Investigation of the influence of a closed loop of longitudinal forces on the dynamics of bending and torsional vibrations of a package working body

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Abstract. It is shown that at bending and torsional fluctuation component design, within which there is closed sidebar internal longitudinal effort, exists its influence upon oscillatory process, defined by correlation of longitudinal acerbity compressed and sprained element and is shown the way of the analytical account of this influence. The decision of the problem is received about изгибных and torsional fluctuation packet worker of the organ with provision for specified phenomena, which has shown the need for the account for this influence under exact calculation.

1 Introduction

In technology, working bodies are used, consisting of disk elements assembled in a package and compressed by a shaft with a force sufficient for its operation as a monolithic body.

The working package body consists of a package of disk elements typed on the shaft and compressed by the latter. The values of the bending and torsional, as well as the longitudinal stiffness of the package of such a working body, can be determined as functions of the magnitude of the compressive force of the package [1].

Its notable feature is that the forces compressing the package of disk elements and tensile forces of the shaft are equal in absolute value and form a closed circuit inside the package working body that performs bending vibrations. Thus, they are internal force factors.

The question of the influence of external longitudinal force factors on the process of bending vibrations is well-studied and covered in the technical literature. The influence of external force factors on torsional vibrations and force factors that make up an internal closed power circuit on bending and torsional vibrations has yet to be studied and there is no information about this in the special literature.

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2 Materials and methods

Let us perform a theoretical analysis of changes in the values of deformations of the package working body and the corresponding changes in the values of deformations of its elements that make up a closed power circuit during oscillations and changes in the values of their potential energies occurring in this case [2].

From the point of view of the resistance of materials under bending deformations, the length of the axial line of the package working body remains unchanged. However, with a more accurate analysis of the deformation pattern, it turns out that the length of the axial line changes during bending and torsional deformations.

This is all the more significant since flexural vibrations that are perceptible and of practical importance occur with amplitudes that are, although small in relation to the length of the package working body, but finite values.

The finiteness of the magnitudes of the amplitudes causes a change in the length of the axial line of the package working body, with its deformations caused by bending and torsional vibrations. Thus, with bending and torsional deformations of the stack working bodies caused by their vibrations, additional longitudinal deformations and longitudinal displacements of their points will take place.

Since the moving points are under the action of compressive or tensile forces, mechanical work will be performed, which will cause a change in the potential energies of the elements of the package working body.

In this case, the magnitude of additional deformation and its sign is the same for the shaft and the package of disk elements. But the work performed in them at the same time, and therefore the changes in the potential energies of the elements will have opposite signs and may not be equal in magnitude.

The opposite of the signs of work and changes in the potential energies of the elements follow from the fact that the shaft is in a state of tension, the package of disk elements is in the form of compression, and the possible inequality in the values of the work is due to the potential inequality of the longitudinal stiffness of the shaft and the package of disk elements. It follows that, for any additional bending deformation of the package working body, the total change in its potential energy is equal to the difference between the changes in the potential energies of the shaft and the package of disk elements and has the sign of the change in the energy of the element of the package working body, which has a greater longitudinal rigidity. It is clear that with the same longitudinal rigidity of the shaft and the package of disk elements, the total change in the potential energy of the package working body is equal to zero.

Note that many small volumes with internal compressive or tensile stresses in a real solid have approximately the same total stiffness.

Therefore, in such a body, the total change in potential energy with additional deformation is zero, and, accordingly, internal stresses do not affect the frequencies of oscillatory processes.

3 Results and discussion

Based on the analysis, an expression was obtained for the value of the total change in the potential energy of deformation during bending and torsional vibrations of the package working body when determining the longitudinal stiffness of the package using the phenomenological method [1,10] in the following form:

$$U_p = \frac{1}{2}\int_0^L (E_b F_b \varepsilon_b^2) dz - \frac{1}{2}\int_0^L \left[\left(1 - e^{-\frac{2A_n N}{N_0}}\right) \frac{(l_p + l_n) E_p F_p E_n F_n}{l_p E_n F_n - l_n E_p F_p} \varepsilon_n^2\right] dz \quad (1)$$

Here:

F_b, F_p, F_n - are the cross-sectional areas of the shaft, working and intermediate disks;

E_b, E_p, E_n - are the moduli of elasticity of the materials of the shaft, working and intermediate disks;

A_n – function of unaccounted for factors in the phenomenological determination of the longitudinal stiffness of the package;

N, N_0 – current and fixed values of the packet compression force;

I_p, I_n – thicknesses of working and intermediate disks;

$\varepsilon_b = \varepsilon_n = \varepsilon$ - the value of additional longitudinal deformation during vibrations of the shaft and package.

Here we recall that the value of the additional deformation during vibrations ε is small, but finite, therefore, the terms containing ε are not excluded from consideration, as is usually done.

To solve the problem for bending vibrations, we express the value of additional deformation as follows

$$\varepsilon_{ben} \approx \frac{1}{2}\left(\frac{\partial u}{\partial z}\right)^2 \tag{2}$$

where: u - displacements in the vertical plane of bending vibrations.

Now, using the expressions obtained in [1,3] for the functions of the longitudinal and bending stiffness of the package, we will compose the equations for the potential and kinetic energies of the package working body during bending vibrations.

Applying the Hamilton principle and the Lagrange equations of the second kind, we obtain the equation of free bending vibrations:

$$E_b J_b \frac{\partial^4 u}{\partial z^4}\left[\frac{(l_p + l_n)E_p J_p E_n J_n}{l_p E_n J_n + l_n E_p J_p}\left(1 - e^{-\frac{2A_u N}{N_0}}\right)\right]\frac{\partial^4 u}{\partial z^4} +$$

$$+\left[\frac{E_b F_b}{4} - \frac{1}{4}\left(\frac{(l_p + l_n)E_p F_p E_n F_n}{l_p E_n J_n + l_n E_p J_p}\right)\left(1 - e^{-\frac{2A_u N}{N_0}}\right)\right]\frac{\partial^4 u}{\partial z^4} +$$

$$+\left(\frac{l_p \rho_p F_p}{l_p + l_n} + \frac{l_n \rho_n F_n}{l_p + l_n}\right)\frac{\partial^2 u}{\partial t^2} + \rho_b F_b \frac{\partial^2 u}{\partial t^2} = 0 \tag{3}$$

Here:

A_u – function of unaccounted for factors in the phenomenological determination of the flexural rigidity of the package;

J_b, J_p, J_n - axial moments of inertia of the cross sections of the shaft, working and gasket disks;

F_p, F_n - function of the cross-sectional areas of the working and intermediate disks;

ρ_b, ρ_p, ρ_n - are the density of the materials of the shaft, working and middle disks.

We take into account the boundary conditions in the experimental stand:

For $z = 0: u = 0$ and for $z = L: u = 0$ and $\frac{\partial^2 u}{\partial z^2} = 0$;

Then the normal function of the problem will take the following form:

$$U = \sin\frac{i\pi z}{L}(A_i \cos\omega_i t + B_i \sin\omega_i t)$$

From this it can be seen that the characteristic equation for determining the frequency of natural vibrations will be written in the form:

$$\sin\frac{i\pi z}{L} = 0$$

An analysis of the results of the calculated determination of the frequencies of free bending vibrations according to (3) of a common package working body - a gin saw cylinder, taking into account the influence of a closed loop of internal longitudinal forces, showed that

the correction in this case can be 0.06% and 0.1%, respectively, at extreme values of the change package compression force.

We turn to the consideration of torsional vibrations.

Let us express the value of the additional longitudinal deformation of the package working body through its angular deformation during torsional vibrations as follows:

$$\varepsilon = \frac{1}{2}\left(\frac{\partial\eta}{\partial z}\right)^2 = \frac{R^2}{2L^2}\left(\frac{\partial\theta}{\partial z}\right)^2, \tag{4}$$

where: $\frac{\partial\eta}{\partial z}$ - the angle of rotation of the rectilinear generatrix during torsional vibrations;

R and L - are the radius and length of the packet.

Using the expressions obtained in [1,4,5] for the functions of the longitudinal and torsional stiffness of the package, we will compose the equations of potential and kinetic energies during torsional vibrations of the package working body and apply the Hamilton principle and the Lagrange equations of the second kind to them, we obtain the equations of free torsional vibrations in the following form:

$$\frac{\partial^2}{\partial z^2}\left[\left(\frac{R^4E_bF_b}{4L^4} - \frac{R^4(l_p+l_n)E_pF_pE_nF_n}{4L^4(l_pE_nJ_{kn}+l_nE_pJ_{kp})}\right)\left(1-e^{\frac{2A_nN}{N_0}}\right)\frac{\partial^2}{\partial z}\right] + \frac{\partial}{\partial z}\left(G_bJ_{kb}\frac{\partial\theta}{\partial z}\right) -$$
$$-\rho_bJ_{pb}\frac{\partial^2\theta}{\partial z\,\partial t} + \frac{\partial}{\partial z}\left[\frac{(l_p+l_n)G_pJ_{kp}G_nJ_{kn}}{l_pG_nJ_{kn}+l_nG_pJ_{kp}}\left(1-e^{\frac{2A_nN}{N_0}}\right)\frac{\partial\theta}{\partial z}\right] -$$
$$-\left(\frac{l_p\rho_pJ_{pp}}{l_p+l_n} + \frac{l_n\rho_nJ_{pn}}{l_p+l_n}\right)\frac{\partial^2\theta}{\partial t^2} = 0 \tag{5}$$

Here: A_k, A_n - are functions of unaccounted-for factors in the phenomenological determination of torsional and flexural stiffnesses;

J_{kb}, J_{kp}, J_{kn} - are the functions of the moments of inertia during torsion of the shaft, working and intermediate disks;

J_{pb}, J_{pp}, J_{pn} - are the polar moments of inertia of the cross sections of the post, working and middle disks;

G_b, G_p, G_n – moduli of elasticity under shear of materials of the shaft, working and intermediate disks.

Under the actual boundary conditions corresponding to the termination of one end with the other end freely supported, we have [6,7]:

at $z = 0$: $\theta = 0$ and at $z = L$: $\theta = 0$ and $\partial\theta/\partial z = 0$;

The accepted conditions correspond to a normal function of the form

$$\theta = \left(\cos\frac{i\pi z}{L}\mathrm{Sh}\frac{i\pi z}{L} - \sin\frac{i\pi z}{L}\mathrm{Ch}\frac{i\pi z}{L}\right)(A_i\cos\omega_i t + B_i\sin\omega_i t)$$

Table 1. The results of calculations of natural frequencies of bending vibrations of the genie saw cylinder, taking into account the influence of a closed loop of internal longitudinal forces.

№	Packet compression force N, kN	$\omega_i(c^{-1})$ - without taking into account the influence of a closed loop of internal longitudinal forces	$\omega_i(c^{-1})$ - taking into account the influence of a closed loop of internal longitudinal forces	The amount of change in the oscillation frequency	
				Absolute $\Delta\omega_i, c^{-1}$	Relative $\delta\omega_i$, %
1	0.0	118.0723	117.7984	0.2739	0.23250
2	5.0	151.0490	150.7242	0.3248	0.21549
3	10.0	161.9897	161.6517	0.3379	0.20905
4	15.0	169.6304	169.2845	0.3459	0.20432

5	25.0	175.6392	175.2878	0.3514	0.20049
6	35.0	180.6422	180.2867	0.3555	0.19721
7	50.0	184.9525	184.5938	0.3587	0.19433
8	75.0	188.7518	188.3906	0.3612	0.19175
9	100.0	192.1564	191.7932	0.3633	0.18940
10	150.0	195.2455	194.8806	0.3649	0.18725

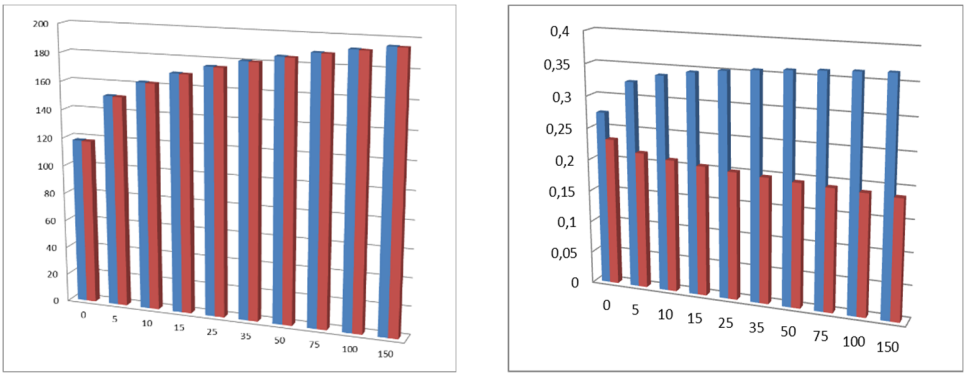
Table 2. The results of calculations of natural frequencies of torsional vibrations of the genie saw cylinder, taking into account the influence of a closed loop of internal longitudinal forces.

№	**Packet compression force N, kN**	**$\omega_i(c^{-1})$ - without taking into account the influence of a closed loop of internal longitudinal forces**	**$\omega_i(c^{-1})$ - taking into account the influence of a closed loop of internal longitudinal forces**	**Correction value**	
				Absolute $\Delta\omega_i$, c^{-1}	**Relative $\delta\omega_i$, %**
1	0.0	1112.7970	1116.3490	3.5520	0.31920
2	5.0	1129.4090	1132.6540	3.2450	0.28732
3	10.0	1163.5720	1166.6280	3.0560	0.26264
4	15.0	1203.2060	1206.0950	2.8890	0.24011
5	25.0	1245.6100	1248.3490	2.7390	0.21989
6	35.0	1289.5130	1292.1150	2.6020	0.20178
7	50.0	1334.1980	1336.6770	2.4790	0.18580
8	75.0	1379.2250	1381.5920	2.3670	0.17162
9	100.0	1424.3130	1426.5770	2.2640	0.15895
10	150.0	1469.2750	1471.4440	2.1690	0.14762

Now we can write an expression for the characteristic equation of the problem in the form:

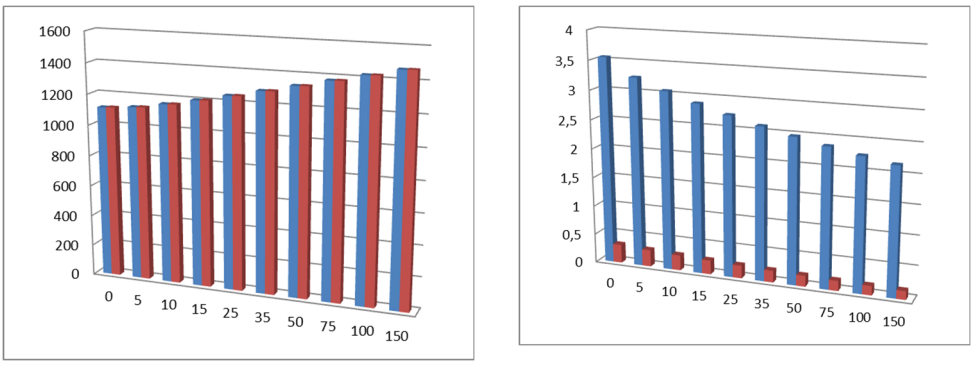
$$cos\frac{i\pi z}{L}Sh\frac{i\pi z}{L} - sin\frac{i\pi z}{L}Ch\frac{i\pi z}{L} = 0$$

Figure 1 shows an intense increase in the natural frequencies of the bending vibrations of the fiber separator saw cylinder occurs with an increase in the compressive force of the package up to 25 kN. A further increase in the compressive force decreases, this is due to the achievement of maximum contact between the end surfaces of the discs and between the saw pads [8,9].



a) b)

Fig. 1. Results of calculations of natural frequencies of bending vibrations. a) difference with and without taking into account the influence of a closed loop of internal longitudinal forces; b) difference in magnitude of change in absolute and relative oscillation frequencies.



a) b)

Fig. 2. Results of calculations of natural frequencies of torsional vibrations. a) difference with and without taking into account the influence of a closed loop of internal longitudinal forces; b) magnitude of absolute and relative corrections.

Figure 2 shows the change in natural frequencies occurs almost linearly and does not depend on the accuracy of the contact surfaces of the discs and between the saw pads.

4 Conclusion

The results of the calculated determination of the frequencies of free bending vibrations (Table 1) and free torsional vibrations (Table 2) of a common package working body - a gin saw cylinder, taking into account the influence of a closed loop of internal longitudinal forces, show:

1. With an increase in the compression force of the package from zero to a maximum value of 150.0 *kN*, the relative correction value decreases from 0.23% and 0.32% to 0.19% and 0.15%, respectively.
2. At a constant value of the compression force of the package, the effect of a closed loop of internal longitudinal forces on the processes of bending and torsional vibrations depends only on the ratio of the longitudinal stiffnesses of the compressed and stretched elements of the package.

The results of the study indicate that this should be taken into account when developing optimal design systems for package structures.

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