Modelling algorithms for learner interaction with training courses

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Abstract. This paper considers learning as a process of mastering a knowledge domain, investigating the interaction of learners with courses of study and the influence of learner actions on the state of mastery of the knowledge space in order to define a learning control function and to model algorithms for constructing the knowledge space. Key aspects of the interaction process, variables that can be changed to customise the learning model, are given. A threshold \( t \) of mastery of a course element is formulated and a scale of mastery level for a particular knowledge element is described. As a result, algorithms for forming, expanding and segmenting the knowledge space were created. The research presented the concept of learning as a guided wave process of knowledge mastery, where the learner's actions correspond to the structure of the knowledge space and are determined by its properties.

Keywords. Modelling, training, interaction, algorithm, learning, mastering, knowledge, course, wave process, knowledge space.

1 Introduction

In modern education, student interaction with the learning material takes place, which is one of the main aspects of effective learning [1]. With the development of technologies and teaching methods, there is a need to develop models capable of describing and predicting this process with a high degree of accuracy [8]. This paper presents a model of interaction with a course of study based on modern theoretical and practical approaches to education and learning. The learner's interaction with the course of study plays a key role in the educational process, determining the effectiveness of learning, achievement of learning objectives and development of learning skills [7]. This process is a complex relationship between the learner and the course content [2]. This study is relevant in considering the dynamics of learner-instructional interaction and the ability to evaluate how effective learning is. Understanding the interaction process allows to improve teaching and learning methodologies and courses, to optimise learning, and to identify various emerging problems [9].

The object of research in this paper is the interaction of the learner with the training course. The study provides a better understanding of this mechanism. The aim of the study is to model the dynamics of the process of interaction between the learner and the training course and to build algorithms for the knowledge space.

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This study brings a new method of modelling interactions with the learning course, and builds algorithms of this interaction. Authors’ hypothesis: the learning process is a controlled wave process of knowledge assimilation, and the level of mastery of the knowledge space can be considered as the energy of the wave. The mastery of the knowledge space means the level of understanding and assimilation. The more knowledge is learnt, the higher the energy of the wave.

The works of many scientists are devoted to the issues of learning, their models and algorithms. John Sweller in his work “Cognitive Load Theory” states that the cognitive load associated with the learning process affects the efficiency of information assimilation. He divides cognitive load into intellectual (related to understanding the material) and extraverted (related to processing external information) [3]. Paul Kirschner, John Sweller and Richard E. Clark discuss the theory of cognitive load and its application in the context of education. The paper discusses principles for designing models of teaching methods to optimise learning [4].

Michelle Riconscente explores the interactions between students and learning materials, as well as the use of technology in educational processes. Bill Cope’s “Multiliteracies: Literacy Learning and the Design of Social Futures” offers an approach to education that recognises the diversity of ways of communicating and presenting information in today’s digital world [5]. Richard E. Clark considers the effectiveness of using technology in learning and develops models of student interaction with courses of study. I.V. Grebenev and E.V. Chuprunov study the relationship between modelling and design in the organisation of the learning process [6].

2 Method

Let’s consider the influence on the process of mastering the training material by manipulating its elements, which determines the level of mastery of the knowledge space presented by the course. Let us introduce the following notations: learner’s actions $\phi$, knowledge space $KS$, element of space $d$. The change of the state of the space $KS$ occurs due to the learner’s actions. The degree of learning determines these actions. This model of interaction with the training course can be described by the equation of the learner’s state change. We will consider this equation on discrete time intervals:

$$\phi_{i+1} = d_{i+1}, \phi_i + KS(\phi_i)$$

$$\phi_{i+1} > \phi_i$$

When building the process for each element of the training course, we define a threshold value $\phi \in \Phi$, which will set the minimum requirements for mastering the element. The learner performs certain actions. Each element $a \in S$ is defined on the scale $\phi \in \Lambda_\phi$, after the realisation of the subsequent action.

$$\phi_{i+1} = d_{i+1}, (\phi_i)$$

$$\phi_{i+1} > \phi_i$$
an indicator of mastery of the element, i.e. the state of the element exceeds the threshold value, it is a mastered element. If the state of element does not exceed the threshold value, it is an undeveloped element: 

\[ a \leq \Phi(a) \leq \Phi(a') \leq \ldots \leq \Phi(a_k) \leq a \in \Delta(a) \]

For such elements, its achievability is determined, taking into account the degree of mastery of previously learnt elements that form maximal chains. The idea of using maximal chains to describe the logic of knowledge acquisition reflects the cognitive nature of this process [10].

The set of elements \( \Delta(a) \), which come before the element, are the main ideal for a certain element. Thus, the maximal chain of the ideal determines the logic of mastering the preceding elements:

\[ \{1, 2, 1, 2\} : (y_1 < y_2 < \ldots < y_k < a) \in \Delta(a) \]

Here \( C \) is the maximal chain of the ideal, \( \lambda \) are elements of the chain.

The ability to learn something is determined by how well all its antecedent components (maximal chains (4)) have been mastered.

The set of values of the states of the components of the maximum circuit is the spread (amplitude) of this circuit

\[ A(c_i) = \{\phi(y_1), \phi(y_2), \ldots, \phi(y_k)\} \]

\[ |A(c_i)| = f(\lambda(y_i)) \]

\[ J : \Phi(a) = \Sigma|A(c_i)| \]

\[ J : \Phi(a) \cap C(a) \rightarrow \Lambda : C(a) = \{y_1 < y_2 < \ldots < y_k\} \]

\[ J : a \notin \Phi(a) \rightarrow J^{-1}(a) \]

\[ \Lambda_{\rho} \subset \Lambda_{\rho} \{\lambda_1, \lambda_2, \ldots, \lambda_m\} \]

\[ \lambda_m = J(\Phi(I) \cap C(a) \rightarrow \Lambda : C(a)) \]

\[ J : a \notin \Phi(a) \rightarrow J^{-1}(a) \]

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\[ J : a \notin \Phi(a) \rightarrow J^{-1}(a) \]
Here $f(\lambda_i y_i)$ is the summation method for $\lambda_i \leq \lambda_j$, $\lambda_i f(\lambda_i y_i)$ are respectively the lower and upper scores obtained from the $\lambda f(\lambda)$ scale defined by $
abla_{\lambda_i \leq \lambda_j} \{\lambda_i, \ldots, \lambda_j\}$, where $i$ varies from 1 to $n$, the size of this circuit; $j$ varies from 1 to $m$, the number of different ways in which the element $a$ can obtain energy from the maximal circuits.

As a result of the performed actions, the learner moves to the state:

$$K\varphi_{++} = d_{++}(\varphi)$$

2 Define the function that controls the learning process. Let us evaluate the degree of completion of the learning process, taking into account all possible states $\Phi(KS \Lambda_{\varphi})$, in which the learner can be, that is, let us take into account the variety and different aspects of states to determine how complete and effective the training has been. Let us select a part of the set:

$$\Phi_{obj}(KS \Lambda_{\varphi}) \subset \Phi(KS \Lambda_{\varphi})$$

Here $\varphi_m \in \Phi_{obj}(KS \Lambda_{\varphi})$

KS ($\varphi_i$) $\subset$ KS ($\varphi_j$) $\subset$ $\cdots$ $\subset$ KS ($\varphi_i$) $\subset$ $\cdots$ $\subset$ KS ($\varphi^{obj}$)

Where $\varphi_i$, $\varphi_j$, $\varphi^{obj}$ are respectively the states of the knowledge space $KS_{\varphi}$ (a certain action has been performed), $KS_{\varphi}$ (a part of the mastered knowledge space of the state $i$), when the learning goal is achieved, there will be a state $\varphi^{obj}$.

The following equality (15) defines the actions $d_{++}$:

$$F(\varphi_i) = d_{++}(\varphi)$$

3. Define the parameters of the learning process model that can be controlled. The parameter $a$ is a numerical score, which reflects the level of knowledge and skills, is controllable. This knowledge and skills are achieved in the learning process. As mentioned in step 2, the goal of the process is to achieve the state $\varphi^{obj}$. All elements of the knowledge space $a$ have a state $a_{\varphi}$. The mastery level of an element is determined by comparing the knowledge and skills of the element with a threshold value $*a_{\varphi}$. If the knowledge and skills of an element exceed this threshold, its level of mastery is considered sufficient or above the set threshold. If they do not reach this threshold, the level of mastery is considered insufficient. Equality to the parameter $*a_{\varphi}$ is the minimum level of mastery. Thus, $*a_{\varphi}$ is a differential.
assessment of the mastery of the course. This assessment can be set equal for all elements of the course:

\[ \phi_a \geq \phi_{\text{obj}} \]

In this case, all elements are of equal importance for mastering the knowledge space. However, some elements may have different significance of the level of information mastery or skill in a particular domain. In a particular case, the lower knowledge level can be defined separately for each element of the training course or for a group of elements that share common characteristics or attributes. Each element or certain types of elements can have its own minimum level of knowledge required for mastery. This makes it possible to individualise the learning process and adapt it to the needs of each learner. In this case,

\[ \phi_a \]

identifies the significance of the element or element type. Achievement of the mastery condition requirement \[ \text{obj} \]

indicates that the knowledge or skills element has been fully mastered. Such a point is determined by a mastery threshold that is set for each specific element of the knowledge space. This threshold is related to \[ (\gamma_i)^{\text{obj}} \], which is the mastery threshold for all previous elements in this knowledge space that are defined as basic for this element. Relationship (17) shows the condition of student movement within the knowledge space, by setting a threshold value to each element:

\[ J^{\min} J_a J_a f y \]

In order to consider a knowledge element as mastered, a certain set value must be reached or surpassed, which is the same for all elements of a given knowledge space:

\[ \forall a \left( J^{\min} J_a J_a \right) \]

Learners can move in the knowledge space by having a certain state on the mastered part of the KS. Advancement or moving forward in the learning process occurs only when there is a certain level of understanding or mastery in the already mastered knowledge areas. If we set the mastery threshold in the form of an inequality:

\[ \forall a \left( J^{\min} J_a J_a \right) \]

in order to consider a course or knowledge area fully mastered, the average level of mastery must be above a given threshold on the mastered space. The mastery threshold is an overall score that determines how well a course has been mastered as a whole. To achieve full mastery of a course or knowledge area, the average knowledge level must exceed this threshold.

Learning can be seen as a dynamic process of knowledge acquisition that is controlled and varies in time and spreads in space. The learning process is a dynamic process that advances and propagates through time and space, similar to waves. A wave carrying energy is considered in the discrete environment of the knowledge space (Figure 1). When something new is learnt, the level of knowledge and skills can be thought of as the energy of the wave, which is determined by the state \[ \phi \]. This energy is determined by how well the new material is learnt. In the process of mastering each knowledge element, the learner transfers some of this energy to the mastering. When an element \[ a \] receives energy from one ray of the maximum circuit, this is the amplitude \[ i Ac \). The amplitude is the sum of knowledge that has been accumulated by all the elements in that chain. It is the amount of knowledge of information or information that is transferred to an element.
Wave propagation: Each ray transfers energy \( J(\varphi) \) to the element, which is called energy flow. When the value of this flow reaches a certain level, which is labelled as a value \( J^* \), the element is ready to be absorbed. The set of all such elements that are ready to be mastered forms the wave front. This means that the energy from each ray is collected together and, when it reaches a certain level, an opportunity for mastering a new element is created. Knowledge acquisition is the process of changing the learner's knowledge and skills. This process is spread in time and space, whereby the sequence of actions can be considered as time. In the process of learning, the learner acquires new knowledge and skills, which is accompanied by energy aimed at changing his/her state of mastery. When one speaks of wave control, it means controlling the spread of knowledge or skills through the entire knowledge space. It is necessary to make sure that the wave front reaches all those involved in the learning, which would ultimately lead to the achievement of the objectives. The movement of this wave of knowledge must be controlled so that it covers the entire area to be learnt.

3. Let us form an algorithm for constructing the knowledge space. Let the training course \( C \) has the following structure:

\[
\text{LC} \leq C \leq \text{LC}
\]

Here \( \text{LC} \) is the training course, \( \{a, b, c, \ldots\} \) is the set of course elements, \( \leq \) is the logical connectivity of the elements. A knowledge lattice is a structure that shows the organisation and hierarchy of the knowledge presented in a course.

Algorithm for building the lattice, correlated to the structure of the training course \( \text{LC} \):

1. Let we have a structure and a set of its min elements for which the condition \( b < a \Rightarrow b = a \);
2. The power of the set of min elements is 1, then the element is the only min element of the lattice \( \text{inf}(a) = \emptyset \Rightarrow \emptyset \);
3. If item 2 is not fulfilled, the element \( \text{inf}(a) = \emptyset \) is attached and the condition \( b < a \Rightarrow b = a \) is fulfilled;
4. Let we have a structure and a set of its maximal elements for which the condition \( b > a \Rightarrow b = a \);
5. If the power of the set of max elements is 1, then the element is the only max element of the lattice

\[ \sup(I_{KS}) = a \]

6. If item 5 is not fulfilled, the element \( \sup(I_{KS}) = a \) is attached and the condition \( b > a \rightarrow b = a \) is fulfilled;

7. To construct a lattice for these pairs \( a \) and \( b \): look for the principal filters and their intersection \( c \), which is a min element, given \( d < c \rightarrow d = c \);

8. Under the singularity condition is an upper bound \( \sup(a, b) = a \oplus b \);

9. If item 8 is not fulfilled, the element \( a \oplus b \) such that the intersection of filters is the main filter, is attached;

10. This element is linked to all min filter intersection elements \( \text{LC} \), where \( \text{LC} \) is the minimal network that represents the organisation and hierarchy of knowledge within a given learning course. In the process of learning knowledge, the nodes or elements of the grid serve as channels of communication between different parts of the knowledge space. They enable the propagation of the wave energy that is transmitted in the learning process. The added nodes represent the exact upper and lower edges of the course substructures that are not mapped to the elements of the learning course. Therefore, these nodes do not contain specific information or knowledge directly related to the content of the training course. Since the wave energy arising from the process of mastering the course elements is transmitted along links (channels), and these nodes do not have such links with the course elements, the wave energy is not channelled to these nodes. The state of the learner does not change in these nodes, because they do not interact directly with the process of mastering specific knowledge.

Consider the process used to add new knowledge or elements to an existing knowledge space. The algorithm enlarges the knowledge space and defines the steps to be performed in order to expand the knowledge set. This could be exploring new topics, learning related concepts, and accepting new information:

1. The new knowledge or elements are highlighted in separate modules \( M_i \) in the training course:

\[ \{M_1, M_2, \ldots, M_n\} \]

2. Links are established between the different modules that reflect the logic with which the modules are to be mastered (Figure 2);

3. Taking into account point 2 and the course structure, the knowledge space is constructed \( \text{LK} \);

4. All modules are considered separately and their structure is built:

\[ \{a, b, c\} \]

5. The knowledge space is decomposed into separate modules, taking into account the parallelism or sequence of learning, and for each module the beginning of learning, \( \text{inf}(KS_i) \), and the end of learning (Figure 3);

6. Module connection is built (Figure 4);

7. Min elements of the main filter are defined: \( \text{min} \), \( \text{min} \), \( \text{min} \)
8. \( M_i \) is the maximum element \( KS^M \), if the set of main filter elements is empty and \( \sup (KS_i) = \sup \).

9. the end of mastering module \( i \) connects to the beginning of mastering module \( j \) for all \( \min M_{jj KS} \) and the main filter set is not empty;

10. max elements are defined:

\[
\max_{i=1}^{\infty} M_i = \ldots M_{\Delta M_i} \ldots M_k
\]

11. \( i \) is the minimum element \( KS^M \), if the set of elements \( \Delta M_i \) is empty and \( \inf (KS_i) = \inf \);

12. the start of module \( i \) is connected to the end of module \( k \) for all \( \forall i KS M \) and the main filter set is not empty.

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**Fig. 2.** Logi of interconnection of modules. 

**Fig. 3.** The individual module and its knowledge space.
Fig. 4. Modular knowledge space of the training course.

The generalised space $\mathcal{KS}$ is organised in such a way that it can be divided into smaller subspaces $\mathcal{M}_i$. If we consider a training course as a knowledge space, each module of this course can be represented by an element of the space $\mathcal{KS}$. If necessary, each module $\mathcal{M}_i$ (element of the space $\mathcal{KS}$) can be further decomposed into smaller subspaces that correspond to narrower topics within each module.

Consider the process used to partition the knowledge space. The algorithm is a method for partitioning a large knowledge domain into smaller and more manageable parts that are easier to learn and understand. For the part to be partitioned, one finds elements for which $\mathcal{C}_i \subset \mathcal{LC}_i$. This will be the smallest lattice corresponding to $\mathcal{C}_i$: $\mathcal{LC}_i \subset \mathcal{LC}$. The algorithm steps are as follows:

1. $a \sqsubseteq b \ldots \mathcal{C}_i \sqsubseteq \mathcal{LC}_i$
2. $\mathcal{C}_i \sqsubseteq a \sqsubseteq b \Rightarrow b = a$
3. $\mathcal{C}_i \sqsubseteq a \sqsubseteq b \Rightarrow a = \sup (\mathcal{KS}_i \Rightarrow \mathcal{KS})$
4. $b \mathcal{C}_i \sqsubseteq a \Rightarrow b \Rightarrow b = a$
5. $\mathcal{C}_i \sqsubseteq a \Rightarrow \sup (\mathcal{KS}_i \Rightarrow \mathcal{KS})$
6. $\mathcal{C}_i \sqsubseteq a \Rightarrow b = a$
7. $b \mathcal{C}_i \sqsubseteq a \Rightarrow \inf (\mathcal{KS}_i \Rightarrow \mathcal{KS})$
8. $b \mathcal{C}_i \sqsubseteq a \Rightarrow b = a$
9. $\mathcal{C}_i \sqsubseteq a \Rightarrow \inf (\mathcal{KS}_i \Rightarrow \mathcal{KS})$

$\mathcal{LC}_i \subset \mathcal{LC}$, the initial element of the structure development (Fig. 5).
3 Results and Discussions

Mastered subset: This is the set of elements that trainees have successfully mastered.

Unlearned subset: This includes elements that trainees have not yet mastered or fully learnt.

Mastery front: This is the set of elements that are in the process of being mastered, i.e., learners are actively working on mastering these elements but have not yet completed this action.

Each of these subsets (mastered, unlearned and front of learning) is formed based on the current state of the learner. It depends on which elements of the course the learners have already learnt, which elements they have not learnt yet, and which elements they are currently actively working on.

The learning front defines the set of all course elements that are currently available for learning. The control function, using the actions, allows you to move the learning front by performing various educational actions. When these actions are performed, the learning front moves towards the undeveloped course elements relative to the already mastered ones. The control function helps learners to focus on course elements that they do not yet know or that require further study by systematically moving towards these undeveloped elements.
4 Conclusions

The results of the study provide a better understanding of the modelling of this process. They have important implications for developing effective learning methods and improving educational practices. Further research in this area can expand the understanding of learning mechanisms and contribute to the development of new approaches to education.

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