Study on modeling of contact interaction in roll modules

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Abstract. The main problem of the mechanics of contact interaction in roll modules is to determine the stress-strain state in the technological zone. The article presents mathematical models of the laws of distribution of contact stresses and analytical dependencies of their main indices, which make it possible to calculate the power parameters of roll modules of technological machines. When developing mathematical models, empirical formulas for the laws of distribution of normal stresses along roll contact curves and a friction stress model were used, considering the effect of external forces, roll drive, and compliance of the roll coating on the distribution of shear stresses. It is shown that normal stresses in the compression zone increase from zero to the maximum value by the harmonic law, and in the recovery zone, they decrease from the maximum value to zero by the parabolic law. It was revealed that in the driven roll, the neutral point is located on the left side of the point of maximum normal stress, and the points that determine the maximum normal stress and compression of the interacting bodies coincide and are shifted from the line of centers towards the supply of the processed material into the contact zone.

1. Introduction
Roll modules are designed to implement various technological processes through the contact interaction of rollers with the material being processed.
Modeling the contact interaction of rolls with the processed material is critical and is a problem that still requires additional research.
The main task when considering the contact interaction of rolls with the processed material is to determine the stress-strain state in the technological zone and, first, the laws of stress distribution (normal and shear) in the contact zone, which is decisive for finding indices that characterize the roll module and technological process.
Despite the extensive material on the study of the contact interaction in roll modules [1-20], its theoretical foundations have not yet been sufficiently developed, and the main and most in-depth studies were conducted only for the case of metal rolling [1-12]. The most complex issue of stress distribution in the contact zone of the roll module, with the rollers covered with elastic coatings, has not been developed.
This study is devoted to modeling the laws of stress distribution in the contact zone of a symmetrical roll module, consisting of a processed material of a thickness of \( \delta_1 \), two driven rolls with radii \( R \) and elastic coatings of thickness \( H \).

2. Materials and Methods
Figure 1 shows the top part of the symmetrical roll module under consideration.
In the technological process, forces are transmitted to the contact zone of the roll module along the roll contact curves; therefore, contact stresses are distributed along these curves [13]. The laws of distribution of contact stresses in a roll module, where the rollers have elastic coatings, primarily depend on the shape of the roll contact curves.
Under the influence of pressure, the interacting bodies (the material being processed and the elastic coatings of the rollers) are deformed. At that, they first are compressed along the contact curves and then, they restore the deformation. Therefore, the contact curve of each roll consists of compression and recovery zones, separated by a point that determines the maximum compression of the interacting bodies. Under static interaction, the point of maximum

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compression lies on the line of centers [14]. Under roller rotation, this point is shifted from the line of centers towards the supply of the processed material into the contact zone [21].

Let the roll contact curve be determined by polar radii \( r_j(\theta_j) \), where \( j \) is the index that denotes the number of the zone of the contact curve.

From Figure 1, it follows that

\[-\alpha_1 + \alpha_3 \leq \theta_2 + \alpha_3 \leq \alpha_2 + \alpha_3,\]

where \( \alpha_1, \alpha_2 \) are the contact angles, \( \varphi_3 \) is the angle defining the point of maximum compression.

The shape of the roll contact curve depends on many factors, in particular, on the rate of deformation of the interacting bodies [13, 20, 22]. The rate of deformation of the interacting bodies is characterized by the ratio of the rate of deformation of the elastic coating to the rate of deformation of the material being processed [20, 22]:

\[ m_j = \frac{d h_{ij}}{dt} \cdot \frac{d h_{mj}}{dt}. \]  

Since in the compression zone (or in the recovery zone) the interaction of the processed material and the roller coating occurs over a short time interval, in the roll module the value of the ratio of deformation rates can be considered constant and be replaced with an average value [20]:

\[ m_{ij} = \frac{h_{ij}}{h_{pj}}. \]

According to Fig. 1, we have

\[ h_{i1} = R - r_i, \quad h_{pj} = r_i - R \frac{\cos(-\alpha_1 + \alpha_3)}{\cos(\theta_1 + \alpha_3)}. \]

Substituting these deformation values into equalities (2) and making some transforms, we obtain

\[ r_1 = \frac{R}{1 + m_{i1}} \left( 1 + m_{i1} \frac{\cos(-\alpha_1 + \alpha_3)}{\cos(\theta_1 + \alpha_3)} \right), \quad -\alpha_1 + \alpha_3 \leq \theta_2 + \alpha_3 \leq 0. \]  

Similarly, we find

\[ r_2 = \frac{R}{1 + m_{i2}} \left( 1 + m_{i2} \frac{\cos(\alpha_2 + \alpha_3)}{\cos(\theta_2 + \alpha_3)} \right), \quad 0 \leq \theta_2 + \alpha_3 \leq \alpha_2 + \alpha_3. \]
3. Results and Discussion
When the processed material interacts with the roller, normal and shear stresses arise in the contact zone. Their magnitude and the law of distribution along the roll contact curve depend on the external forces acting on the roll, the deformation properties of interacting bodies, and the friction properties of surfaces [13].

The deformation properties of the material of each interacting body are characterized by a compression curve under contact interaction and a curve that determines the deformation recovery (shape). These curves define the relationship between stress and strain of material under compression and recovery. In general, the stress-strain relationship of the material of the interacting bodies is nonlinear.

Roller machines process materials such as fabrics, cotton, leather, paper, wool, etc. The rollers are coated with rubber and polyurethanes of various grades, technical cloth, made from felt of various types. In the technical literature and publications of researchers involved in determining stress and relative deformation under compression and recovery in such materials, the following formulas are given [23-30]:

\[ \sigma_{pj} = B_j e_{kj} \quad \sigma_{ij} = A_j e_{ij}, \]

where \( B_j, k_j \) – are the deformation coefficients of the material being processed, \( A_j, s_j \) – are the deformation coefficients of the roller coating.

From Figure 1, it follows that at each point of the compression zone of the roll contact curve, the following equality holds between the normal stress and the compression stress:

\[ n_1 = \sigma_{pj} \cos \beta_1, \]

where \( \beta_1 \) – is the angle between the normal line and the radius-vector of the points of the compression zone of the roll contact curve, equal to [14]:

\[ \beta_1 = \arctg \frac{r_1'}{r_1}. \]

Considering expressions (4) and (8), we obtain

\[ \tan \beta_1 = \frac{m_{a1} \cos(\alpha_1 + \alpha_3)}{\cos(\theta_1 + \alpha_3) + m_{a1} \cos(-\alpha_1 + \alpha_3)} \tan(\theta_1 + \alpha_3). \]

Next, considering expressions (3), (4), and (6), from equality (7), we obtain:

\[ n_1 = C_1 \left( 1 - \frac{\cos(-\alpha_1 + \alpha_3)}{\cos(\theta_1 + \alpha_3)} \right)^{k_1} \frac{1}{\sqrt{1 + \tan^2 \beta_1}}, \]

where

\[ C_1 = B_1 \left( \frac{2R}{\delta_j (1 + m_{a1})} \right)^{k_1}, \quad \tan \beta_1 = \frac{m_{a1} \cos(-\alpha_1 + \alpha_3)}{\cos(\theta_1 + \alpha_3) + m_{a1} \cos(-\alpha_1 + \alpha_3)} \tan(\theta_1 + \alpha_3). \]

Likewise, we obtain:

\[ n_2 = C_2 \left( 1 - \frac{\cos(\alpha_2 + \alpha_3)}{\cos(\theta_2 + \alpha_3)} \right)^{k_1} \frac{1}{\sqrt{1 + \tan^2 \beta_2}}, \]

where

\[ C_2 = B_2 \left( \frac{2R}{\delta_2 (1 + m_{a2})} \right)^{k_1}, \quad \tan \beta_2 = \frac{m_{a2} \cos(\alpha_2 + \alpha_3)}{\cos(\theta_2 + \alpha_3) + m_{a2} \cos(\alpha_2 + \alpha_3)} \tan(\theta_2 + \alpha_3). \]

To assess the level of impact of the roll pair on the processed material, indices such as average stress \( n_{av} \) and maximum stress \( n_{max} \) are used. The last index, according to many researchers, is determining for roll modules [32, 33].

It is known [13], that the point defining the maximum of normal stresses is shifted from the line of centers in the direction opposite to the movement of the material being processed.

Let this point be determined by the angle \(-\alpha_4 + \alpha_5\). Applying the condition of maximum for function \( n_1(\theta_1) \) defined by equality (10) and assuming that \( \sin(-\alpha_4 + \alpha_5) \approx -\alpha_4 + \alpha_5, \cos(-\alpha_4 + \alpha_5) \approx 1 \), we obtain \( \alpha_4 = \alpha_5 \).
Thus, the points that determine the maximum normal stresses and compression of the interacting bodies coincide. Under rotating rollers, these points are shifted from the center line towards the supply of the processed material into the contact zone, and under static interaction of the rollers, they are on the line of centers.

Then, from equalities (10) and (12), we obtain:

\[ n_{\text{max}} = C_1 \left( \frac{(\alpha_4 - \alpha_3)^2}{2} \right)^{k_1}, \quad n_{\text{max}} = C_2 \left( \frac{(\alpha_2 + \alpha_3)^2}{2} \right)^{k_2}. \]  (14)

Equating the right-hand sides of dependencies (12), we obtain:

\[ \alpha_4 = \alpha_3 = \frac{C_1 a_1^{2k_1} 2^{k_1} - C_2 a_2^{2k_2} 2^{k_2}}{k_1 C_1 a_1^{2k_1 - 1} 2^{k_1 + 1} + k_2 C_2 a_2^{2k_2 - 1} 2^{k_2 + 1}}. \]  (15)

Formulas (10) and (12) determine the laws of distribution of normal stresses along the roll contact curves. These formulas have difficulties for further use, for example, to determine the power parameters of the roll module. Therefore, we consider the problem of replacing formulas (10) and (12) with simplified empirical ones, considering indices \( n_{\text{max}} \) and \( \alpha_3 (\alpha_4) \), given in formulas (14) and (15). To do this, we use the results given in [13, 20], which provide graphs and empirical formulas for the following possible laws of distribution of normal stresses along roll contact curves - elliptical, parabolic, harmonic, and exponential laws.

An analysis of the comparison of the left and right parts of the graphs of dependencies (10) and (12) and possible laws of distribution of normal stresses showed that formulas (10) and (12) can be replaced, respectively, with the empirical formulas of the harmonic and parabolic laws with indices \( n_{\text{max}} \) and \( \alpha_3 \).

As a result of replacement, we obtain:

\[
 n(\theta) = \begin{cases} 
 \frac{n_{\text{max}}}{2} \left(1 + \cos \left( \frac{\alpha_1 + \alpha_3}{\alpha_3 - \alpha_1} \pi \right) \right) - \alpha_1 + \alpha_3 \leq \theta + \alpha_3 \leq 0, \\
 n_{\text{max}} \left(1 - \left( \frac{\alpha_2 + \alpha_3}{\alpha_2 + \alpha_3} \right)^2 \right), & 0 \leq \theta + \alpha_3 \leq \alpha_2 + \alpha_3.
\end{cases} \]  (16)

The distribution of shear stresses also depends on many factors, the main one being the friction stress model, which determines the relationships between normal and shear forces [3]. There are various models of friction stress [34,35,36], and many of them were developed in metal rolling. However, a correct model of friction stresses for the stick zone in metal rolling has not yet been created. Therefore, friction stress models for metal rolling can be considered as an assumption used only for sliding zones [34]. Currently, the most acceptable value for roll modules in the sliding and sticking zones is the friction stress modulus [35]:

\[ \tau_j = (a_j \times (\theta_2 + \alpha_3 - \beta_j) + D)n_j, \]  (17)

in which the effect of external forces and roller drive on the distribution of shear stresses is taken into account by the dynamic coefficient \( D \), and the compliance of the roller coating - by angle \( \beta \).

As a result of transforming equality (17) considering equalities (11) and (13), we have

\[ \tau_j = (a_j \times (\theta_2 + \alpha_3) + D)n_j, \]  (18)

where

\[ a_1 = \frac{1}{1 + m_{a1} \cos(-\alpha_1 + \alpha_3)}, \quad a_2 = \frac{1}{1 + m_{a2} \cos(\alpha_2 + \alpha_3)}. \]  (19)

From equalities (16) and (18), we find

\[
 \tau(\theta) = \begin{cases} 
 \frac{n_{\text{max}}}{2} \left(1 + \cos \left( \frac{\alpha_1 + \alpha_3}{\alpha_3 - \alpha_1} \pi \right) \right) \left(a_1 n_2(\theta_1 + \alpha_3) + D\right) - \alpha_1 + \alpha_3 \leq \theta + \alpha_3 \leq 0, \\
 n_{\text{max}} a_2 \left(1 - \left( \frac{\alpha_2 + \alpha_3}{\alpha_2 + \alpha_3} \right)^2 \right) \left(a_2 n_2(\theta_2 + \alpha_3) + D\right), & 0 \leq \theta + \alpha_3 \leq \alpha_2 + \alpha_3.
\end{cases} \]  (20)
At the neutral point, and accordingly, at the neutral angle \((-\alpha_5 + \alpha_3)\), the shear stress is zero [31]. From formula (20), it follows that

\[
\alpha_5 = \alpha_3 + \frac{D}{a_1}.
\]  

(21)

Under static interaction of the rolls \(D = 0\), therefore, \(\alpha_5 = \alpha_3 = 0\).

Thus, the neutral point of the driven roll is in the sticking zone on the left side of the point of maximum normal stress. Under static interaction of rolls, this point coincides with the points of maximum normal stress and compression of the interacting bodies and is located on the line of centers.

4. Conclusions

1. Modeling the laws of stress distribution in the contact zone of a symmetrical roll module was conducted in the following order:
   1. The shapes of the roll contact curves were determined.
   2. The laws and basic indices of the distribution of normal stresses were obtained.
   3. The laws of distribution of normal stresses were approximated by empirical formulas with the indices obtained.
   4. The laws of distribution of shear stresses were determined using the most appropriate friction stress models available at present.

Based on the simulation, mathematical models (16) and (20) were obtained that described the laws of distribution of normal and shear stresses along the roll contact curves of a symmetrical roll module. The main parameters of these models were determined by expressions (14), (15), and (21). The resulting models could be used in the design of roller technological machines, in particular, to determine the power parameters of the roll modules of such machines.

Based on the analysis of the graphs of the models obtained, the following was revealed:
- normal stress in the compression zone increases from zero to the maximum value by the harmonic law, and in the recovery zone, it decreases from the maximum value to zero by the parabolic law;
- shear stresses change signs at the neutral point, which in the driven roll is located on the left side of the point of maximum normal stresses;
- the points that determine the maximum normal stresses and compression of the interacting bodies coincide and, under the roll rotation are shifted from the line of centers towards the entrance of the processed material into the contact zone.

References


