Copulas and hydro-economic models for assessing the impacts of climate change in irrigated agriculture

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Abstract: Climate change significantly affects water resources through alterations in rainfall patterns, reduced rainfall quantities, rising temperatures, and prolonged drought cycles. Consequently, the escalating demand for water coupled with diminishing water availability is anticipated to lead to a critical water scarcity issue in the future unless proactive and innovative strategies are implemented.The impact of climate change on water resources is globally recognized as an increasingly vital concern, given its intricate interconnections with various sectors, notably agriculture, energy, and the provision of drinking water. Consequently, the scientific community has dedicated substantial efforts to devise optimal water resource management strategies in response to the challenges posed by climate change. Over the past four decades, hydro-economic models (HEM) have been instrumental in proposing solutions to adapt to evolving climatic conditions.This paper presents a newhydro-economic optimization model accounting for climatic uncertainties. The problem is formulated as a chance-constrained program in which the dependence structure between hydrologic and meteorologic variables is modeled using copula theory. The novelty of the approach lies in the capacity of the model to optimize the water resources, taking into account the dependence between agronomic, socio-economic, and hydrologic systems and climatic uncertainties.

Keywords: Climate change, hydro-economic, Copula, optimization, uncertainties.

1. Introduction

In numerous regions globally, water supply faces significant uncertainties, posing a substantial challenge for optimal water resources management [1]. Water scarcity has been identified as a prominent global risk, attracting considerable attention from researchers and policymakers [2]. Climate change further compounds these challenges by intensifying both water scarcity and water-related hazards, such as floods and droughts, with consequential impacts on agricultural production and incomes. Over the past few decades, HEMs have been devised to evaluate the impacts of climate change and suggest adaptation strategies. These models, specifically designed for regional water resource management, confront various uncertainties associated with climate change that accumulate during the decision-making process [3]. Effectively addressing these uncertainties becomes a pivotal task in water resource modeling, aiming to enhance the formulation of more efficient policies for water resource management.

This paper introduces a hydro-economic optimization model that explicitly accounts for climatic uncertainties. The problem is framed as a chance-constrained program, where the dependence structure between hydrologic and meteorologic variables is modeled using copulas theory [4, 5]. This innovative approach aims to provide a robust framework for managing water resources by incorporating and addressing the inherent uncertainties associated with climate change impacts on hydrology and meteorology.

The next section introduces the Hydro-economic Models (HEM) for the assessment of climate change impacts in the agriculture sector. Section 3 provides a theoretical background and the proposed methodology for modeling dependencies between hydroclimaticvariables using copulas. Section 4 presents a stochastic hydro-economic optimization model accounting for climatic uncertainties. The novelty of the approach lies in the capacity of the model to optimize the water resources taking into account the dependence between agronomic, socio-economic, and hydrologic systems, and climatic uncertainties.

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2. Hydro-economic Models

The interdisciplinary nature of water resourceproblems requires the incorporation of several components into a coherent modeling framework [6]. In recent decades, this objective has been accomplished through the development of hydroeconomic models[7]. In some applications, they are called integrated hydrologic-economic model[8,9], holistic water resources economic models [10], and other nominations.

Hydro-economic modeling, indeed has its roots in the 1960s and 1970s, particularly in arid regions like Israel and the southwestern United States. Early contributors like Jacob Bear, Oded Levin, Rogers, Smith, Gisser, and Mercado played crucial roles in establishing the conceptual framework for regional-scale integrated water management models [7,11,12].

HEM is a modeling approach that integrates engineering, environmental, and economic considerations in the management of water resources systems at a regional scale [7]. The central concept is to put economic principles into practice by incorporating them into the core of water resource management models. This model optimizes the allocation of water across time and space, considering diverse physical, economic, environmental, and institutional constraints. The optimization objective functions typically involve maximizing net benefits (gross benefits derived from water use minus costs) or, similarly, minimizing costs such as water scarcity costs, capital investment costs, and operational costs. Solutions to optimization models can be found through analytical methods like mathematical programming, dynamic optimization, or heuristic search techniques such as evolutionary algorithms, or a combination of these methods.The core idea of hydro-economic modeling is to integrate economic principles into water resource management models, thereby optimizing the allocation of water resources across different uses, considering various constraints. These constraints include physical, economic, environmental, and institutional factors [7].The optimization objective functions in hydro-economic models typically revolve around maximizing net benefits, which are derived by subtracting costs from gross benefits associated with water use. The costs considered may include water scarcity costs, capital costs of investments, and operating costs. By incorporating economic concepts at the center of water resource management models, hydro-economic modeling aims to provide a comprehensive approach to water allocation.

To solve HEMs, analytical methods such as mathematical programming and dynamic optimization can be utilized. Additionally, heuristic search techniques, including evolutionary algorithms, or combinations of these methods, are applied to find optimal solutions. These optimization models are essential tools for decision-makers in water resource management, allowing them to make informed decisions that balance economic, environmental, and engineering considerations.

3. Copula for hydroclimatic applications

Hydrological data are frequently multidimensional, necessitating the simultaneous modeling of multiple variables. The copula approach offers a means to connect any two marginal distributions to their bivariate distribution. In simpler terms, by having knowledge of only the marginal distributions of two random variables, it is theoretically possible to construct their joint distribution. Copulas have found widespread application in various fields, notably in finance [13-17] and hydrology [18-22, 4-5].

Copulas were first defined in 1959 by Sklar (1959) [23]. They are known for their ability to model the joint behavior of two or more variables using their marginal distributions.

Theorem: Sklar (1959): Let H be a two-dimensional distribution function with marginal distribution functions F and G. Then there exists a copula C such that:

$$H(x,y) = C(F(x),G(y)) = C(u,v) \quad (1)$$

Conversely, for any distribution functions F and G and any copula C, the function H is a two-dimensional distribution function with marginals F and G. Furthermore, if F and G are continuous, C is unique.

Choosing the Appropriate Copula fitting

The following algorithm [7,5] allows to identify the appropriate copula for modeling the joint distribution of two random variables(X, Y). Let $(X_1, Y_1) \dots \dots \dots (X_n, Y_n)$be a sample of (X, Y).

Step 1 : Employ graphical methods, such as Chi-plots or K-plots, to determine the copula family fitting the data.

Step 2 : Apply a semi-parametric method based on the Deheuvels copula [24] and the Mean Square Error, to choose the most suitable copulas among those families,.

Step 3 : Reduce the set of copulas acquired in Step 1 by applying Akaike's Information Criterion (AIC).

Step 4: Use goodness of fit tests (Cramer–von Mises and Kolmogorov–Smirnov tests) to select the best copula to fit the data.

Generally hydrologic time series are serially correlated, they are not immediately ideal for copula modeling.Using the Box-Jenkins approach [25], the models adjusted to $X_t$and $Y_t$yield correlated residuals,ε_{X_t} and ε_{Y_t}, that are not autocorrelated.Finding the optimal copula for modeling the joint distribution of $(\varepsilon_{X_t}, \varepsilon_{Y_t})$ is the proposed concept for modeling the dependance between X_t and Y_t.In this instance, the predicted values are obtained from the conditional copulas on the residuals.

The conditional copula proposed by Nelsen [26]C_X is defined as:

$$P(Y \leq y|X \leq x) = P(V \leq v|U \leq u) = \frac{\partial C(u,v)}{\partial u} = C_X(F(x), G(y)) \qquad (2)$$

Where$G(y) = v and F(x) = u$.

We present below some of the most common copula families.

Elliptical copulas, particularly exemplified by Gaussians and t-copulas, are proficient in representing symmetric dependence. The t-copula, in particular, stands out for its capability to effectively capture tail dependence, offering a valuable feature for modeling extreme values.

In cases where tail dependence is a critical consideration, Archimedean copulas come into play. Among them, the Clayton, Frank, and Gumbel copulas are noteworthy examples. The Frank copula displays symmetry, whereas the Clayton and Gumbel copulas exhibit asymmetry. Specifically, the Clayton copula demonstrates stronger dependence in the lower tail, while the Gumbel copula excels in capturing upper tail dependence. This array of copula types provides a versatile toolkit for modeling diverse dependence structures, allowing for a nuanced representation of relationships across different segments of the distribution.Vine (R-vine) copulas are a statistical approach for evaluating composite risk. These copulas provide a flexible modeling framework for high-dimensional data. A vine-based multivariate probability mass function is constructed from bivariate copula building blocks arranged in a tree structure to explore multiple dependencies.

In the field of hydrology, Erhardt et al. [27] showed a remarkable link between vine-copula parameters and station distances in a study focused on daily mean temperature. Vernieuwe et al. [28] took a comprehensive approach by incorporating all relevant connections between storm variables using vine copulas. Musafer et al. [29] proposed a method utilizing vine copulas to estimate prediction uncertainty within an environmental context. Pereira et al. [30] employed vine copulas to achieve a stochastic simulation of stream-flow scenarios.In a more recent development, El Hannoun et al. [4] utilized R-vine copulas to model the co-dependencies of five reservoirs, emphasizing hydrologic design and cascade reservoir management in the basin of the Saint-John River. This research highlights the versatility and applicability of vine copulas in addressing various aspects of hydrological studies, ranging from temperature relationships to storm variables, prediction uncertainty, stream-flow scenarios, and reservoir co-dependencies.

4. Copulafor a Hydro-Economic Model for Climate Change Impact Assessment in irrigated sector: Problem formulation and solution approach

In the context of climate change, extreme weather events pose significant threats to agricultural systems, with agricultural drought risk being a prominent concern [3]. This type of risk, characterized by the likelihood of crop failure, can lead to substantial economic losses [31]. Therefore, it is recommended to conduct a comprehensive assessment, based on a probabilistic approach, that combines the evaluation of both the drought hazard and its impacts on crop production [32].

In this section, we present the formulation of a specific stochastic program designed for HEMs, focusing on assessing the impacts of climate change and adaptation strategies within the agricultural sector. This formulation aims to address the uncertainties inherent in climate change scenarios. The model is tailored to replicate the decision-making processes of farmers within a climate change context. The objective function maximizes the expected profit at the watershed level.

The hydro-agro-economic model developed is a non-linear mathematical programming optimization model at the watershed level and Combines hydrological, agronomic, and economic components.The model takes into account various constraints categorized into hydrological, agricultural, and resource availability constraints. For hydrological constraints, in addition to conventional constraints, such as temporal and spatial water balance equations, the model incorporates specific water constraints.

The mathematical program includes meteorological parameters, crop yield, crop price, and production cost. Including these random variables introduces variability into the objective function, denoted as π, making it a stochastic variable. As a result, the optimization process is conducted in a stochastic-order sense, recognizing the uncertainties associated with meteorological conditions and crop outcomes. This approach allows for a more comprehensive and realistic assessment of the hydro-agro-economic system at the watershed level.

$$\begin{cases} Max\ E(\pi) = E(S) - E(cost) \\ \\ Sc \quad Ax \leq b \quad x \geq 0 \end{cases} \tag{3}$$

$$cost = \sum_{crop}^{m} \left(qx_{crop} + \frac{1}{2}Qx^2\right) \tag{4}$$

$$\mathrm{S} = \sum_{crop}^{m} x_{crop} \times P_{crop} \times Y_{crop} \tag{5}$$

$$E(\pi) = \left(\sum_{crop}^{m} x_{crop} \times \left(C\left(P_{crop}, Y_{crop}(H,T,R,\mathrm{I})\right) + E(P_{crop}) \times E\left(Y_{crop}\right)\right) - \sum_{crop}^{m} \left(qx_{crop} + \frac{1}{2}Qx^2\right) \tag{6}$$

Where

π: objective function (agricultural profit)
$Cost$: quadratic cost
S : selling watershed
x_{crop}: aera cultivate per crop
M: number's crop
q: (N×1) vector of parameters
Q: (N×N) symmetric, positive (semi-) definite matrix of parameters

P_{crop} : crop price
Y: yield production
T : temperature
H: Humidity
R : Rainfull
I: irrigation per crop and per hectare per m^3
P_i : price of water irrigation per m^3
C(P_{crop}, Y_{crop}): copula function

The Positive Mathematical Programming Method (PMP) approach, proposed by Howitt [33], serves as a means to calibrate the models. The core concept of PMP is to align the new optimization of the mathematical programming model with economic and social realities, as observed in the database set [34-36]. For reasons of computational simplicity and the absence of compelling arguments for other types of functions, a quadratic cost function is often utilized, with exceptions noted, such as in the work by Paris and Howitt [37].

5. Conclusion

This article introduces a new formulation of hydro-economic optimization model designed to address climatic uncertainties for effective water resource management at the watershed level. The mathematical program incorporates meteorological values, crop yield, crop price, and production cost as random variables, rendering the agriculture profit objective function a complex random variable that requires maximization.To handle this complexity, the problem is formulated as a chance-constrained program, integrating copulas theory to model the dependence structure between hydrologic and meteorologic variables. The novelty of this approach lies in the model's ability to optimize water resources while considering the interdependencies among agronomic, socio-economic, and hydrologic systems, and accounting for climatic uncertainties.

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