# Evaluation of Airspace Operation Safety Level based on PMS

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**Abstract.**The point merge (PM) flight procedure is a new type of flight procedure of performance -based navigation (PBN). The capacity of the point merge procedure is evaluated through the parameters of the point merge flight procedure. Through the parameters of the point merge flight procedure and the parameters of the aircraft flying in the point merge flight procedure, the collision risk between two aircraft in the merge area and the overall collision risk of the aircraft in the point merge procedure are calculated. The evaluation methods and calculation formulas of these three parameters are summarized, and a reasonable method of airspace operation safety level evaluation based on point merge system (PMS) is sought. The inferred method of airspace operation safety level evaluation based on PMS is to compare the operation capacity with the calculated theoretical capacity, and compare the collision risk between two aircraft in the fusion area calculated according to track data and radar data and the overall collision risk of aircraft in PMS with the target safety level specified by China's air transportation industry.

## 1 INTRODUCTION

Point merge technology has attracted more and more attention in our country, and the research of point merge technology is gradually becoming the mainstream to solve the problem of large busy airports in our country. The ICAO cites point merge technology as part of the aviation system block upgrade and as a technology to support continuous descent operations (CDO). Based on the existing Precision Area Navigation (P-RNAV) technology, point merge technology optimizes and improves the traffic flow of the aircraft in the airspace, and relies on its unique sequencing arc and fusion zone to complete the orderly aircraft into the instrument approach program at intervals.

In 2016, Chaulmianbo et al.[1] designed a runway point merge program based on the basic concept of point fusion and considered the practical requirements of China's flight program design and control interval, and evaluated the modified program. The evaluation showed that the static capacity and descent gradient of the point merge program had been significantly improved. In 2021, starting with the use of point fusion, Bian Xiaofeng et al.[2] introduced the working principle, components and related technologies of PMS.

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Since the point merge airspace is located in the terminal area, the evaluation of the operational safety level of the airspace based on point merge is similar to that of the terminal area. In 2014, Wang Zhenfei[3] established a flight conflict risk calculation model for cross-traffic flow, converging traffic flow flight conflict and five-sided parallel flight conflict in the approach phase, which has certain reference value. Li Yike and other scholars[4] identified different types of risk scenarios based on parallel operation mode, and proposed a safety risk modeling method based on parallel approach operation and a risk probability measurement method based on Monte Carlo simulation experiment. In 2021, Yue Ruiyuan et al.[5] proposed a vertical collision risk assessment method based on the improved Event model for the aircraft collision risk at two adjacent altitude levels on the flight segment, and the resulting collision risk was in line with the safety target risk level stipulated by the International Civil Aviation Organization. The feasibility of quantitative assessment of flight collision risk was verified.

In 2019, Mitici Mihaela Blom, and AP Henk[6] conducted a comparative evaluation of previously proposed mathematical models for estimating air traffic collision and collision probability. Advances in evaluating air traffic collision and collision probability using uniform criteria and the use of directional mathematical models for air traffic collision and collision probability estimation in various air traffic applications are described, providing insight into the capabilities and limitations of mathematical models when evaluating future ATM designs.

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## 2 CAPACITY EVALUATION OF POINT MERGE SYSTEMS

### 2.1 Sort arc capacity evaluation

Capacity evaluation of sorting arc. Generally, the point merge flight program has two sort arcs, namely, the outer sort arc and the inner sort arc. According to the ideal case of maximum flow, the interval between aircraft on the sort arc is the minimum approach radar interval. However, due to the requirement of safety redundancy and tangential effect, the distance between the front and rear aircraft entering the sequence arc is larger than the minimum distance of the approach radar in practice, and the distance between the front and rear aircraft is uncertain.

Starting from a sequencing arc, assuming that this sequencing arc has a total of n segments, a total of k aircraft arriving before and after, the evaluated point fusion has a total of m sequencing arcs. Let Rs be the number of flights running simultaneously on a single sort arc, and let Rsi (i=1,... m) is the single capacity of the i th sort arc in the point fusion system, and Rq is set to be the number of flights accommodated on a single sort arc.

When $\frac{\sum_1^{k-1} D_{k-1}}{k-1} \geq L_R$,

$$R_q = \frac{S_n(k-1)}{\sum_1^{k-1} D_{k-1}} \quad (1)$$

Where Si (i=1, ... , n) represents the available arc length of the i th ordering arc; Di (i=1,… k-1) indicates the actual interval between the i aircraft and the preceding aircraft (i.e. i-1) during the point-of-entry fusion procedure, both in kilometers.

When $\frac{\sum_1^{k-1} D_{k-1}}{k-1} < L_R$,

$$R_q = \frac{S_n}{L_n} \quad (2)$$

Where LR represents the approach radar interval, in kilometers.

From this, we can calculate the capacity of each sort arc in one hour:

$$R_s = \frac{P_n}{C_a} + R_q \quad (3)$$

Where $P_i$ (i=1,... ,n) represents the proportion of the flow rate of the sequence arc in the i segment to the total flow rate; $C_i$ (i=1,… k-1) indicates the flight arrival concentration of the i aircraft and the preceding aircraft (i.e. i-1). The latter is measured in hours.

The theoretical capacity of the ordering arc of a point fusion system is

$$CAP_1 = \sum_1^n \left(\frac{P_n}{C_a} + R_q\right) \quad (4)$$

### 2.2 Merge area capacity evaluation

When the number of aircraft is small, the controller can issue instructions to the aircraft to fly directly to the fusion point. When there are more aircraft, we should follow the operation method of point fusion to command, and make full use of the sequence arc and distance ring contained in the program. In the operation process, the relationship between the number of flights in the fusion zone and the upper capacity of the point merge program is of great significance to the safety evaluation of the point merge flight program.

The static capacity of the merge area is

$$C_{static} = \frac{L}{L_R} \quad (5)$$

In the formula， Cstatic round down. Where L represents the length of the envelope of the point merge program, in kilometers.

The flight time of the aircraft from the inner sequence arc to the convergence point is

$$T_{inside} = \frac{L}{V_{pms}} \quad (6)$$

Where Vpms represents the envelope length of the point merge program, in kilometers.

The flight time of the aircraft from the outer sequence arc to the convergence point is

$$T_{outside} = \frac{L+L_d}{V_{pms}} \quad (7)$$

$L_d$ represents the horizontal distance of two sorting arcs in kilometers.

Assuming that when the front plane passes through the first distance loop satisfying the approach radar interval, there is just a rendezvous point on the sort arc where the aircraft can fly directly, then the maximum capacity per hour is

$$CAP_2 = \frac{1}{\left(\frac{L}{V_{pms}} + \frac{L_R - L_D}{V_{pms}}\right)} * C_{static} \quad (8)$$

### 2.3 Merge point capacity evaluation

The theoretical capacity of the merge point mainly depends on two aspects: on the one hand, the capacity of the runway, in the approach, that is, the landing capacity, usually expressed by the five-sided landing interval, and on the other hand, the capacity of the entire point merge system for five-sided service flights.

The choice of landing interval on the five sides of the runway is

$$T_{max} = \max\left\{T_0, \frac{60L_R}{V_{pms}}\right\} \quad (9)$$

In the formula, To represents the landing time interval on the five sides of the runway, in minutes.

When calculating the hourly capacity accepted by the fusion point, the limitations that need to be considered are the minimum hourly capacity of the five-sided interval, the time interval, and the fusion zone interval, respectively. Therefore, the capacity of the fusion point is

$$CAp_3 = \min\left\{\frac{V_{app}}{L_{land}}, \frac{60}{T_{max}}, \frac{V_{pms}}{L_R}\right\} \quad (10)$$

Where Vapp denotes the average speed of the aircraft on the five sides of the approach in kilometers per hour. Lland denotes the five-sided landing distance interval of the runway in kilometers. Tmax denotes the integrated time interval of the fusion point when the five-side interval is measured in time, in minutes.

For point fusion procedures at airports implementing independent parallel approach modes, when a point fusion system provides the flow of arriving flights for n runways operating in an independent parallel approach mode, the mode will typically have 1/n of the five-side intervals less than the approach radar intervals when compared to a single runway operating mode, so the capacity is limited by the approach radar intervals in this case.

Assuming that the number of runways that can be used for independent parallel approaches is n, the choice of the landing time interval on the five sides of the runway is then

$$T_{max}=\max\left\{\frac{T_0}{n},\frac{60L_R}{V_{pms}}\right\} \quad (11)$$

The capacity of the merge point at this point is

$$CAp_3=\min\left\{,\frac{60}{T_{max}},\frac{nV_{app}}{L_{land}},\frac{V_{pms}}{L_R}\right\} \quad (12)$$

### 2.4 Point Merge Program Capacity Evaluation

Combining all the above capacity constraints on sorting arcs, fusion zones, and fusion points accordingly, i.e., $CAP_1$, $CAP_2$, $CAP_3$, the capacity of the entire point merge program is

$$CAP_{pms}=\min\{CAP_1,CAP_2,CAp_3\} \quad (13)$$

In the airspace operation safety level assessment based on point merge, the limitation of the capacity of the point fusion program is manifested as the limitation of the peak hourly sorties of the point merge program, and the actual operation sorties shall not reach or be higher than the theoretical capacity, or else it can be regarded that the operation of the point merge program in this unit of time is unreasonable or unsafe. On this basis, for the consideration of safety margin, a safety capacity or warning capacity should be set on the basis of the original theoretical capacity, and after reaching this capacity, it is necessary to consider whether to implement traffic control and other means to ensure the safe operation of the point merge flight program.

## 3 POINT MERGE SYSTEM COLLISION RISK ASSESSMENT

### 3.1 Collision risk between two aircraft in the fusion zone

Aircraft usually meet the specified interval during the flight of point fusion procedure, but due to many factors such as navigation position error, human factors of controllers and pilots, wind shear environmental factors, etc., it will lead to deviation from the expected flight trajectory, and ultimately lead to the flight position error of aircraft, which ultimately leads to the possibility of the emergence of the two aircrafts are less than the specified interval or even collision between them.

Aircraft flight position error is the distance from the baseline when the aircraft flies according to the flight procedure, i.e., the distance between the actual flight position of the aircraft and the planned position, and these errors will affect the flight status of the aircraft, which will lead to the risk of inter-aircraft collision.

Firstly, we study the longitudinal collision risk, the longitudinal direction is denoted by x, and the variance of the longitudinal error is denoted by σx2. Assuming that these factors affecting the positional error are random, the longitudinal error can be considered to obey a normal distribution, which can be expressed as:

$$f(x)\sim N\left(\mu_X,\sigma_X^2\right) \quad (14)$$

From the above equation, the longitudinal error of aircraft i at time t can be introduced as:

$$\varepsilon_{ix}(t)\sim N\left(\mu_{ix},\sigma_{ix}^2\right) \quad (15)$$

Where i = 1,2, denoting the 1st and 2nd aircraft, respectively, and the x-direction denotes the longitudinal direction.

The longitudinal distance of aircraft i from a reference point at time t is denoted by dix(t). The actual position of aircraft i in the longitudinal direction can be denoted as Xi(t)=dix(t)+ εix(t). The actual longitudinal separation between the two aircraft can be denoted as:

$$X_1(t)-X_2(t)=\left(d_{1x}(t)+\varepsilon_{1x}(t)\right)-\left(d_{2x}(t)+\varepsilon_{2x}(t)\right) \quad (16)$$

The significance of d1x-d2x is the lateral distance between the two airplanes, and because of ε1x (t)~N (μ1x, $\sigma_{1x}^2$), ε2x (t)~N (μ2x,$\sigma_{2x}^2$), it can be obtained that ε1x (t)-ε2x (t)~N (μ1x+μ2x, $\sigma_{1x}^2+\sigma_{2x}^2$), then the actual distance between the two airplanes longitudinally at time t can be expressed as:

$$X_1(t)-X_2(t)=\Delta X(t)+\varepsilon_{1x}(t)-\varepsilon_{2x}(t)\sim N\left(\Delta X(t)+\left(\mu_{1x}-\mu_{2x}\right),\sigma_{1x}^2+\sigma_{2x}^2\right) \quad (17)$$

Let the fuselage length of the airplane be g, the wingspan be w, and the fuselage height of the airplane be h. The longitudinal collision risk of the two aircraft at moment t can be obtained as:

$$P_x(t)=\frac{1}{\sqrt{2\pi\left(\sigma_{1x}^2+\sigma_{2x}^2\right)}}\int_{-g}^{g}\exp\left[-\frac{\left(x-(\Delta X(t)+\mu_{1x}-\mu_{2x})\right)^2}{2\left(\sigma_{1x}^2+\sigma_{2x}^2\right)}\right]dx \quad (18)$$

Then by the same reasoning, we can get the lateral collision risk and vertical collision risk between the two aircrafts at moment t, respectively:

$$P_y(t)=\frac{1}{\sqrt{2\pi\left(\sigma_{1y}^2+\sigma_{2y}^2\right)}}\int_{-w}^{w}\exp\left[-\frac{\left(y-\left(\Delta Y(t)+\mu_{1y}-\mu_{2y}\right)\right)^2}{2\left(\sigma_{1y}^2+\sigma_{2y}^2\right)}\right]dy \quad (19)$$

$$P_z(t)=\frac{1}{\sqrt{2\pi\left(\sigma_{1z}^2+\sigma_{2z}^2\right)}}\int_{-h}^{h}\exp\left[-\frac{\left(z-(\Delta Z(t)+\mu_{1z}-\mu_{2z})\right)^2}{2\left(\sigma_{1z}^2+\sigma_{2z}^2\right)}\right]dz \quad (20)$$

The collision risk between two aircraft is the product of the longitudinal risk, the lateral risk and the vertical collision risk, then the collision risk between two aircraft at moment t is:

$$P(t)=P_X(t)\times P_Y(t)\times P_Z(t) \quad (21)$$

### 3.2 Overall collision risk for aircraft within the point merge program

#### *3.2.1 Aircraft collision probability analysis within the merge zone*

It is assumed that there are N airplanes flying in the airspace from the sorting arc to the merge point in the point merge flight program, and the motions of the airplanes are independent of each other and obey a uniform distribution. The operation of the airplanes in the point merge airspace is decomposed into vertical and horizontal velocities. For the convenience of the study, it is assumed that the two velocities do not affect each other, are relatively independent of each other, and that the motion of the airplanes does not respond to collisions.

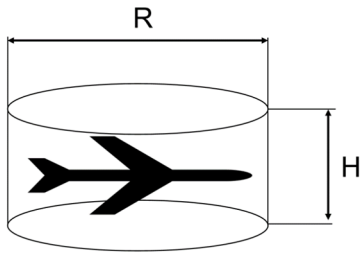


**Fig 1** Cylinder representing the volume of an airplane

As shown in Fig. 1, each aircraft is represented as a cylinder, with H as the height of the fuselage, R as the length of the fuselage and the maximum value of the wingspan of the aircraft.

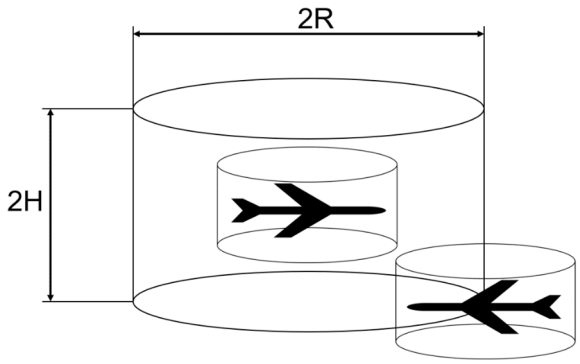


**Fig. 2** Collision between two airplanes

The definition of a collision needs to be clarified, i.e., a collision is judged to have occurred when the center of the cylinder of one aircraft enters a cylinder twice the size of the straight cylinder of the other aircraft, as shown in Figure 2. The definition of the number of collisions per unit of time for an airplane is based on the same principle as the calculation of the less-than-interval risk.

When the two aircraft R, H parameters are the same when an aircraft in a unit of time the number of collisions is

$$\left(\pi R^2 E|V_{rver}|\right)+4RHE(V_{rhor})\times\frac{n}{V_{pms}}a \qquad (22)$$

Where n indicates the number of aircraft in the fusion area, in units of racks; R indicates the maximum value of fuselage length and aircraft wingspan, H indicates the height of the fuselage, in units of meters; Vpms indicates the volume of airspace in the fusion area, in units of square meters; α indicates the correction coefficient of the risk of collision; $V_r$ indicates the relative speed, Vrver indicates the vertical relative speed, and Vrhor indicates the horizontal relative speed, in units of meters per second.

Since there are a total of n aircraft in the fusion area of the point fusion flight program, the total collision probability of all is

$$CR = (\pi R^2 E|V_{rver}|)+4RHE(V_{rhor})\times\frac{n^2}{2V_{pms}}a(23)$$

When the types of aircraft are different (i.e., when the values of R and H are not the same), the collision rate between the two types of aircraft is

$$CR'=\left(\pi R^2 E|V_{rver}|\right)+4RHE(V_{rhor})\times\frac{n_1\times n_2}{V_{pms}}a(24)$$

Where $n_1$ denotes the number of the first type of aircraft in the convergence part of the point fusion program, $n_2$ denotes the number of the second type of aircraft in the convergence part of the point merge program, in units of aircraft; R denotes the maximum value of the length of the calculated fuselage and the wingspan of the aircraft, and H denotes the height of the fuselage of the aircraft being evaluated, in units of meters; Vr denotes the relative speed of the aircraft being evaluated, Vrver denotes the relative vertical speed of the aircraft being evaluated, and Vrhor denotes the relative horizontal speed of the aircraft being evaluated, in units of meters per second.

### *3.2.2 Calculus Analysis*

This method utilizes Matlab for calculations and simulations. Let the descent rate of the aircraft in the fusion zone be 1.8% (the trajectory is from the outer sorting arc to the fusion point), the collision risk correction is 1.83e-8, the fuselage length of the aircraft is 38 meters, the fuselage height is 13 meters, and the wingspan of the aircraft is 34 meters. The volume of airspace in the merge zone is taken as the volume of airspace within 150 meters above and below the height of the merge zone procedure.

The results of the calculations based on equation 4-4 are shown in the table 1 below:

**Table 1** Crash Risk Calculations

| Aircraft traffic entering procedures (sorties/hour) | Collision risk (accidents/flight hour) |
|---|---|
| 20 | 4.00e-9 |
| 30 | 9.00e-9 |
| 40 | 3.20e-8 |

Aircraft traffic entering the point merge procedure is within 20-40 sorties/hour, and the aircraft collision risk meets our prescribed safety target level of 1.5e-7 accidents/flight hour.

First, in the point-merge flight procedure, it is mentioned that a number of factors such as navigation position errors, human factors of air traffic controllers and pilots, and wind shear may cause aircraft to deviate from their intended flight paths, thus increasing the likelihood of collisions between aircraft. By investigating the longitudinal collision risk, the longitudinal position errors of the aircraft were considered, which were assumed to conform to a normal distribution. By calculating the actual longitudinal distance between the airplanes and by taking into account the airplane's fuselage length, wingspan, and fuselage height, the longitudinal collision risk between two airplanes at a given point in time can be obtained. Similarly, the calculation of lateral collision risk and vertical collision risk is mentioned.

Next, the method of analyzing the overall collision risk of all aircraft in a point-merge flight procedure is presented. It is assumed that there are N airplanes flying in the point-merge region, and the motions of these airplanes are independent of each other and obey a uniform distribution. To facilitate the study, the motion of the aircraft is decomposed into vertical and horizontal velocities and it is assumed that these two velocities are independent of each other and that the motion of the aircraft does not react to a collision. By representing the airplane as a cylinder and defining the determination

conditions of collision, the number of collisions of the airplane per unit time can be calculated. According to the number of different types of airplanes, the size of the airplanes, and the relative speeds and other parameters, the collision rate between different types of airplanes can be calculated.

Finally, based on a specific case study. Taking the standard instrument approach to runway 33/34 of Shenzhen Bao'an Airport as an example, the parameters such as the speed and descent rate of the aircraft in the merging area are given, and the collision risk under the flow of different approaching airplanes is calculated. According to the calculation results, the collision risk of an airplane meets the predefined safety target level when the approaching aircraft flow is between 20-40 sorties/hour.

## 4 CONCLUDE

1. The three indicators, namely, the capacity assessment of the merge procedure, the collision risk assessment between two aircraft within the merge area, and the overall collision risk assessment of aircraft within the point merge procedure, can be used to assess the level of safety of airspace operations based on point merge. In practice, the airspace operation safety level of the point fusion program can be assessed from the perspective of these three indicators based on the static data parameters of the point merge as well as the historical radar data.

2. Based on the relative position mathematical model of the two aircraft in the merge area, the position correction of the collision risk of the two aircraft can effectively improve the calculation accuracy, taking into account the safety and efficiency of air traffic.

3. The overall collision risk of aircraft in the point merge procedure at runway 33/34 of Shenzhen Airport is evaluated and the results show that the probability of collision risk is in line with the target safety level for transportation aviation in China.

4. Shortcomings and prospects.In the article, only the collision risk of two airplanes in the merging area is evaluated, and the collision risk of multiple airplanes flying in the merging area at the same time is not considered. The collision risk assessment method for multiple aircraft can be further investigated and corresponding solutions can be proposed to ensure flight safety in high-density traffic situations.

In addition, the study in the article is limited to the air transportation industry in China, and it can be expanded to other countries and regions to compare the safety level of point merging systems in different regions and propose corresponding improvement measures.

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