1 Introduction

In technical and technological processes, the movement of fluids flowing through channels and pipes in an oscillatory flow depends on the rheological properties of fluids, which may cause unconventional hydrodynamic phenomena in some cases. Therefore, this article deals with the specific problem of the oscillatory flow of rheological complex fluids in a flat channel. The main goal is to study the movement of rheological complex fluids based on simplified mathematical models and to compare the obtained results with the existing hydrodynamic laws in the oscillating flow of a Newtonian fluid, and as a result, to determine the characteristics that differ from it, to develop a new hydrodynamic system depending on the hydrodynamic characteristics of rheological complex fluids is to create effects.

Stationary oscillating flows, in which transition processes do not appear in known fluid flows, are of particular interest in science, in the fields of agriculture related to fluid movement, and in technological processes. In such processes, even if the fluid movement occurs in a stationary mode, since there is an oscillatory motion, the considered process consists of a periodic function of time. In this case, it is considered that the fluid fluctuations occur in the same state in each period. Therefore, when solving problems related to the oscillating flow of a liquid, periodic functions of time can be used, this feature makes it much easier to solve the system of differential equations representing the oscillating flow of
rheological complex fluids, and in such cases it is not required to set the initial conditions for solving the system of differential equations.

Despite the fact that the cases considered as Newtonian fluids are sufficiently studied in the research studies of oscillatory flows in channels and pipes [1-3], very few research studies have been devoted to the flows of rheological complex fluids in this field. In this field, the research works conducted by applying Maxwell's generalized models to the oscillating flow of complex rheological fluids are insufficient. It is known that Maxwell's generalized models play an important role in characterizing the vibrational behavior of water-dissolved polymer solutions, turbid water mixtures, petroleum products, and other similar fluids. Therefore, in this article, based on Maxwell's generalized models, the problems of oscillating flows of rheological complex fluids in flat channels are studied.

2 Statement of the Problem and Solution Method

The distance between the walls of the flat channel in the formation of the problem $2h$, the channel length $L$ is defined as

Here $L$ is large enough that in this $h/L \ll 1$, the condition is assumed to be fulfilled. In such cases, the flow is stabilized and the transverse velocity value is equal to zero. And channel axes are defined as follows, $x$ the axis is directed along the middle of the channel, in the horizontal direction and is called the longitudinal axis, $y$ and the arrow $x$ is taken in the vertical direction perpendicular to the axis and is called the vertical axis. The change of the pressure gradient over time in a sinusoidal form is defined as follows

$$\frac{-\partial p}{\partial x} = \left( -\frac{\partial p}{\partial x} \right) + \left( -\frac{\partial p}{\partial x} \right) e^{i\omega t}$$

Notations are the same as in works [6-10]. Since the flow is symmetrical about the longitudinal axis of the flat channel, the boundary conditions are formulated as follows:

$$y = h \quad \text{at} \quad u = 0, \quad y = 0 \quad \text{at} \quad \frac{\partial u}{\partial y} = 0$$

In this problem, no initial conditions are required since the stabilized oscillatory flow of the fluid is considered. In general case (1), the change in pressure gradient can be given in complex form, where its real part represents the solution of the problem

$$\frac{-\partial p}{\partial x} = \left( -\frac{\partial p}{\partial x} \right) + \left( -\frac{\partial p}{\partial x} \right) e^{i\omega t}$$

The pressure gradient consists of two parts, i.e., the first part is constant and the second part is time-varying periodic function. Therefore, the solution of equation (2) is sought as

$$\begin{cases}
\frac{\partial u}{\partial t} = -\frac{\partial p}{\rho \partial x} + \frac{\partial \tau}{\rho \partial y} \quad \frac{\partial p}{\partial y} = \tau = \tau_s + \tau_p \\
\tau_s = \eta_s \frac{\partial u}{\partial y} + \lambda \frac{\partial \tau_p}{\partial t} + \tau_p = \eta_p \frac{\partial u}{\partial y} \quad \eta = \eta_s + \eta_p
\end{cases}$$
The sum of the following two functions. The first terms of the equations express the velocities in the stationary flow, the experimental stress and pressure change, and the second terms express the velocities in the oscillating flow and the experimental stress and pressure gradient change.

\[
\begin{align*}
  u &= u_0 + u_1 e^{i\omega t}, \\
  \tau &= \tau_0 + \tau_1 e^{i\omega t},
\end{align*}
\]

\[
\begin{align*}
  \frac{\partial p}{\partial x} &= \frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = \tau_s + \tau_p, \\
  \tau_s &= \eta_s \frac{\partial u}{\partial y}, \\
  \eta = \eta_s + \eta_p.
\end{align*}
\]

In this case, the solutions corresponding to the first term of the solution are the solutions defining the state of stationary flow, and the solutions corresponding to the real part of the second term are the solutions characterizing the flow in which oscillations occur. Based on this approach, it should be noted that the rheological complex fluid becomes a Newtonian fluid when studying the stationary flow of a rheological complex fluid that does not depend on time. Therefore, the stationary flow of the liquid consists of a Poiseuille flow. Oscillating flows of rheological complex fluids, the system of differential equations corresponding to the second terms of the unknowns in substitutions (2) looks like

\[
\begin{align*}
  i\omega \tau_p + \tau_p &= \eta_p \frac{\partial u}{\partial y}, \\
  \eta &= \eta_s + \eta_p.
\end{align*}
\]

Chand the conditions of ownership are defined as follows:

\[
\begin{align*}
  u &= h \quad \text{at} \quad y = 0, \\
  \frac{\partial u}{\partial y} &= 0.
\end{align*}
\]

Using the equations from the second to the fifth equation in the resulting system of equations (5), we create an equation of this form to determine the test voltage:

\[
\begin{align*}
  \frac{\partial^2 u}{\partial y^2} - \frac{\rho \omega}{\eta \eta^*(i\omega)} u &= \frac{\rho \omega}{\eta \eta^*(i\omega)} \frac{\partial p}{\partial x}, \\
  \eta^* i\omega &= \eta_s + \eta_p, \\
  X &= + Z, \\
  \eta^* = \eta^* + \eta \lambda.
\end{align*}
\]

\[
\begin{align*}
  u &= C_1 x + C_2 y + C_3 x i\alpha \sqrt{\eta^* \omega h} + C_4 x i\alpha \sqrt{\eta^* \omega h}, \\
  \eta &= \eta_s + \eta_p.
\end{align*}
\]
Since the non-homogeneous part of the equation is constant, its solution is sought in this form:

\[ u_A \cdot \frac{\partial u}{\partial y} \]

is the solution of the inhomogeneous part of equation (8).

\[ u = \frac{1}{\rho \omega} \frac{\partial p}{\partial x} \]

\[ u = C_i \sqrt{i \alpha \omega h} \cdot \sqrt{i \alpha \omega h} + \frac{1}{\rho \omega} \frac{\partial p}{\partial x} \]

\[ C = \frac{y}{\eta \omega} \]

\[ u_i \mid h = C \]

\[ \alpha = \frac{\sqrt{\omega \cdot h}}{v} = \frac{\eta}{\rho} \]

\[ u_i \mid = \frac{h}{\eta} + \frac{1}{\eta} \frac{\partial p}{\partial x} \left[ \text{real} \left( i \alpha \sqrt{i \alpha \omega h} \right) \right] \]

\[ \frac{u_i \mid}{u_i} = \frac{\partial p}{\partial x} \left[ \text{real} \left( i \alpha \sqrt{i \alpha \omega h} \right) \right] \]

\[ \frac{1}{\rho \omega} \frac{\partial p}{\partial x} \left[ \text{real} \left( i \alpha \sqrt{i \alpha \omega h} \right) \right] \]
3 Numerical calculation results and analysis

\[
\frac{u'(y)}{u_0} = \left( -\frac{v^2}{h} \right) + \Lambda W
\]

Where

\[
\Lambda = -\frac{\partial p}{\partial x}
\]

\[
W = \frac{\alpha ot}{\alpha } \left[ \left( \bar{A} \bar{M} \bar{y} \bar{M} \bar{y} - \bar{B} \bar{M} \bar{y} \bar{M} \bar{y} \right) \times \bar{W} \right]
\]

\[
\bar{W} = \bar{A} \bar{B} \bar{M} \bar{M} \bar{y}
\]

\[
y = \frac{y}{h}
\]

\[
i \alpha \left( \frac{\partial}{\eta \left( i \omega \right)} \right) = \left( -\frac{\alpha}{\sqrt{\phi + n\pi}} + \frac{\alpha}{\sqrt{\phi + n\pi}} i \right) \bar{G}_i + \bar{G}_i = -\frac{\alpha}{\sqrt{\phi + n\pi}} \bar{G}_i + \bar{G}_i
\]

\[
= -\frac{\alpha}{\sqrt{\phi + n\pi}} \left[ \bar{G}_i + \bar{G}_i - \bar{G}_i - \bar{G}_i \right] = -\frac{\alpha}{\sqrt{\phi + n\pi}} \bar{G}_i - \bar{G}_i
\]

\[
\left( \frac{\alpha}{\sqrt{\phi + n\pi}} \bar{G}_i + \frac{\alpha}{\sqrt{\phi + n\pi}} \bar{G}_i \right) = \bar{M} \bar{y} \bar{M} \bar{y} \bar{G}_i + \bar{G}_i \bar{G}_i = \bar{G}_i - \bar{G}_i
\]

\[
\bar{G}_i = \sqrt{\bar{G}_i + \bar{G}_i} \times \frac{\phi + n\pi}{\eta \left( i \omega \right)} \bar{G}_i = \sqrt{\bar{G}_i + \bar{G}_i} \times \frac{\phi + n\pi}{\eta \left( i \omega \right)} \bar{G}_i
\]

\[
\varphi = \arctg \frac{G_i}{G_i} = \arctg \frac{\varphi + n\pi}{\eta \left( i \omega \right)}
\]

\[
G_0 = \frac{\varphi + De X \alpha}{\alpha} X \alpha + iDe X \alpha - X \alpha = G_0 + G_i
\]

\[
G_0 = \frac{\varphi + De X \alpha}{\alpha} X \alpha + iDe X \alpha - X \alpha = G_0 + G_i
\]
These coefficients look very simple for a Newtonian fluid in particular:

\[
\eta \ast (i\omega) = \frac{\eta_p}{\eta} + \frac{\eta_p}{\eta} i\alpha = X + i\alpha
\]

\[
\therefore iDeX \alpha = \frac{\eta_p}{\eta} X = \frac{\eta_p}{\eta} i\alpha = \frac{\eta_p}{\eta} X + Z
\]

\[
De = \frac{\lambda \eta}{\rho h^2} \quad \alpha = \frac{\omega}{h}
\]

\[
\vec{A} = \frac{\lambda \eta}{\rho h^2} \quad \vec{B} = \frac{\lambda \eta}{\rho h^2} \quad \vec{y} = \frac{y}{h}
\]

\[
\alpha = \sqrt{\frac{\omega_h}{v}} \quad i\alpha = M_i h \quad \vec{M} = \frac{\alpha}{\sqrt{\omega}} \quad \vec{M} = \alpha
\]

\[
T_\alpha = \frac{\rho h^2}{\eta}
\]

\[
De = \frac{\lambda \eta}{\rho h^2} = \frac{\lambda}{T_\alpha}
\]

\[
\alpha = \frac{\rho h^2}{\eta}
\]

\[
De < \frac{\rho h^2}{\eta}
\]

\[
\alpha t =
\]

Fig. 1. Longitudinal speed generator on a flat channel cross-section do not vibrate in stream ($\omega t = 0, \alpha_0 = 1$ when; 1 line Newton fluid, 2-4 lines $De$ distribution of rheological complex liquid at different concentrations).
Fig. 2. Longitudinal speed generator on a flat channel cross-section do not vibrate in stream ($\omega t = 0, \alpha_0 = 2.5$ when; 1 line Newton fluid, 2-4 lines $De = 1$ distribution of rheological complex liquid at different concentrations).

Fig. 3. Longitudinal speed generator on a flat channel cross-section do not vibrate in stream ($\omega t = 0, \alpha_0 = 3$ when; 1 line Newton fluid 2-4 lines $De = 1$ distribution of rheological complex liquid at different concentrations).
Fig. 4. Longitudinal speed generator on a flat channel cross-section do not vibrate in stream \((\omega t = 0, \alpha_0 = 5)\) when; 1 line Newton fluid, 2-4 lines \(D_e = 1\) distribution of rheological complex liquid at different concentrations.

The occurrence of vibrational effects in the oscillating flow of a viscous fluid is due to fluid inertia and viscous fluid friction. At large values of the vibration frequency, the presence of inertia is important. Under the influence of the pressure gradient change, at large values of the oscillation frequency, due to the presence of the viscous properties of the liquid, the velocity changes in the layers close to the wall occur faster than in the core of the flow \[1, 14-16\]. As a result, the fluid velocity in front of the wall can change its direction due to the change in the pressure gradient sign, while the movement in the flow core due to inertia continues in the previous "old" direction. The occurrence of such a situation occurs in the profiles of velocity distribution in an oscillating flow \(M\) is explained by the appearance of a similar profile.

3 Conclusion

The article deals with the oscillatory flow of rheological complex fluids in a flat channel, where the rheological complex fluid is formed from a mixture of two fluids, the first fluid is taken as a Maxwell fluid and the second is a Newtonian fluid. Both fluid mixtures are represented in the form of a homogeneous model of a two-component fluid. Here, the differential equations of motion of homogeneous fluid mixtures were created, and the problem of the oscillating flow in a flat channel was solved analytically. Formulas for determining speed profiles are derived in the solution. With the help of derived formulas, distribution graphs of velocity profiles on the cross-section of the channel were determined depending on the change of the vibration frequency parameter, and appropriate hydrodynamic conclusions were drawn based on the research results.

References


