

Time Series Prediction on Population Dynamics

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Abstract. Predicting the time series is a challenging topic mainly on the era of big data. In this research, data taken from population dynamics of one dimension of logistic map with various parameters that leading the system into chaos. Various machine learning methods is employed for predicting the time series data such as Long Short-Term Memory (LSTM), Gated Recurrent Unit (GRU) and 1 Dimension of Convolution Neural Network (1D CNN). Several data sizes were considered: 1000, 10000, 50000, 100000 and 1 million points of time series data. As evaluation metric, Root Means Square Error (RMSE) is used to assess the accuracy of each method. The result indicating that the LSTM has the smallest RMSE value among all the three machine learning methods.

1 Introduction

Nowadays, predicting time series data is a popular data driven decision for simply helping human better life. Long-Short Term Memory (LSTM) is a type of neural network specially for sequential data. LSTM has three gates namely reset gates, update gate and output gate. Various studies have been done by utilizing LSTM for predicting the traffic flow [1]–[4], blood pressure [5], depression [6], air quality [7], battery useful life [8] and stock price [9].

The alternative for LSTM is called Gated Recurrent Unit (GRU) with simpler architecture without the output gate. GRU has been employed for predicting electricity generation's planning and operation [10], Jiuxianping landslide [11], traffic flow [12], healthcare [13], wind speed [14], beating heart surgery [15] and stock price [16].

Convolution Neural Network (CNN) is a type of neural network mainly used for image processing and predicting the sequential data. CNN has been utilized for predicting human interaction [17], long-term terror [18], healthcare [19], stock price [20], flood [21], and finance [22].

Predicting the population dynamics has been studied in various researches [23]–[30], but rarely using the sequential architecture of neural network such as LSTM, GRU and CNN. In this study the population dynamics model will be predicted by all of those neural network methods.

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2 Materials and methods

Without competition, population of certain species will follow discrete exponential growth $P_{n+1} = \alpha P_n$, where P_n is the number of species population at time n and α is the birth rate of the population. This population model represents the abundant of food resources and living of spaces for all the species population. It is a good approach on early stage of population development. But in a long term, when the population grows exponentially, at certain time, there will be a competition amongst the population because the food and space available is getting scarce compare to the increase number of population. For this reason, for a long run, the model can be written as $P_{n+1} = \alpha P_n - \beta P_n P_n$. Where β is the rate of competition among the population itself for the limitation of food and space availability.

Then the previous population dynamics will be transformed by letting $x_n = \frac{\beta}{\alpha} P_n$ to get a new formula as written by:

$$x_{n+1} = \alpha x_n (1 - x_n) \quad (1)$$

Where $\{x_n\}$ can be represented as the normalization of the number of certain species population at time n . This formula famously known as logistic map which have properties:

1. The sequence of $\{x_n\}$ is bounded on $0 \leq x_n \leq 1$ in $0 \leq \alpha \leq 4$.
2. There are two critical points $x^* = 0$ and $x^* = 1 - \frac{1}{\alpha}$.
3. The critical point $x^* = 0$ will be stable in $0 \leq \alpha \leq 1$.
4. The critical point $x^* = 1 - \frac{1}{\alpha}$ will be stable in $1 \leq \alpha \leq 3$.
5. There will be 2-period solution for $3 < \alpha \leq 1 + \sqrt{6}$.
6. There will be 4-period solution for $1 + \sqrt{6} < \alpha \leq 3.54409$.
7. Period doubling solution for $3.54409 < \alpha \leq 3.56995$.
8. Most likely chaos for $3.56995 < \alpha \leq 4$ with $3k$ -period for some windows as k is natural numbers.

From those of all properties, there is an interesting part when the solution is bounded but there is unstable critical point in the range of $3.56995 < \alpha \leq 4$. It means the sequence of $\{x_n\}$ will be in the range of $[0,1]$ without any chance to converge to certain critical point. All of the properties above can be seen in the limit set picture as presented on figure 1.

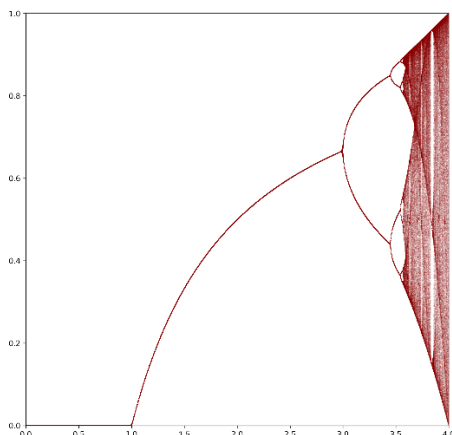


Fig. 1. Limit set of $x_{n+1} = \alpha x_n (1 - x_n)$ with all of its properties

The time series data of the population dynamics will be taken by running the logistic map with $\mu = 4 - \varepsilon$, where ε is a very small positive number near zero. At first the sequence will

be run for 100000 times to make it stable on its range, then the amount of 1000, 100000, and 1000000 data taken for further time series prediction analysis. To be noticed, because of the system of (1) is sensitive to initial condition, the timeseries data will be different for every run of computational analysis.

This data that extracted from the logistic map simulation then divided into three groups namely training, validation and testing which contains 80%, 10% and 10% of the data consecutively. After that, the data taken will be analyzed by three methods: LSTM, GRU and 1 Dimension CNN.

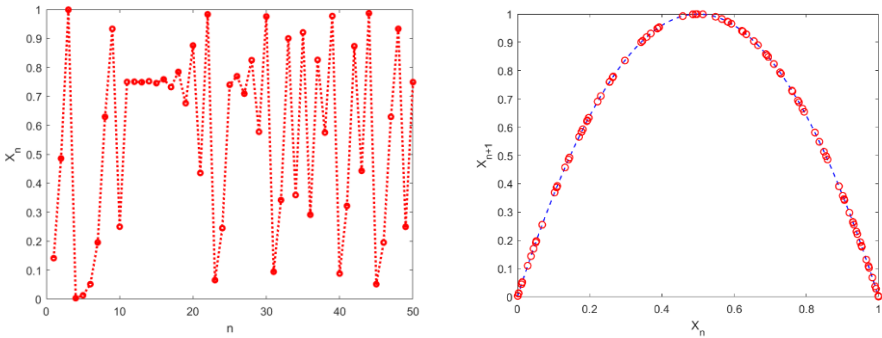


Fig. 2. On the left is the time series data sample taken after the 100000 times running. Then on right is the density of data taken on the bounded range $0 \leq x_n \leq 1$.

LSTM or Long-Short Term Memory is a typical of neural networks with memory unit for processing the sequential data. This type of neural network is the advancement of previous architecture called RNN or Recurrent Neural Network which contains no memory unit. It means LSTM will have a good memory collection compared to RNN. There are three gates on LSTM namely reset gates, update gate and output gate. The weakness of LSTM is the data will be processed on sequence and cannot compute parallel data processing when passing all those gates.

Meanwhile GRU or Gated Recurrent Unit has almost the same architecture with LSTM but without the output gate. To transfer information from previous computation result, the hidden state is utilized by GRU. This simpler architecture makes the time of computation will be faster but sacrificing the accuracy.

At last, CNN or Convolution Neural Network mainly used on image processing by examining visual representations. Image itself contains pixels and one pixel to its neighbors has correlation and needs to remember the patterns. For those reasons, one dimension of CNN can be utilized for processing sequential data.

In this research, the data processing employs Python 3.9.7 with Keras and Tensorflow modules. The data extracted from the logistic map will be trained with the three methods above then each of the model will be utilized to predict the testing data.

3 Results and discussion

Time series data taken from equation (1) by running it until $n = 1000000$. Then data taken into several categories namely 1000, 10000, 50000, 100000 and 1000000 first data. Each category will be divided into three groups: train data consists of 80% of the first data of each category, validation data consists of 10% remaining data after the first data, then testing data contains the last 10% of data. The data that will be predicted is the testing data which contains the last 10% of each category.

The initial condition is $x_1 = \sqrt{2} - 1$ with $\varepsilon = 0$, which means the data will be spread on $0 < x_n < 1$ for every n is natural number from 1 to 1000000. In the neural network architecture, there are two hidden layers employed consists of 64 and 8 neurons consecutively with Relu activation function. The result of various data prediction by using LSTM can be seen on figure 3.

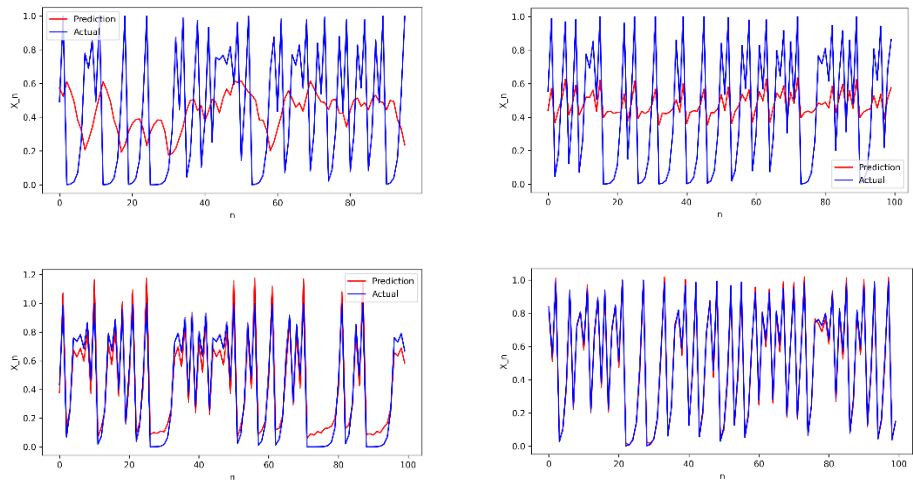


Fig. 3. All the figures are the prediction results of the last 10% of data in each category. For convenience, only the last 100 data for each category is plotted. The upper left is the data prediction from 1000 data, upper right is the data prediction from 10000 data, the bottom left is from 50000 data and the bottom right is from 100000 data. The blue line is the plot of real data and the red line is the prediction.

In figure 3, the least amount data provided the harder LSTM to make a good prediction. Starting from 50000 data the prediction already follows the main data so that the RMSE is getting smaller. All of the RMSE value can be found in the table 1.

Table 1. The numbers of Data with Root Means Square Errors (RMSE).

Data	RMSE
1000	0.394903088989
10000	0.286226350609
50000	0.075545737921
100000	0.022378630071
1000000	0.001982162095

Knowing the prediction is better when the number is data big enough, then the 1000000 is predicted using all the three methods: LSTM, GRU and 1D CNN. All the three prediction results can be seen on figure 4.

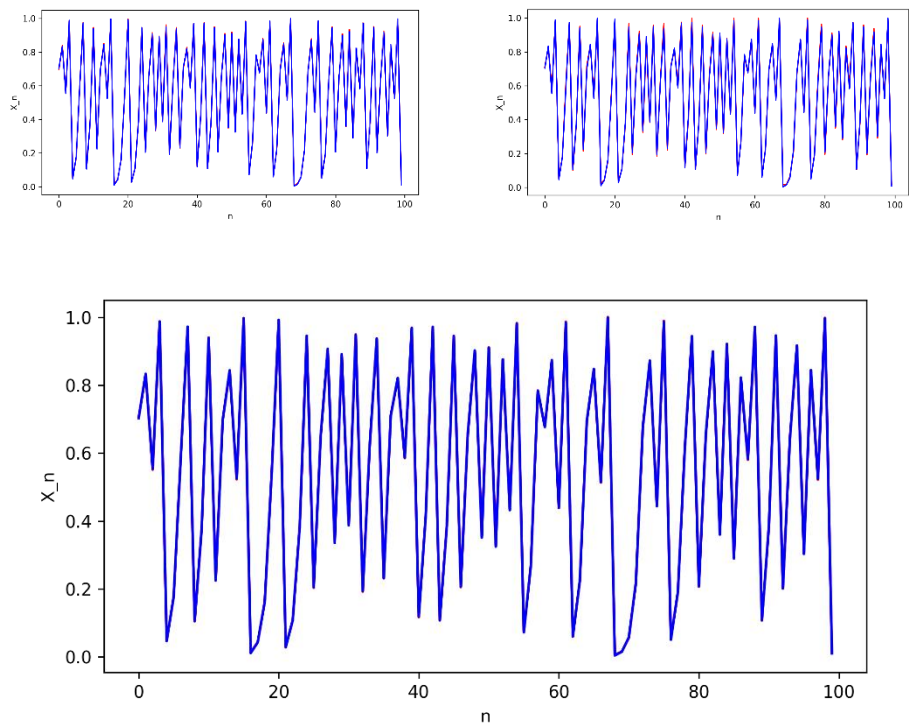


Fig. 4. The prediction results of GRU (upper left), 1D CNN (upper right) and LSTM (bottom). For the inconvenient, only the last 100 data of its prediction ranges is plotted.

All the three methods have good prediction where almost all the red lines covered by the blue lines. Actually, all the plotted data on figure 4 consist of red line as the predicted result and the blue line as the real data. In the result of GRU and 1D CNN prediction still appear the red line on some occasion, but in the LSTM result the red line almost perfectly covered by the blue line. The RMSE can be seen on table 2 where the LSTM has the least RMSE compared all of the three methods. Then GRU has smaller RMSE compares than 1D CNN.

Table 2. Root Means Square Errors (RMSE) of LSTM, GRU and CNN 1D for 1000000 data.

LSTM	GRU	CNN 1D
0.001982162095	0.0047150845189	0.0107348571565

4 Conclusion

In small size data, all the three methods have struggled to make a good prediction. In case the data is getting bigger, all the three methods make a better prediction through the smaller RMSE. Meanwhile when the data is big enough, from all the three methods, LSTM outperforms the rest. Then followed by GRU and the last is 1 D CNN as shown by the RMSE value. The interesting question here is how many is called a big for all the three methods will predict appropriately? This question can be answered by various experiment with different data sizes.

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