On the method of approximation and quantization of information transmission through communication channels

Mamirjon Turdimatov, Mohira Xusanova, Xursanoy Sadirova, Sultonali Abdurakhmonov, and Inom Bilolov

1 Fergana Branch of Tashkent University of Information Technologies, Fergana, Republic of Uzbekistan
2 Fergana Polytechnic Institute, Fergana, Republic of Uzbekistan
3 Fergana Branch of Tashkent University of Information Technologies, Fergana, Republic of Uzbekistan

Abstract. This article discusses the issues of information transmission over communication channels based on an adjacent algorithm for signal approximation and sampling. The nature of the communication channel is described, which is a set of technical means for transmitting messages from one point in space to another. It is proved that the transmission is most often carried out in conditions of unavoidable interference. From this point of view, to eliminate gaps, an approximation method is used for a continuous message converted into an electrical form of the primary $x(t)$, which can be converted into a discrete form (sequence of numbers) using the process of taking samples at certain intervals. It is proved that from a practical point of view, the intervals are taken to be the same and equal to a certain value. The mathematical expectation of quantization noise is used to determine interference and signal quality. Legendre's interpolation polynomial is used to reconstruct the signals. The computational procedure for adaptive sampling can be implemented in hardware. The introduction substantiates the relevance of the publication of this scientific article based on the analysis of the available scientific results of scientific research in the field of signal processing and transmission by quantization and approximation. The summary lists the results obtained in the framework of scientific research and discusses their theoretical and practical significance.

1 Introduction

In the modern form of information technology, the issues of the approximation method and quantum transmission of information through communication channels are mainly considered from the point of view of modern technology and technologies for processing information through communication channels. The main attention is paid to the reception and processing of signals from the point of view of practical application in microprocessor devices [1–3]. Approximation and quantization of signals in communication channels were
2 Materials and methods

A communication channel is a set of technical means for transmitting messages from one point in space to another. This transmission is most often carried out in conditions of unavoidable interference. In this point of view, an approximation method is used to eliminate gaps for a continuous message of the primary function $x(t)$, converted into an electrical form, a discrete form (sequence of numbers) can be transformed using the process of taking samples at intervals $\Delta t_1, \Delta t_2, \Delta t_3, \ldots$ From a practical point of view, the intervals are taken to be the same equal to some $\Delta t_B = \sum_{k} \delta(t + kT_B)$

$$u_{k_B}(t) = \sum_{k} u(t)\delta(t + kT_B)$$

If we approximate by $U(k\Delta t)$ the quantization in time. The quantization result can be written as

$$u_k(t) = \sum_{i} \delta(t) u(t)\delta(t + kT_B)$$

where $\delta(t)$ is the delta function.

As a result of interference, each sent element can be identified by the recipient as $y_k$ and $y_k \neq x_i$. This process happens by errors. Similarly, a continuous message $x(t)$ can be taken as

$$y(t) \neq ax(t - \tau)$$

For all or some moments of time, where $a$ and $\tau$ are constants, usually not significant in terms of the amount of information. From the point of view of information theory, the physical device of the communication channel is not essential. The channel properties are fully described by the transition probability matrix $P(x_i/y_k)$ or $P(y_k/x_i)$, where $P(x_i/y_k)$ is the probability of sending element $x_i$ if the received element $y_k$ is fixed, and $P(y_k/x_i)$ is the probability of receiving element $y_k$ if the element $x_i$ is fixed. It is assumed that new elements cannot be created under the influence of interference, therefore,

$$\sum_{k=1}^{M} P(y_k/x_i) = 1, \sum_{i=1}^{M} P(x_i/y_k) = 1$$

If there is no interference, then all diagonal elements $P(x_k/y_k)$ or $P(y_k/x_k)$ are equal to one, and the rest are zero. With very high interference, all elements of the matrices can be approximately the same.

3 Methods of solving the problem

$$\Delta u \ll U_{max} - U_{min}$$

$$P_i = P_U(u_i)\Delta u$$
\[ M[\xi_i] = P_i \int_{u_{i-\frac{1}{2}}}^{u_{i+\frac{1}{2}}} (u - u_i) \, du = \frac{1}{2} P_i \left[ (u_{i+\frac{1}{2}} - u_i)^2 - (u_{i-\frac{1}{2}} - u_i)^2 \right] \] (6)

\[ D[\xi_i] = P_i \int_{u_{i-\frac{1}{2}}}^{u_{i+\frac{1}{2}}} (u - u_i)^2 \, du = \frac{1}{3} P_i \left[ (u_{i+\frac{1}{2}} - u_i)^3 - (u_{i-\frac{1}{2}} - u_i)^3 \right] \] (7)

\[
\frac{dD[\xi_i]}{du_i} = P_i \left[ (u_{i+\frac{1}{2}} - u_i)^2 - (u_{i-\frac{1}{2}} - u_i)^2 \right] = 0
\] (8)

\[ \pm (u_{i+\frac{1}{2}} - u_i)^2 = \pm (u_{i-\frac{1}{2}} - u_i)^2 \] (9)

\[ u_{i+\frac{1}{2}} = u_{i-\frac{1}{2}} \] (10)

\[ u_i = -\frac{u_{i+\frac{1}{2}} + u_{i-\frac{1}{2}}}{2} \] (11)

\[ u_{i+\frac{1}{2}} = u_i + \frac{\Delta u}{2} \] (12)

\[ u_{i-\frac{1}{2}} = u_i - \frac{\Delta u}{2} \] (13)

\[ D[\xi_i] = \frac{1}{3} P_i \left[ (u_{i+\frac{1}{2}} - u_i)^3 - (u_{i-\frac{1}{2}} - u_i)^3 \right] = \] (14)

\[
\frac{1}{3} P_i \left[ \frac{\Delta u^3}{8} - \left( -\frac{\Delta u}{8} \right)^3 \right] = \frac{p_i \Delta u^3}{12} = (p_i \Delta u)^2 \frac{\Delta u^2}{12}
\] (15)

\[ D[\xi] = \frac{\Delta u^2}{12} \] (16)
The variance of the distribution $P_U(u)$ over the interval $\Delta u$.

Now we can evaluate the accuracy of the quantized message by the ratio $P_c/P_w$, where $P_c$ is the average power of the message, $P_w$ is the power of the quantization noise. The quantization noise power is calculated by the formula (17):

$$P_w = \sigma^2 \xi = \frac{1}{\Delta u} \int \frac{\xi^2}{\Delta u} d\xi = \frac{\Delta u^2}{12}$$ (17)

$$P_c = \sigma^2_u = \frac{1}{2U_M} \int_{-U_M}^{+U_M} u^2 du = \frac{U_M^2}{3}$$ (18)

$$P_k = \frac{P_c}{P_w} = \left(2 \frac{U_M}{\Delta u}\right)^2$$ (19)

The general method of interpolation is to find the function $f(t)$ passing through the sample values at the moments of counts $t_0, t_1, \ldots, t_n$ and minimally evading at intermediate times. Usually the interpolation function $f(t)$ is among the power polynomials [9, 10]. In particular, the Legendre interpolation polynomial can be found as (20).

$$f(t) = \sum_{k=0}^{n} u(t_k) \times \prod_{k \neq k+1} (t - t_k - t_{k+1})^{(t_k - t_{k+1})} \prod_{k \neq t_k} (t - t_k)^{(t_k - t)}$$ (20)

$$u(t) = \sum_k u(kT_B) \prod(T_B + kT_B)$$ (21)

$$\prod(t) = \begin{cases} 1, & 0 < t < T_B, \\ 0, & \text{otherwise}. \end{cases}$$ (22)

$$u(t) = \sum_k \left[u(kT_B) + \frac{u[(k + 1)T_B] - u(kT_B)}{T_B}\right] \prod(t + kT_B)$$ (23)
which means the interpolation of $u(t)$ values between each two samples by straight lines. Sometimes these two types of interpolation are mistakenly called interpolations by polynomials of zero and first degree, polynomials (12) and (14), have infinite degree. However, for the optimality of sampling on an approximation basis, it is necessary that in each case the norm of approximation of the approximating function to the original process should be Chebyshev. Such requirements are met by uniform Chebyshev splines of different types with defect $m$, the approximation parameters of which are determined by algorithms [11, 12].

The use of algorithms [13, 14] of uniform approximation by Chebyshev splines makes it possible to obtain a minimum number of samples for each of the types of splines. The computational procedure for adaptive sampling can be implemented by hardware [14]. However, it requires a slightly larger number of actions when processing a single report than conventional algorithms. Therefore, it is desirable to use a microprocessor [6, 8] with high speed and sufficient buffer memory in the compression scheme. This will make it possible to effectively use the algorithm of uniform approximation by Chebyshev splines in systems with preliminary accumulation and subsequent accelerated transmission of compressed messages [5, 14–16]. Therefore, to optimize the overall task we have set, we take the discretization’s on the segment $[a; b]$ given a grid:

$$\{ f_1, f_2, \ldots, f_n \}$$

At the nodes of this grid, the values $f_1, f_2, \ldots, f_n$ are known. Consider a class of functions (24).

$$W^2_{2}[a, b]$$

It is characteristic of these functions that their second derivative is summable with the square on $[a, b]$ (25).

$$\int_a^b f_n^2(x) \, dx < \infty$$

$$\sigma(x_i) = f_i$$

$$\|\sigma''\|_{L^2}^2 = \int_a^b [\sigma^n(x)] \, dx \to min$$

$$(x_i, f_i) = \bar{1}, \bar{N}$$

$$\|\sigma''\|_{L^2}^2$$
\begin{align*}
E(\sigma) &= \|T\sigma\|_Y^2 \\
X &= W_2^2[a, b], T = \frac{d^2}{dx^2}, Y = L_2[a, b]
\end{align*}

\begin{align*}
\sigma(x_i) &= f_i, i = \overline{1, N} \\
A_\sigma &= f
\end{align*}

\begin{align*}
\|T\sigma\|_Y^2 \to \min
\end{align*}

\begin{align*}
A(f, x) &= f_i(1 - \xi(x))^2 \ast (1 + 2\xi(x)) + f_{i+1}\xi(x) \ast (3 - 2\xi(x)) + m_i h_1(x) \ast \\
&\ast (1 - \xi(x))^2 - m_{i+1} h_1 \xi(x)^2 \ast (1 - \xi(x))
\end{align*}

\begin{align*}
h_1 &= x_{i+1} - x_i, \xi(x) = \frac{x - x_i}{h_i},
\end{align*}

\begin{align*}
x - \text{the point at which it is calculated } A(f, x)
\end{align*}

\begin{align*}
m_i &= A(f, x), i = \overline{0, N}
\end{align*}

\begin{align*}
A'(f, x_i + 0) = A'(f, x_i - 0)
\end{align*}

\begin{align*}
\lambda_i m_{i-1} + 2m_i + \mu_i m_{i+1} = b_i, i = \overline{1, N - 1}
\end{align*}

\begin{align*}
\mu_i &= h_{i-1} (h_{i-1} + h_i)^{-1}, \lambda_i = 1 - \mu_i
\end{align*}

\begin{align*}
e_i &= 3\left( M_i \frac{f_{i+1} - f_i}{h_i} + \lambda_i \frac{f_i - f_{i+1}}{h_{i-1}} \right)
\end{align*}
\[ A'(f, a) = f'(a), A'(f, b) = f'(b) \]  
(41)

\[ A'(f, a) = f'(a), A'(f, b) = f'(b) \]  
(42)

\[
\begin{aligned}
2m_0 + \mu_0 m_i &= c_0, \\
\lambda_i m_{i-1} + 2m_i + \mu_i m_{i+1} &= c_i, i = 1, N - 1 \\
\lambda_N m_{N-1} + 2m_N &= c_N
\end{aligned}
\]  
(43)

\[ \mu_0^* = \lambda_N m_1 = c_0 \]  
(44)

\[ c_0^* = 2f_0' \]  
(45)

\[ c_N^* = f_N \]  
(46)

\[ \mu_0^* = \lambda_N m_1 = 1 \]  
(47)

\[ c_0^* = 3 \cdot \frac{f_1 - f_0}{h_0} - \frac{h_0}{2} f_0' \]  
(48)

\[ c_N^* = 3 \cdot \frac{f_N - f_{n-1}}{h_0} n \]  
(49)

\[ y_i = v_i v_{i+1} + u_i \]  
(50)

\[ i = 1, N - 1 \]  
(51)

\[ (a_1 + c_i v_{i-1})y_i + b_i y_{i+1} = d_i - c_i u_i - 1 \]  
(52)

### 4 Discussion

In conclusion, we can say that the popularity of spline approximation methods is explained by the fact that they serve as a universal tool for modeling functions and, in comparison with other mathematical methods, with equal information and hardware costs, they provide greater accuracy of calculations [5, 10].

The tasks of developing methods, algorithms of hardware and software for fast search and identification of local features of signals are relevant. Signal analysis and recovery forms the basis of the processes of solving the problems of processing geophysical and seismic signals, processing the results of bench tests, image processing and others.

The requirements of high performance of computing systems used in these areas can be met both through the development of new methods and algorithms for digital signal processing, and with the help of multiprocessor parallel-pipelined computing [6, 11].

Generalized spectral methods and methods of spline functions are widely used to solve problems of signal analysis and reconstruction.

### 5 Conclusion
The algorithm of uniform approximation by Chebyshev splines in systems with pre-accumulation is fully described by the matrix of transient probabilities; an algorithm has been developed to optimize the method of approximation of the obtained polynomials and algorithms for uniform approximation by Chebyshev splines, which will allow obtaining the minimum number of samples for each of the types of splines; the difference from the usual algorithm is a microprocessor with high performance and sufficient buffer memory; estimates of the fidelity of message quantization are developed—the average power of the message and the power of the quantization noise; the computational procedure for adaptive sampling can be implemented in hardware.

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Statement of interest, disclosure of information

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