinskaya trough of the Predpatomic marginal sagging near its intersection with the inner region of the Siberian Platform. This deposit is characterized by low reservoir temperature, which is close to the equilibrium temperature of hydrate formation at sufficiently high reservoir pressure. Under these conditions, there is a danger

of gas hydrate formation directly near the bottomhole and in the wellbore, which can lead either to a decrease in its capacity or to its complete plugging. The initial parameters for the computational experiment were taken from [12]: well length is 2480 m; its diameter is 0.146 m; permafrost capacity is 680 m; initial formation temperature and pressure are  $T_0 = 286.35$  K,  $p_0 = 18.835$  MPa; gas constant  $R = 438.3$  J/(kg·K); critical parameters of gas mixture  $T_c = 195.376$  K,  $p_c = 4.471$  MPa; empirical coefficients  $a = 6.635$  K,  $b = 182.951$  K; temperatures of the rocks at the bottomhole and wellhead level –  $T_{e0} = 286.48$  K,  $T_s = 271.15$  K; geothermal gradients in thawed and frozen zones of rocks  $\Gamma_1 = 0.0074$  K/m,  $\Gamma_2 = 0.0029$  K/m.

In Figures 1–3, surface 1 corresponds to the results of solving the problem using the half division method, and surface 2 represents the results of solving the inverse problem using the algorithm [12]. The pressure at the wellhead varied from 7 MPa to 14 MPa. In preliminary calculations it was obtained that the pressure and temperature of the formation in the bottomhole zone for such a short time of operation of the well until its complete plugging by hydrates practically do not change. Thus, at the bottomhole these parameters can be considered constant. It can be seen from Figure 1–3 and Table 1 that when solving the problem by the half division method, the time of gas withdrawal is 1.2–1.3 times longer than in the reverse problem [12]. The lower the wellhead pressure at constant bottomhole pressure (i.e., the higher the pressure drop), the shorter the well operation time and the smaller the relative deviation between the results of the calculation of the "forward" and inverse problems (see Table 1).

**Table 1.** Well operating time as a function of wellhead pressure  $p_y$ , in minutes.

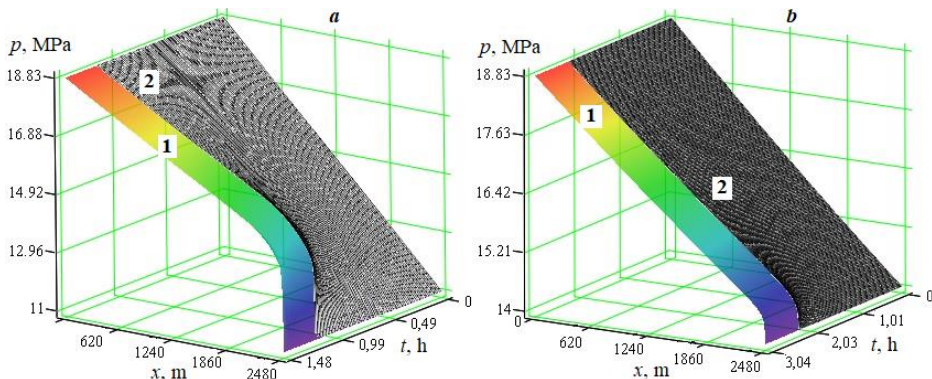
Wellhead pressure $p_y$ , MPa	14	13	12	11	10	9	8	7
"Direct" problem	182	119	94	81	72	64	59	54
Inverse problem based on [12]	138	93	75	65	58	52	48	44
Relative deviation, %	31.9	28.0	25.3	24.6	24.1	23.1	22.9	22.7



**Fig. 1.** Dynamics of gas temperature change in the well at wellhead pressure: *a* – 11 MPa; *b* – 14 MPa; 1 – solution of the problem by the method of half division; 2 – solution of the inverse problem by the algorithm [12].

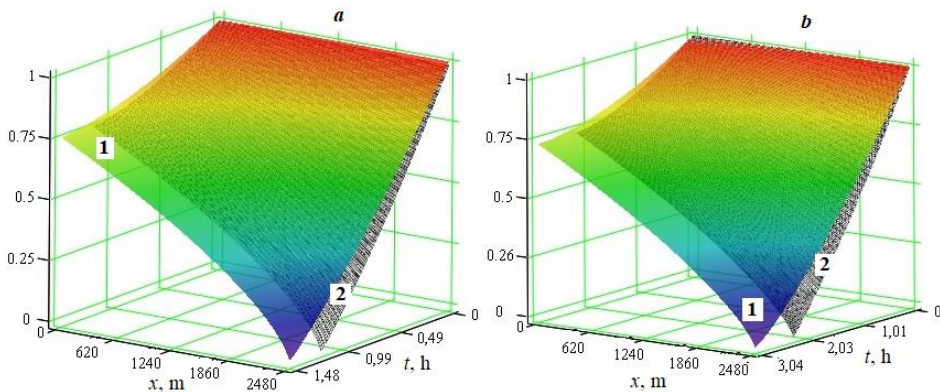
It should be noted that at all operating modes of the well hydrate deposition occurs along its entire borehole and hydrate plug is formed at the wellhead ( $x = 2480$  m), because the temperature of gas along the well is lower than the equilibrium temperature of hydrate formation  $T_h$ . As a result of thermal interaction with the surrounding rocks and throttling,

the temperature of gas at a given wellhead pressure of 11 MPa decreases to 265.96 K and at 14 MPa to 269.26 K (see Figure 1).



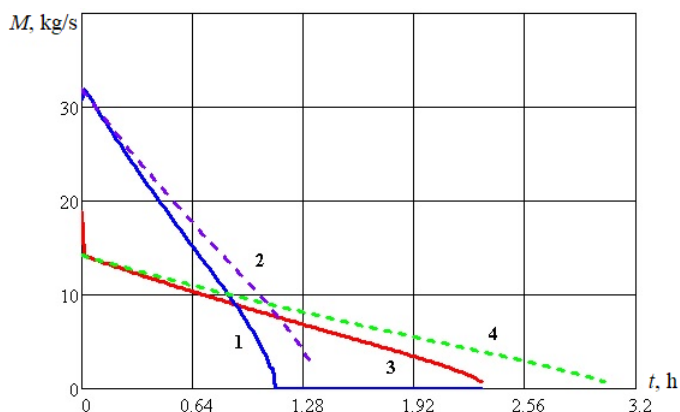
**Fig. 2.** Dynamics of gas pressure change in the well at wellhead pressure: *a* – 11 MPa; *b* – 14 MPa; 1 – solution of the problem by the method of half division; 2 – solution of the inverse problem by the algorithm [12].

The decrease in temperature and cross-sectional area is accompanied by a sharp pressure drop in the wellhead zone (see Figure 2). Hydrate formation occurs throughout the wellbore (see Figure 3), but this process is the most intensive in the upper part of the wellbore, approximately corresponding to the thickness of permafrost. If at the beginning of well start-up the flowing section was free from hydrates (i.e.  $S = 1$ ), at a constant wellhead pressure of 11 MPa, complete plugging of the well with hydrates occurs after 1.35 hours when solved by the half division method (vs. 1.08 hours [12]), and at a pressure of 14 MPa - after 3.03 hours (vs. 2.3 hours [12]). At the same time at the bottom of the well ( $x = 0$  m) 25÷28% of the flowing section will be blocked.



**Fig. 3.** Dynamics of the borehole cross-section change at wellhead pressure: *a* – 11 MPa; *b* – 14 MPa; 1 – solution of the problem by the method of half division; 2 – solution of the inverse problem by the algorithm [12].

In the case of gas withdrawal mode at given constant bottomhole and wellhead pressures with hydrate layer formation, the mass flow rate decreases to zero in a short time. The rate of decrease depends on the pressure drop between bottomhole and wellhead (compare curves 1, 3 and 2, 4 in Figure 4). The greater the differential pressure (the lower the wellhead pressure), the higher the mass flow rate at the moment of well start-up and the faster the well plugging occurs. At a small pressure difference, the gas does not have time to cool down significantly, and the contribution of the throttling effect is small.



**Fig. 4.** Dynamics of mass flow rate change in the well: 1, 2 – at wellhead pressure of 11 MPa; 3, 4 – at wellhead pressure of 14 MPa; 1, 3 – solution of the inverse problem according to the algorithm [12]; 2, 4 – solution of the problem by the method of half division.

It was obtained that at constant downhole pressure, the implementation of the proposed algorithm for solving the problem by the method of half division leads to an increase in the duration of the process of plugging the well with hydrates, and, accordingly, to an increase in the total amount of produced gas (compare the corresponding areas of the figures bounded by the curves and coordinate axes in Figure 4).

## 4 Conclusion

The comparison of the results of calculation of the main parameters in the well during hydrate formation (changes in gas pressure and temperature, as well as the cross-section along the length of the well and in time), obtained by the method of half division, with the numerical results of solving the inverse problem [12] is presented. The analysis of the hydrate layer growth dynamics in the well shows that the formed hydrate plug is located in the wellhead zone. It is formed within a few minutes after the start of the well and this time will be longer in the case of solving the problem by the method of half division. In this case, the total amount of produced gas increases, numerically equal to the area of the figure under the curve of mass flow rate dependence on time. The developed algorithm for solving the problem can provide an opportunity to predict with a sufficient degree of accuracy the dynamics of gas mass flow rate (or flow rate) in the well and the total accumulated production before the well is plugged with hydrates.

## Acknowledgements

The research was performed as part of the state order of the Ministry of Science and Higher Education of the Russian Federation No. 122011100157-5.

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