Application of a modified mathematical model "consumer – resource" to justify the periods of treatment of potato late blight with fungicides

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Abstract. Potato late blight is a harmful disease. Treatment of fields with fungicides is the main method of combating this phytopathology. The work presents a modified mathematical model "consumer - resource" - this is a system of partial differential equations, which contains a step function with the help of which fungicidal treatment can be successfully modeled. The system was solved numerically. Based on the modeling results, it is possible to successfully simulate the development of leaf damage during the growing season without and with fungicide treatment. Thus, knowing the duration of action of the drugs and assuming how intensively late blight will develop, it is possible to model different combinations of fungicides and spraying times. This circumstance can be successfully implemented into decision support systems for agricultural enterprises.

1 Introduction

Potatoes are one of the most important agricultural crops in many countries. Its cultivation is hampered by many pests and diseases, including late blight. The characteristic features of this phytopathogen are rapid development and spread to neighboring plants, great harmfulness, significant dependence on weather conditions and the ability to survive for a long time in an unfavorable environment. To combat it, different strategies are used:

- Territorial remoteness of fields
- Careful selection and processing of seed material
- Treatment of fields with fungicides.

This work is devoted to modeling the latter strategy. Note that modern models take into account weather data: daily or hourly temperature fluctuations, relative humidity, and sometimes precipitation [2-5]. There are models that additionally use biological information: a potato variety, a popular oomycete strain in the region and its characteristics [6, 7]. Currently, tools are gaining popularity - decision support systems (DSS), which allow recommending fungicidal treatments [8-11]. Most of these systems assess the favorableness of weather conditions for the development of the disease and, depending on the level of danger, make recommendations for treatment. Such DSS may use information from earlier applications of the fungicide. The vulnerability of such models is that the actual state of the

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plants is not considered - the degree of damage to the tops and infection points on the field are not taken into account. The purpose of this work is to present a model of disease development over time with external regulation and to obtain recommendations for the optimal use of fungicidal treatments in terms of quantity and duration.

2 Mathematical model of the distribution of "consumer" on a non-renewable "resource" with the possibility of external regulation of the damage caused

The work [1] presents a model for the development of a "consumer" (late blight) on a fixed "resource" (potato tops). The modified model, which is presented in this work for modeling fungicide treatment, has the form:

$$\begin{gathered}\frac{\partial u}{\partial t} = \beta u\left(F_{drug}(t)\frac{v}{a+v} - \frac{u}{k_1}\right) + D\frac{\partial^2 u}{\partial x^2} \\ \frac{\partial v}{\partial t} = -c_2 u\frac{v}{a+v}\end{gathered} \tag{1}$$

where $u(x,t)$ is consumer linear density; $v(x,t)$ is linear resource density; β is specific population growth rate; a is half-saturation parameter. It should be noted that in this case the equation $v/(a+v)$ characterizes the share of the resource consumed for consumer reproduction; $D\frac{\partial^2 u}{\partial x^2}$ - random process of movement of a pathogen across an area (diffusion displacement); D - coefficient characterizing consumer mobility; $\beta u\left(\frac{v}{a+v} - \frac{u}{k_1}\right)$ - term describing the logistic population model, which takes into account the capacity of the environment (k_1); $\beta u\left(F_{drug}(t)\frac{v}{a+v} - \frac{u}{k_1}\right)$ - modified term that takes into account fungicide treatment ($F_{drug}(t)$).

From the moment of application, the fungicide inhibits the development of the pathogen for a certain period of time. We will assume that

$$F_{drug}(t) = \begin{cases}\alpha, & if\ t \in [t_1, t_2] \\ 1, & if\ t \notin [t_1, t_2]\end{cases}, \tag{2}$$

where t_1 - start of processing, t_2 - expected expiration date of the fungicide, α - degree of pathogen suppression ($\alpha = 0$ - pathogen reproduction is completely suppressed, $\alpha = 1$ - reproduction is not suppressed).

Border conditions

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \left.\frac{\partial u}{\partial x}\right|_{x=1} = 0$$

describe the situation, the pathological process does not go beyond the territory (the consumer does not cross the border).

Let us set the initial conditions:

$$\begin{cases} v(x,0) = 1\ if\ 0 \le x \le l \\ u(x,0) = u_0\ if\ 0 < x \le \tilde{x},\ \ \tilde{x} < 0.01l \end{cases}$$

At the initial moment of time, throughout the entire area, the density of the "resource" has a given value, which we will consider equal to 100%. At the initial moment of time, the consumer occupies a linear habitat of less than 1% of the territory under consideration. The stability of the system solution in the absence of a pathogen at $\alpha = 1$ is considered in [1].

3 Numerical experiment

Strategies for introducing fungicide to control potato late blight are considered. The introduced treatment rules depend on the day the treatment begins, the number of treatments

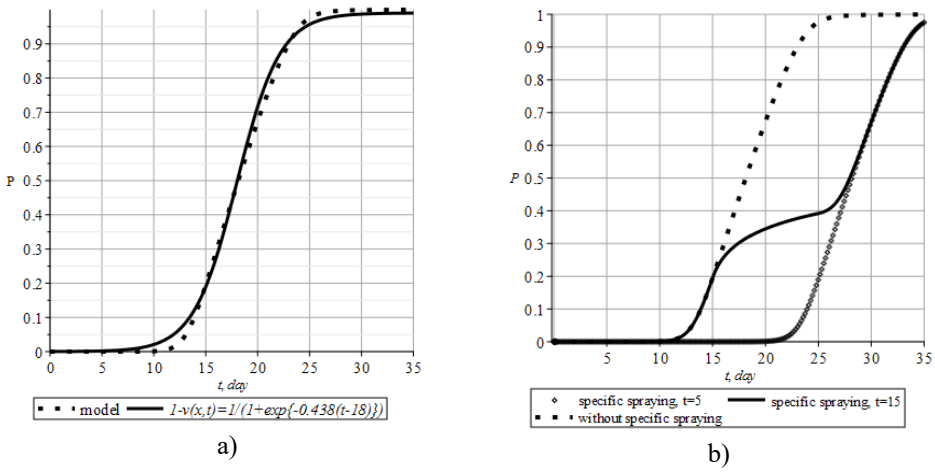
and their duration (Table 1). The values were selected randomly from the specified intervals. Number of tests N = 1000. The numerical experiment was carried out using standard functions of the Maple package for mathematical calculations.

System parameter values: $\alpha = 0$; $\beta = 1.4$; $a = 0.05$; $c_2 = 0.115$; $k_1 = 1$; $D = 0.0001$.

Table 1. Time restrictions imposed on treatments in the experiment.

number of treatments	fungicide application intervals	maximum period of action of the fungicide (in days)
1	[1,30]	10
2	[1,14], [15,25]	7, 10
3	[1, 12], [13,24], [25,30]	7, 10, 10

Treatment 1: The loss rate on the 30th day from the onset of the disease when applying the strategy was not lower than 80%. An optimal treatment lasting 10 days was obtained, starting on days 6–8 from the moment of infection.



a) b)

Fig. 1. Results of modeling the proportion of tops affected ($P = 1 - v(x,t)$): a) without processing; b) single processing $\alpha = 0$ if $t \in [5, 14]$(circle line) or $t \in [15, 24]$ (solid line).

Fig. 1 a): the simulation result is compared with the approximating function of experimental data $1 - v(x,t) = 1/(1 + exp\{-0.438(t - 18)\})$ [1]. Fig. 1 b): the characteristic bend in the curve of the proportion of tops affected by late blight when treated with fungicides is clearly visible, as noted in [13].

Treatment 2: Losses at the end of the period under review are 7–10%. The best results are when you start the first treatment on days 2–5, and the second on 15–17. The duration of treatments is 6–7 and 9–10 days, respectively. If the first treatment begins later, the optimal start of the second is shifted to days 17–19. In this case, the defeat will be 10–17%.

Treatment 3: The strategy showed the best results - on the 30th day no more than 5% losses. The optimal ones turned out to be: the first seven-day treatment starting on days 1–3, the second eight-day treatment on days 13 or 14, and the third three-day treatment starting on days 25 or 26. If 10% losses are acceptable or the onset of the disease is missed, it is possible to shift the first treatment and increase the duration of the second and third by 1-2 days while maintaining the same day of onset.

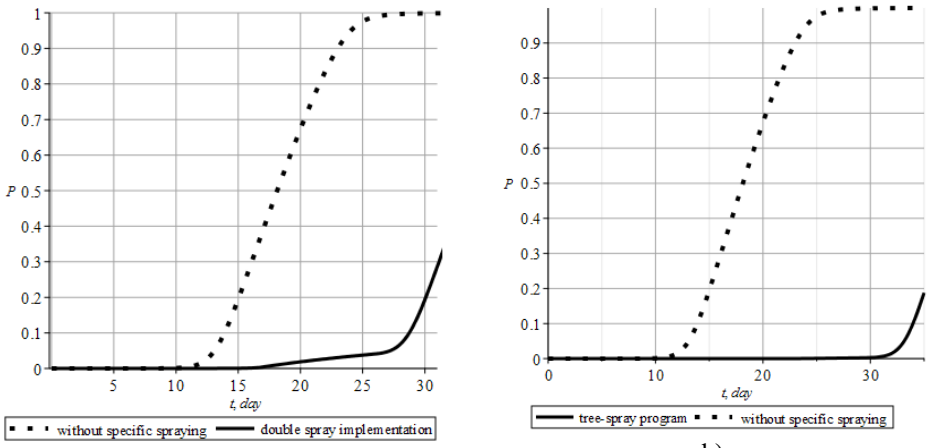


Fig. 2. Results of modeling the proportion of tops affected ($P = 1 - v(x,t)$): a) double processing $\alpha = 0$ при $t \in [7,13]\&[17,26]$ (single line) ; b) triple processing $\alpha = 0$ if $t \in [3,10]\&[13,21]\&[24,29]$(solid line).

As a result of the experiment, the optimal processing periods for each strategy in terms of volume, duration and efficiency were identified. Intervals for the day the treatments begin allow minimizing damage to the tops even if the development of the disease was not detected in the early stages.

4 Modeling a routine treatment scheme for potato fungicide

Routine treatment of a plant with fungicides is a regular treatment that is carried out after a certain period of time (7-10 days). The main initial data are the results of monitoring the development of topsoil damage when exposed to late blight without treatment and treatment with Dithane (the active ingredient is mancozeb) in the UK in 2013 [12]. Processing dates for the observed period (from August 4 to September 3): August 8,16,23,30; September 6, 14, 21, 28. The indicated dates correspond to the simulated processing times (in days): 5, 13, 20, 27, 34, 42, 50, 57. The remaining model parameters are identical to paragraph 4 of this article. The simulation results are presented in Figure 3.

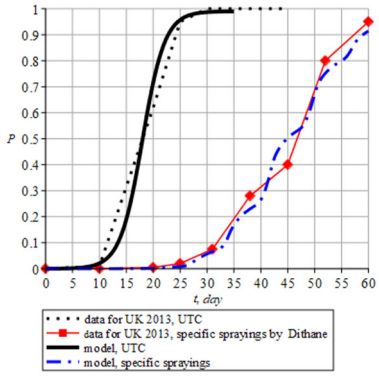


Fig. 3. Results of modeling the proportion of tops affected ($P = 1 - v(x,t)$) during routine treatment (8 times during the observed period) and comparison with field data on monitoring the development of late blight in the UK in 2013 and the results of treatment with Dithane fungicide.

The origin of coordinates corresponds to the date August 4, 2013. During modeling, for the best agreement with experimental data, the duration of action of the fungicide was equal to 5 days. The maximum relative error in the proportion of tops affected was 25% on day 45, the average for the entire period under review was 5%.

5 Conclusion

A model (1) of treating potato late blight with fungicides has been constructed. The model is based on partial differential equations. Fungicidal treatment is modeled by introducing a step function (2). Note that the function F_drug (t) models a simplified effect of a fungicide in practice. Despite this limitation, the results of the numerical solution of system (1) give acceptable results with the experimental curves of the development of the disease given in various sources [12-14]. The presented model can be implemented in decision support systems for agricultural farms. This approach can help in optimizing the choice of fungicidal treatment drugs and, as a result, reduce enterprise costs.

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