Oscillatory flow of rheological complex fluid in a flat channel

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Abstract. The article examines the oscillatory flow of rheological complex fluids in a flat channel, in which the rheological complex fluid was obtained based on the Maxwell model, and its mixture is described based on the Newtonian fluid model. Both mixtures of fluids are represented as a homogeneous model of a two-component liquid. In this case, differential equations of motion of homogeneous liquid mixtures are given. Based on this equation, the problem of oscillatory flows of rheological complex fluids in a flat channel is solved analytically. When solving, formulas are given for determining the longitudinal velocity. Using the obtained formulas, graphs of the longitudinal velocity distribution over the cross section of the channel are determined depending on the change in the oscillation frequency parameter, and with their help, appropriate conclusions are drawn.

1 Introduction

In technical and technological processes, the movement of fluids flowing through channels and pipes in an oscillatory flow depends on the rheological properties of fluids, which may cause unconventional hydrodynamic phenomena in some cases. Therefore, this article deals with the specific problem of the oscillatory flow of rheological complex fluids in a flat channel. The main goal is to study the movement of rheological complex fluids based on simplified mathematical models and to compare the obtained results with the existing hydrodynamic laws in the oscillating flow of a Newtonian fluid, and as a result, to determine the characteristics that differ from it, to develop a new hydrodynamic system depending on the hydrodynamic characteristics of rheological complex fluids is to create effects.

Stationary oscillating flows, in which transition processes do not appear in known fluid flows, are of particular interest in science, in the fields of agriculture related to fluid movement, and in technological processes. In such processes, even if the fluid movement occurs in a stationary mode, since there is an oscillatory motion, the considered process consists of a periodic function of time. In this case, it is considered that the fluid fluctuations occur in the same state in each period. Therefore, when solving problems related to the oscillating flow of a liquid, periodic functions of time can be used, this feature makes it much easier to solve the system of differential equations representing the oscillating flow of

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rheological complex fluids, and in such cases it is not required to set the initial conditions for solving the system of differential equations.

Despite the fact that the cases considered as Newtonian fluids are sufficiently studied in the research studies of oscillatory flows in channels and pipes [1-3], very few research studies have been devoted to the flows of rheological complex fluids in this field. In this field, the research works conducted by applying Maxwell's generalized models to the oscillating flow of complex rheological fluids are insufficient. It is known that Maxwell's generalized models play an important role in characterizing the vibrational behavior of water-dissolved polymer solutions, turbid water mixtures, petroleum products, and other similar fluids. Therefore, in this article, based on Maxwell's generalized models, the problems of oscillating flows of rheological complex fluids in flat channels are studied.

2 Statement of the Problem and Solution Method

The distance between the walls of the flat channel in the formation of the problem $2h$, the channel length L is defined as Here L is large enough that in this $h/L \ll 1$, the condition is assumed to be fulfilled. In such cases, the flow is stabilized and the transverse velocity value is equal to zero. And channel axes are defined as follows, x the axis is directed along the middle of the channel, in the horizontal direction and is called the longitudinal axis, y and the arrow x is taken in the vertical direction perpendicular to the axis and is called the vertical axis. The change of the pressure gradient over time in a sinusoidal form is defined as follows

$$\left(-\frac{\partial p(t)}{\partial x}\right)=\left(-\frac{\partial p_0}{\partial x}\right)+\left(-\frac{\partial p_1}{\partial x}\right)\cos\omega t \tag{1}$$

Here $\left(-\frac{\partial p_0}{\partial x}\right)$ - pressure gradient in stationary flow; $\left(-\frac{\partial p_1}{\partial x}\right)$ - the pressure gradient representing the change in the amplitude of oscillation in the oscillating flow; ω - vibration frequency; t - time [4-5].

The system of oscillating differential equations of rheological complex fluid, the simplified system of differential equations of motion is expressed as follows [6-10]

$$\begin{cases} \dfrac{\partial u}{\partial t}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}+\dfrac{1}{\rho}\dfrac{\partial \tau}{\partial y}, \quad \dfrac{\partial p}{\partial y}=0, \tau=\tau_s+\tau_p, \\ \tau_s=\eta_s\dfrac{\partial u}{\partial y}, \quad \lambda\dfrac{\partial \tau_p}{\partial t}+\tau_p=\eta_p\dfrac{\partial u}{\partial y}, \eta=\eta_s+\eta_p \end{cases} \tag{2}$$

Notations are the same as in works [6-10]. Since the flow is symmetrical about the longitudinal axis of the flat channel, the boundary conditions are formulated as follows:

$$y=h \ at \ u=0, \ y=0 \ at \ \partial u/\partial y=0 \tag{3}$$

In this problem, no initial conditions are required since the stabilized oscillatory flow of the fluid is considered. In general case (1), the change in pressure gradient can be given in complex form, where its real part represents the solution of the problem

$$\left(-\frac{\partial p(t)}{\partial x}\right)=\left(-\frac{\partial p_0}{\partial x}\right)+\left(-\frac{\partial p_1}{\partial x}\right)e^{i\omega t}.$$

The pressure gradient consists of two parts, i.e., the first part is constant and the second part is time-varying periodic function. Therefore, the solution of equation (2) is sought as

a sum of the following two functions. The first terms of the equations express the velocities in the stationary flow, the experimental stress and pressure change, and the second terms express the velocities in the oscillating flow and the experimental stress and pressure gradient change

$$u = u_0(y) + u_1(y)e^{i\omega t},\ \tau = \tau_0(y) + \tau_1(y)e^{i\omega t},\ \frac{\partial p(t)}{\partial x} = \frac{\partial p_0}{\partial x} + \frac{\partial p_1}{\partial x}e^{i\omega t} \quad (4)$$

In this case, the solutions corresponding to the first term of the solution are the solutions defining the state of stationary flow, and the solutions corresponding to the real part of the second term are the solutions characterizing the flow in which oscillations occur. Based on this approach, it should be noted that the rheological complex fluid becomes a Newtonian fluid when studying the stationary flow of a rheological complex fluid that does not depend on time. Therefore, the stationary flow of the liquid consists of a Poiseuille flow. Oscillating flows of rheological complex fluids, (4) the system of differential equations corresponding to the second terms of the unknowns in substitutions (2) looks like

$$\begin{cases} i\omega u_1 = -\dfrac{1}{\rho}\dfrac{\partial p_1}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial \tau_1}{\partial y},\ \tau_1 = \tau_S + \tau_p,\ \tau_S = \eta_S \dfrac{\partial u_1}{\partial y}, \\ i\lambda\omega\tau_p + \tau_p = \eta_p \dfrac{\partial u_1}{\partial y},\ \eta = \eta_s + \eta_p \end{cases} \quad (5)$$

Chand the conditions of ownership are defined as follows:

$$y = h\ at\ \ u_1 = 0, y = 0\ at \frac{\partial u_1}{\partial y} = 0 \quad (6)$$

Using the equations from the second to the fifth equation in the resulting system of equations (5), we create an equation of this form to determine the test voltage:

$$\tau_1(y) = \eta\left(\frac{\eta_s}{\eta} + \frac{\eta_p}{\eta}\frac{1}{1+i\omega\lambda}\right)\frac{\partial u_1}{\partial y}. \quad (7)$$

As a result of putting the found value (7) of the test voltage into the first equation of the system of equations (5), this equation is formed:

$$\frac{\partial^2 u_1}{\partial y^2} - \frac{\rho i\omega}{\eta\eta*(i\omega)}u_1 = \frac{1}{\eta\eta*(i\omega)}\frac{\partial p_1}{\partial x}. \quad (8)$$

Here

$$\eta*(i\omega) = \left(\frac{\eta_s}{\eta} + \frac{\eta_p}{\eta}\frac{1}{1+i\omega\lambda}\right) = \left(X_1 + Z_1\frac{1}{1+i\omega\lambda}\right)$$

Fundamental solutions of the homogeneous part of the defined equation (2.8)

$$\cos\left(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}}\frac{y}{h}\right) \quad \text{and} \quad \sin\left(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}}\frac{y}{h}\right)$$

consists of functions, and the general solution of the homogeneous part is found in the following form:

$$u_1(y) = C_1(x)\cos\left(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}}\frac{y}{h}\right) + C_2(x)\sin\left(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}}\frac{y}{h}\right). \quad (9)$$

Since the non-homogeneous part of the equation is constant, its solution is sought in this form:

$u_1^* = A$, in this case $\frac{\partial^2 u_1^*}{\partial y^2} = 0$ is the solution of the inhomogeneous part of equation (8). $u_1^* = \frac{1}{\rho i\omega}(-\frac{\partial p_1}{\partial x})$ will be Taking these into account, the general solution to the problem is determined as follows

$$u_1(y) = C_1(x)\cos(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}}\frac{y}{h}) + C_2(x)\sin(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}}\frac{y}{h}) + \frac{1}{\rho i\omega}(-\frac{\partial p_1}{\partial x}) \quad (10)$$

The unknown integral coefficients in the found solution (10) are found from the boundary condition (6). (10) on both sides of the solution y if we take the derivative according to and set it equal to zero $C_2 = 0$ it follows that $y = h$ and when $u_1(x,h) = 0$ from this condition C_1 coefficient is found.

$$C_1 = -\frac{1}{\rho i\omega}(-\frac{\partial p_1}{\partial x})\frac{1}{\cos(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}})} \quad (11)$$

C_1 and C_2 - as a result of putting the values of the coefficients in the solution (10), this solution is created:

$$u_1(y) = \frac{1}{\rho i\omega}(-\frac{\partial p_1}{\partial x})\left(1 - \frac{\cos(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}}\frac{y}{h})}{\cos(i^{\frac{3}{2}}\alpha_0\sqrt{\frac{1}{\eta^*(i\omega)}})}\right) \quad (12)$$

Here $\alpha_0 = \sqrt{\frac{\omega}{\nu}}h,\ \nu = \frac{\eta}{\rho}$.

Using (4) for the found solution (12), the final solution is determined as follows:

$$u_1(y,t) = \frac{h^2}{\eta}(-\frac{\partial p_1}{\partial x})\left[real\left[\frac{1}{i\alpha_0^2}\left(1 - \frac{\cos\left(i^{\frac{3}{2}}\alpha_0(\frac{1}{\eta^*(i\omega)})^{\frac{1}{2}}\frac{y}{h}\right)}{\cos\left(i^{\frac{3}{2}}\alpha_0(\frac{1}{\eta^*(i\omega)})^{\frac{1}{2}}\right)}\right)e^{i\omega t}\right]\right] \quad (13)$$

Dividing the solution (13) for the velocity by its maximum steady-state velocity yields a dimensionless solution of the following form:

$$\frac{u_1(y,t)}{u_{1\max}^0} = 2(-\frac{\partial \overline{p}_1(x)}{\partial x})real\left[\frac{1}{i\alpha_0^2}\left(1 - \frac{\cos\left(i^{\frac{3}{2}}\alpha_0(\frac{1}{\eta^*(i\omega)})^{\frac{1}{2}}\frac{y}{h}\right)}{\cos\left(i^{\frac{3}{2}}\alpha_0(\frac{1}{\eta^*(i\omega)})^{\frac{1}{2}}\right)}\right)e^{i\omega t}\right] \quad (14)$$

Here $(-\frac{\partial p_1}{\partial x})/(-\frac{\partial p_{10}}{\partial x}) = -\frac{\partial \overline{p}_1}{\partial x}$, $u_{1\max}^0 = \frac{1}{2\eta}\left(\frac{p_1^0 - p_1^L}{L}\right)h^2$. - Newtonian fluid is the maximum speed of stationary flow.

3 Numerical calculation results and analysis

As the formula (14) found as a result of solving the problem of the oscillating flow of a rheological complex fluid in a flat channel contains a complex variable, it is necessary to separate its real and abstract parts

$$\frac{u(y,t)}{u_{0\max}} = \left(1 - \frac{y^2}{h^2}\right) + 2\Lambda W \tag{15}$$

Here is the first term on the right $\left(1 - \frac{y^2}{h^2}\right)$ - the stationary component of the speed, the second term ΛW and oscillation is the quantity that determines the change in velocity in the flow, where $\Lambda = (-\frac{\partial p_1}{\partial x})$. W By splitting the real part of the expression, it becomes:

$$2W = 2\frac{\sin\omega t}{\alpha_0^2} - \frac{2}{\alpha_0^2\left(\overline{A}^2 + \overline{B}^2\right)}\Big[\left(\overline{A}\cos M_1\overline{y}\, ch\vec{M}_1\overline{y} - \overline{B}\sin M_1\overline{y}\, sh\vec{M}_1\overline{y}\right)\times \tag{16}$$

$$\times\cos\omega t + \left(\overline{B}\cos M_1\overline{y}\, ch\vec{M}_1\overline{y} + \overline{A}\sin M_1\overline{y}\, sh\vec{M}_1\overline{y}\right)\sin\omega t\Big].$$

Although the given formula (16) looks similar to the formula given in [2,5], here $\overline{A}$, $\overline{B}$, M_1, $\vec{M}_1$ coefficients are fundamentally different from each other, these coefficients are determined as follows for a rheological complex liquid:

$$\overline{A} = \sin M_1 sh\vec{M}_1, \quad \overline{B} = \cos M_1 ch\vec{M}_1, \quad \overline{y} = \frac{y}{h},$$

$$i^{\frac{3}{2}}\alpha_0\left(\frac{1}{\eta^*(i\omega)}\right)^{\frac{1}{2}} = \left(-\frac{\alpha_0}{\sqrt{2}} + \frac{\alpha_0}{\sqrt{2}}i\right)(\overline{G}_1 + \overline{G}_2 i) = -\frac{\alpha_0}{\sqrt{2}}(1-i)(\overline{G}_1 + \overline{G}_2 i) =$$

$$= -\frac{\alpha_0}{\sqrt{2}}\left[(\overline{G}_1 + \overline{G}_2) - (\overline{G}_1 - \overline{G}_2)i\right] = -\frac{\alpha_0}{\sqrt{2}}(\overline{\overline{G}}_1 - \overline{\overline{G}}_2 i) =$$

$$\left(-\frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_1 + \frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_2 i\right) = M_1 + \vec{M}_1 i, \; \overline{\overline{G}}_1 = \overline{G}_1 + \overline{G}_2, \overline{\overline{G}}_2 = \overline{G}_1 - \overline{G}_2,$$

$$M_1 = -\frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_1, \quad \vec{M}_1 = \frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_2, \quad \sqrt{\frac{1}{\eta^*(i\omega)}} = \sqrt{G_1 + G_2 i} = \overline{G}_1 + \overline{G}_2 i,$$

$$\overline{G}_1 = \sqrt{\sqrt{G_1^2 + G_2^2}}\cos\frac{\varphi + 2n\pi}{2}, \quad \overline{G}_2 = \sqrt{\sqrt{G_1^2 + G_2^2}}\sin\frac{\varphi + 2n\pi}{2}, \; n = 0,1$$

$$\varphi = arctg\frac{G_2}{G_1}, \qquad \frac{1}{\eta^*(i\omega)} = \frac{1 + De^2 X_1\alpha_0^4 + iDe\alpha_0^2(1 - X_1)}{1 + De^2 X_1^2\alpha_0^4} = G_1 + G_2 i$$

$$G_1 = \frac{1 + De^2 X_1\alpha_0^4}{1 + De^2 X_1^2\alpha_0^4}, \qquad G_2 = \frac{De\alpha_0^2(1 - X_1)}{1 + De^2 X_1^2\alpha_0^4}$$

$$\eta*(i\omega)=(\frac{\eta_s}{\eta}+\frac{\eta_p}{\eta}\frac{1}{1+iDe\alpha_0^2})=(X_1+Z_1\frac{1}{1+iDe\alpha_0^2})=$$

$$\frac{1+iDeX_1\alpha_0^2}{1+iDe\alpha_0^2},\ \frac{\eta_s}{\eta}+\frac{\eta_p}{\eta}=1\ X_1=\frac{\eta_s}{\eta},\ Z_1=\frac{\eta_p}{\eta},X_1+Z_1=1,$$

$$De=\frac{\lambda\eta}{\rho h^2},\qquad \alpha_0^2=\frac{\omega}{\nu}h^2.$$

These coefficients look very simple for a Newtonian fluid in particular:

$$\bar{A}=\sin M_1 sh\bar{M}_1,\quad \bar{B}=\cos M_1 ch\bar{M}_1,\quad \bar{y}=\frac{y}{h},$$

$$\alpha_0=\sqrt{\frac{\omega}{\nu}}h,\quad i^{\frac{3}{2}}\alpha_0=M_1+\bar{M}_1 i,\ M_1=-\frac{\alpha_0}{\sqrt{2}},\ \bar{M}_1=\frac{\alpha_0}{\sqrt{2}}.$$

Velocity change in the oscillating flow of a rheological complex fluid De-Debor number or varies depending on the elasticity number and it should be noted that it differs from the velocity of a Newtonian fluid. In this $T_{st}=\frac{\rho h^2}{\eta}$ with the characteristic time of the shear wave propagation of the viscous fluid in the channel, the number of elasticity is related to each other, i.e. $De=\frac{\lambda\eta}{\rho h^2}=\frac{\lambda}{T_{st}}$ due to the fact that in a flat channel elastic viscous fluid oscillating flows have two numbers: the number of elasticity De and Reynolds number of oscillation $\mathrm{Re}_\omega=\frac{\rho\omega h^2}{\eta}$ can be characterized by If $De<<1$, then the elasticity of the fluid does not affect the profile of the velocity distribution, that is, the velocity distribution in the oscillating flow is almost the same as in the Newtonian fluid. In Figure 1 $\omega t=0$ distribution of the longitudinal velocity depending on the frequency of oscillations in the oscillating flow of Newtonian and rheological complex fluids along the section of the channel [12-13] is shown. It can be seen from the figure that in Newtonian fluids in an oscillating flow, at increased values of the frequency of oscillations, the amplitude of the velocity oscillation decreases.

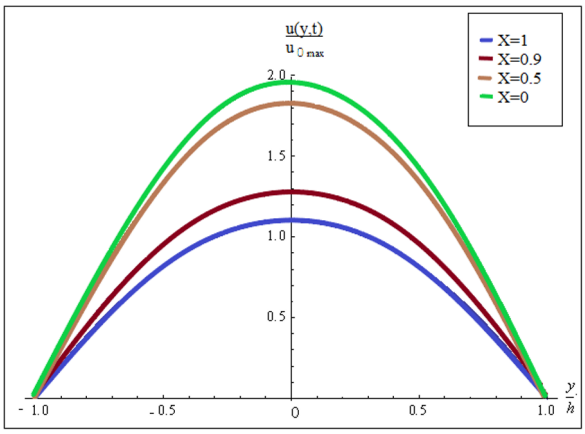

Fig. 1. Longitudinal speed generator on a flat channel cross-section do not vibrate in stream ($\omega t=0, \alpha_0=1$when; 1 line Newton fluid, 2-4 lines $De=1$distribution of rheological complex liquid at different concentrations).

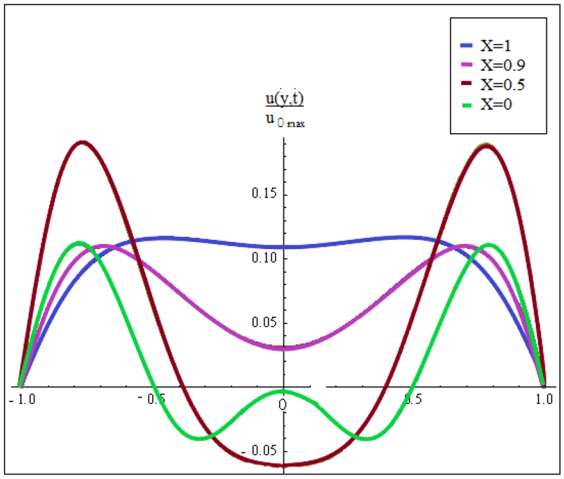

Fig. 2. Longitudinal speed generator on a flat channel cross-sectiondo not vibrate in stream ($\omega t = 0, \alpha_0 = 2.5$ when; 1 line Newton fluid, 2-4 lines $De = 1$distribution of rheological complex liquid at different concentrations).

In the oscillating flow of rheological complex fluids, the opposite occurs, i.e., the amplitude of the speed oscillation increases when the frequency of oscillations approaches a number. This situation is illustrated in graphs corresponding to lines 2-4 of Figure 1. But in a rheological complex fluid flow$De \geq 2$ when$M$- it is observed that similar velocity distributions appear even at positive values of the oscillation number, which are not large compared to the velocity distributions in Newtonian fluid flow (Fig. 2).

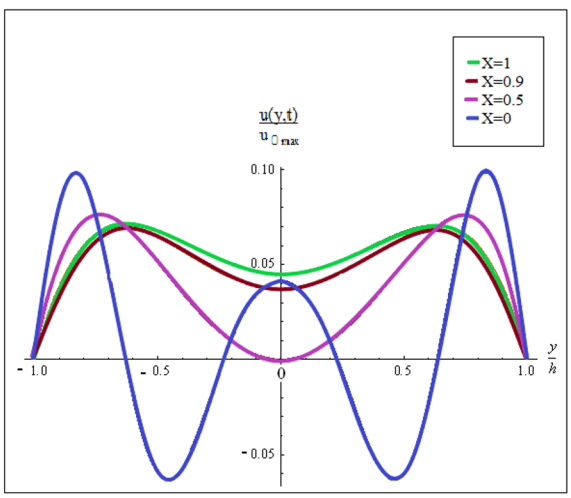

Fig. 3. Longitudinal speed generator on a flat channel cross-sectiondo not vibrate in stream ($\omega t = 0, \alpha_0 = 3$when; 1 line Newton fluid 2-4 lines$De = 1$distribution of rheological complex liquid at different concentrations).

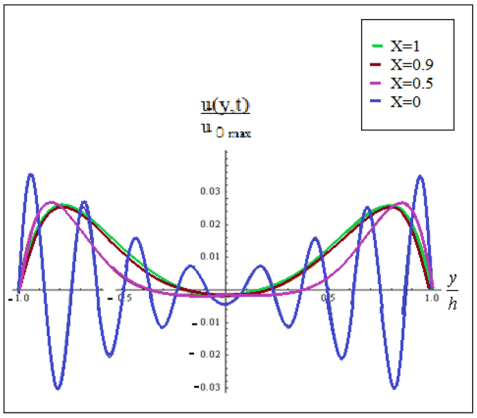

Fig. 4. Longitudinal speed generator on a flat channel cross-sectiondo not vibrate in stream ($\omega t = 0, \alpha_0 = 5$when; 1 line Newton fluid, 2-4 lines$De = 1$distribution of rheological complex liquid at different concentrations).

In a rheological complex fluid flow, longitudinal velocity distributions can occur differently in different sections, and even flows in opposite directions can be formed (Figures 3 and 4). The occurrence of vibrational effects in the oscillating flow of a viscous fluid is due to fluid inertia and viscous fluid friction. At large values of the vibration frequency, the presence of inertia is important.

Under the influence of the pressure gradient change, at large values of the oscillation frequency, due to the presence of the viscous properties of the liquid, the velocity changes in the layers close to the wall occur faster than in the core of the flow [14-16]. As a result, the fluid velocity in front of the wall can change its direction due to the change in the pressure gradient sign, while the movement in the flow core due to inertia continues in the previous "old" direction. The occurrence of such a situation occurs in the profiles of velocity distribution in an oscillating flow$M$– is explained by the appearance of a similar profile.

3 Conclusion

The article deals with the oscillatory flow of rheological complex fluids in a flat channel, where the rheological complex fluid is formed from a mixture of two fluids, the first fluid is taken as a Maxwell fluid and the second is a Newtonian fluid. Both fluid mixtures are represented in the form of a homogeneous model of a two-component fluid. Here, the differential equations of motion of homogeneous fluid mixtures were created, and the problem of the oscillating flow in a flat channel was solved analytically. Formulas for determining speed profiles are derived in the solution. With the help of derived formulas, distribution graphs of velocity profiles on the cross-section of the channel were determined depending on the change of the vibration frequency parameter, and appropriate hydrodynamic conclusions were drawn based on the research results.

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