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Kinematics and traction properties of the V-belt transmission

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Abstract. The wide application of V-belt transmissions in various types of machinery requires analysing the operation of these transmissions, including consideration of their kinematic and force parameters. In this work it is executed with application of the spatial theory of interaction of a V-belt with pulleys of the transmission - the mechanism of movement of an element of a belt in a groove of a pulley, its sliding and forces acting on an element of a belt are considered. The methodology of kinematic and force parameters determination for V-belt transmission is given.

1 Introduction

V-belt transmissions are widely used in agricultural machinery, in drives of aggregates of automobiles and tractors, in various mechanisms of industrial purpose.

Theoretical and practical issues of the operation of these gears are considered in detail in the studies of Pronin B.A. [1], Virabov R.V. [2, 3], Herbert W. [4], as well as, for example, in works [5-9].

Among a large number of such studies, we can single out the works of Virabov R.V., in which the spatial theory of interaction of the V-belt with the pulleys of the transmission is applied to analyse the operation of the V-belt transmission.

In the V-belt transmission, unlike the flat-belt transmission, the normal forces between the pulley and the belt when the latter runs over the pulley are developed in the process of belt sliding into the pulley groove. This is accompanied by transverse compression of the belt and its sliding against the pulley, where both the normal forces and the friction forces that prevent the belt from sliding are simultaneously increasing.

2 Materials and methods

The realisation of circumferential tangential forces in contact with the pulley of the V-belt element is only realised as a result of the rotation of the total friction forces arising already during the push-in of the advancing belt element, with their direction approaching the circumferential direction [3]. The belt element slides into the pulley groove under the action

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of the force $Fd\alpha$ (Fig. 1), the shear force for this element is the tension difference dF in its boundary cross-sections.

If the velocity of the belt branch running into the pulley is equal to the circumferential velocity of the pulley at the design radius, the belt element is pushed into the groove of the driving pulley by the force of $F_1 d\alpha$ (for idler pulley $F_2 d\alpha$), will reach the corresponding initial design radius $R_{1H}(R_{2H})$, travelling radially in the pulley groove, without a circumferential component in relation to speed. With this movement, frictional forces act on the belt element only in the axial plane of the pulley and the tension in its boundary cross-sections remains the same. In this case, there is no force shearing the element, and there is no reason for circumferential components of friction forces to arise in contact between the element and the pulley as the element moves across the entire girth arc.

If there is a difference between the velocity of the advancing belt branch and the circumferential velocity of the pulley at the initial design radius, the belt element, as it slides into the pulley groove, will reach the initial design radius with a circumferential component of relative velocity. This will give rise to circumferential components of the friction forces between the belt element and the pulley with a corresponding tension differential dF in the boundary cross-sections of the element. As a result, at the initial design radius the element will be under the influence of not only the thrust force $Fd\alpha$, but also the shear force dF (Fig. 1).

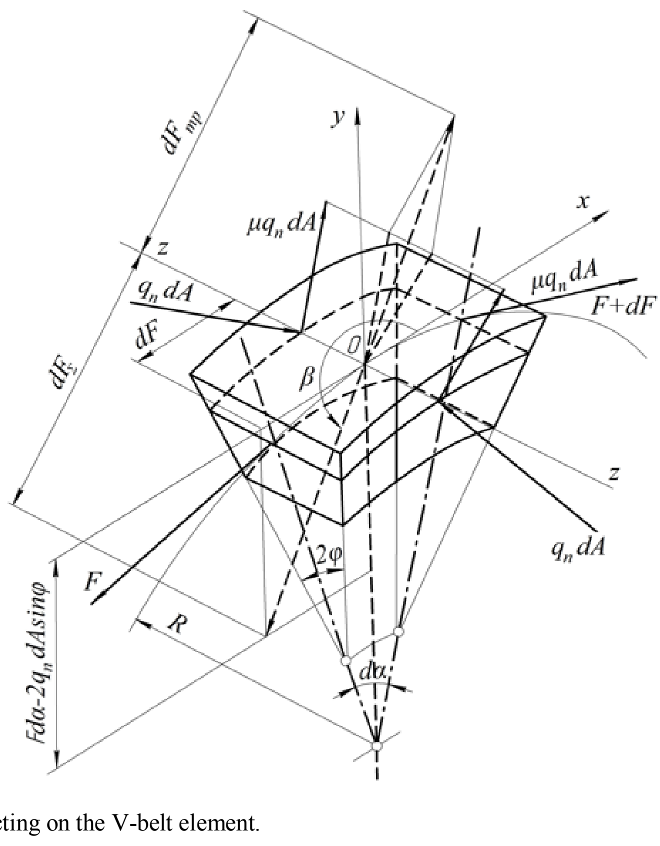

Fig. 1. Forces acting on the V-belt element.

According to the spatial theory of the wedge stated in [10], the element in this position is not in equilibrium, because the equidistance of active forces dF in combined action of forces$Fd\alpha$ and dF are greater than the equalities of the active forces under the action of the push-in force alone. Therefore, the element will continue to be pushed in at an angle to the radius even when the element is moving with the pulley on the circumferential arc. During this push-in, the circumferential components of the friction forces will cause a change in the belt tension with a corresponding change in its absolute velocity.

The belt movements relative to the pulley on the girth arcs of the driving and driven pulleys due to the different changes in belt tension are of a different nature [3]. On the driven pulley, where the belt tension increases and the driving $Fd\alpha$ and shearing dF element of force increases monotonically and the radial component of the active forces (Fig. 1) is directed towards the centre of the pulley over the entire girth arc. Therefore, here the sliding of the belt element is accompanied by its sliding into the pulley groove over the entire girth arc.

The radial component of the equidistant active force for the element is equal to $Fd\alpha - 2q_n dA \sin\varphi$, where q_n defines pressure on lateral surfaces of the element, dA is the lateral surface area of the element, which can be assumed constant on each pulley and approximated by the belt height and the corresponding initial radius:

$$dA = \frac{hR_H}{\cos\varphi}. d\alpha$$

The girth arc can be divided into two parts on the drive pulley, where the belt tension decreases. In the first part of the arc, the sliding of the belt element under the action of the increasing shear force and the decreasing insertion force is accompanied by the element sliding into the pulley groove, because the radial component of the increasing equidistance of the active forces of the element during insertion is directed towards the centre of the pulley. However, as the belt tension decreases and the pressure on its lateral surfaces increases, this radial component decreases and may become zero at some point in the girth arc. The sliding of the belt element in the pulley groove under the action of the shear force limiting for the given position will cause a further reduction of the belt tension with a corresponding decrease of the force pushing in the element $Fd\alpha$. This should cause the belt to extend during its sliding relative to the pulley, since the radial component of the active forces of the element is directed away from the centre of the pulley.

To determine the axial forces on the pulleys of the V-belt transmission, the equations obtained in [2, 11], which describe the process of circumferential force transmission by the V-belt from the position of the spatial theory of the wedge, can be used.

The following notations are adopted in this paper:

$R_{1(2)}$ is design design radius (the radius of the pulley where the groove width is equal to the width of the neutral layer of the undeformed belt). Here and in the following, index 1 refers to the drive pulley, index 2 to the driven pulley;

$r_{1H(2H)}$ is the radius of the neutral layer of the belt in the grooves of the drive and driven pulleys at idle speed (radius of the neutral layer of the V-belt element in the pulley groove, obtained from the condition of purely radial pushing of the element into the pulley groove);

r, dr is current radius of the neutral belt layer location and its increment;

b_p – design belt width (width of the neutral layer of the undeformed belt);

h – belt height;

A – V-belt cross-sectional area;

E_p – reduced modulus of elasticity of the V-belt in longitudinal pressure;

E_c – reduced modulus of elasticity of the V-belt in transverse compression;

φ – pulley groove profile angle;

f – friction coefficient between belt and pulley materials;

P– specific pressure on the working surface of the belt;

Ψ defines traction coefficient;

τ, $d\tau$ – the current circumferential displacement of the belt element relative to the pulley and its incremental movement;

ξ_1, ξ_2 – relative speed loss when the belt runs over the gear pulleys;

α – main girth arc (full girth arc minus entry and exit arcs);

$d\alpha$ – elementary girth arc angle, girth angle increment;

F_0 – belt pre-tensioning force (initial tension);

F_1, F_2 – belt tension in drive and driven branches;

F, dF – belt tension force at the current point of the girth arc and its increment;

$F_{z1(2)}, dF_z$ – axial force on the driving and driven pulleys and its increment.

The present method of calculating V-belt gears in order to determine the axial forces acting on the girth arcs of the gear pulleys assumes the following input data for the calculation:

$R_{1(2)}, \varphi,\ f, F_{1(2)},\ b_p,\ h, A, E_c,\ \xi_{1(2)}, \alpha_{1(2)}$;

The following systems of equations can be used in the calculation:

a) for the driving unit:

$$dF = (-\frac{1}{\cos\left(\frac{\varphi}{2}\right)}\sqrt{4h^2 \cdot r_{1H}^2 \cdot \left(f^2 - tg^2\frac{\varphi}{2}\right)p^2 + 4tg\frac{\varphi}{2} \cdot h \cdot r_{1H} \cdot p \cdot F - F^2})d\alpha; \quad (1)$$

$$d\tau = \left[1 - \frac{r}{r_{1H}\,(1-\xi_1)}\left(1 + \frac{F_1 - F}{E_p A}\right)\right]d\alpha; \quad (2)$$

$$dr = \frac{d\tau(2tg\frac{\varphi}{2} \cdot r_{1H} \cdot h \cdot p - F)}{dF}d\alpha; \quad (3)$$

$$p = \frac{E_c(R_{1(2)} - r)\sin\varphi}{b_p} + \frac{F\sin\varphi}{4h \cdot r_{1H}}; \quad (4)$$

$$dF_z - (\frac{h \cdot r_{1H} \cdot p}{cos^2\frac{\varphi}{2}} - \frac{F\sin\varphi}{4h \cdot r_{1H}})d\alpha. \quad (5)$$

b) for the driven unit:

$$dF = (\frac{1}{\cos\left(\frac{\varphi}{2}\right)}\sqrt{4h^2 \cdot r_{2H}^2 \cdot \left(f^2 - tg^2\frac{\varphi}{2}\right)p^2 + 4tg\frac{\varphi}{2} \cdot h \cdot r_{2H} \cdot p \cdot F - F^2})d\alpha; \quad (6)$$

$$d\tau = \left[1 - \frac{r(1-\xi_2)}{r_{2H}}\left(1 + \frac{F_{12} - F}{E_p A}\right)\right]rd\alpha; \quad (7)$$

$$dr = \frac{d\tau(2tg\frac{\varphi}{2} \cdot r_{2H} \cdot h \cdot p - F)}{dF}d\alpha; \quad (8)$$

$$p = \frac{E_c(R_2 - r)\sin\varphi}{b_p} + \frac{F\sin\varphi}{4h \cdot r_{2H}}; \quad (9)$$

$$dF_z = (\frac{h \cdot r_{2H} \cdot p}{cos^2\frac{\varphi}{2}} - \frac{F \cdot \sin\frac{\varphi}{2}}{2})d\alpha. \quad (10)$$

It is obvious that, summing up the elementary axial forces dF_z it is possible to determine the total value of the axial force compressing the belt within the main section of the girth arc of each pulley.

To integrate the differential equations included in these systems, either initial or boundary conditions are required, i.e. values of the required value at both ends of the interval (section) of the arc of girth in which the problem of integration of the corresponding equation is considered.

When solving the problem, assuming that the tension force of the branches of the gear we calculate is of known value, the belt tension at the beginning of the main girth arc will be assumed to be equal to the tension in the advancing branch. We consider the belt tension at the end of the girth arc to be equal to its tension in the runaway branch. That is, when integrating equations (1) and (6), the boundary equations for the belt tension values are as follows:

a) for driving pulley: $F = F_1 \ \ at\ \alpha = 0$ and $F = F_2 \ \ at\ \alpha = \alpha_1$

b) for driven pulley: $F = F_2 \ \ at\ \alpha = 0$ and $F = F_1 \ \ at\ \alpha = \alpha_2$

Transferable circumferential force F_{circ} is defined as $F_{circ} = F_1 - F_2$, and the traction coefficient

$$\Psi = \frac{F_1 - F_2}{2F_0} = \frac{F_{circ}}{2F_0}.$$

With automatic belt tensioning, selecting the traction ratio Ψ at a given belt pre-tension value F_0 unambiguously determines also its tension in the transmission branches:

$$F_1 = F_0 \cdot (1 + \Psi) \qquad\qquad F_2 = F_0 \cdot (1 - \Psi).$$

The transmitted circumferential force during automatic tensioning is equal to:

$$F_{circ} = F_1 - F_2 = 2\,\Psi F_0.$$

The radius of the V-belt element in the pulley groove at the beginning of the main girth arc section of each pulley is determined by the following formula:

$$r_{1H(2H)} = \frac{R_{1(2)}}{2} + \sqrt{\frac{R_{1(2)}^2}{4} - \frac{F_{1(2)} \cdot b_p}{4h \cdot E_c \sin\frac{\varphi}{2}} \left(\frac{1}{f\cos\frac{\varphi}{2} + \sin\frac{\varphi}{2}} - \sin\frac{\varphi}{2}\right)} \tag{11}$$

In the process of solving the systems of differential equations, the values of ξ_1 for the driving pulley or ξ_2 – for the driven pulley, at which the boundary conditions for the belt tension force F will be fulfilled. That is, for a given load, it is necessary to find such a value of the relative loss of run-up velocity ξ_1 or ξ_2, at which the belt tension varies within the arc of the girth of the pulley we calculate, respectively from F_1 to F_2 or from F_2 to F_1.

The solution of the system of differential equations is performed in the following sequence. Based on the conditions that at $\xi_{1(2)}$ different from zero, the belt element, after running over the pulley, slides into the groove at an angle to the radius with a corresponding change in belt tension in its boundary sections, the belt tension increment was specified as dF centrally located at $d\alpha$ or a derivative $dF/d\alpha$ at $\alpha = 0$.

Then according to the formula:

$$p = \frac{-F_{1(2)} tg\frac{\varphi}{2} + \sqrt{f^2 F_{1(2)}^2 + \cos^2\frac{\varphi}{2}\left(f^2 - tg^2\frac{\varphi}{2}\right)\left(\frac{dF}{d\alpha}\right)^2}}{4h \cdot r_{1H(2H)} \cdot \left(f^2 - tg^2\frac{\varphi}{2}\right)} \tag{12}$$

specific pressure on the lateral (working) surface of the element is determined. With known p according to the formula

$$r = R_{1(2)} + \frac{b_p \cdot F}{4h \cdot r_{1H(2H)} \cdot E_c} - \frac{p \cdot b_p}{E_c \sin\varphi} \tag{13}$$

the radius of the belt element in the pulley groove is calculated, and then by the formula

$$\frac{dr}{d\alpha} = \frac{r - r_{1H(2H)}}{d\alpha} \quad (14)$$

the derivative of the radius by the girth angle.

The obtained values allow us to determine the derivative of the circumferential displacement by the girth angle according to the formula

$$\frac{d\tau}{d\alpha} = \frac{\frac{dr}{d\alpha} \cdot \frac{dF}{d\alpha}}{2tg\frac{\varphi}{2} \cdot r_{1H(2H)} \cdot h \cdot p - F} \quad (15)$$

The speed loss when the belt runs over the driving pulley is then determined as follows

$$\xi_1 = 1 - \frac{1}{1 - \frac{1}{r_{1H}} \frac{d\tau}{d\alpha}} \quad , \quad (16)$$

or for a driven pulley:

$$\xi_2 = \frac{1}{r_{2H}} \frac{d\tau}{d\alpha}. \quad (17)$$

Knowing the values F,r, τ,$\xi_{1(2)}$ at $\alpha = d\alpha$ we can proceed to the integration of differential equations, i.e., to the solution of the set problem.

Further it is necessary to find such a value $\frac{dF}{d\alpha}$ and $\xi_{1(2)}$ respectively, that at $\alpha = \alpha_{1(2)}, i.e.$, at the end of the main girth arc, when calculating the drive pulley, the belt tension was equal to F_2, and at the end of the girth arc of the idler pulley when calculating the idler pulley F_1.

3 Conclusions

A mechanism of belt element movement in the pulley groove, its sliding and forces acting on the belt element are considered in the presented work with application of the spatial theory of interaction of the V-belt with the pulleys of the transmission. The methodology of determination of kinematic and force parameters of V-belt transmission is given.

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