Calculation of the wave field in a layered soil foundation using the grid method

Dmitrii Sizov[1*] *and Elvira* Egereva[1]

[1]Moscow State University of Civil Engineering, Department of Structural and Theoretical Mechanics, 129337 Yaroslavskoe highway, 26, Moscow, Russia

Abstract. The article contains a solution to the problem of propagation of waves in a soil massif arising from the movement of subway trains. This problem often arises when predicting vibration levels in buildings located near metro lines. The problem is considered in a two-dimensional formulation, taking into account the actual location of soil layers with different physical and mechanical properties. The soil is modeled using the assumption of its linear deformability, which, to a first approximation, is acceptable due to small vibration displacements. To solve the problem, the finite difference method is used, which allows taking into account the various physical properties of individual soil layers and the structure of subway tunnels and at the same time is one of the simplest numerical methods. To obtain a solution, the direct integration method is used using an explicit scheme, which requires setting sufficiently small time steps. The results are presented as graphs of vibration displacements obtained at various points in time. The graphs illustrate the different nature of the wave field at different load frequencies. The dynamic load is specified in the form of vibrations of the walls of the subway tunnel.

1 Introduction

In the practice of modern construction, the problem of constructing buildings in the zone of influence of vibration arising from the movement of subway trains through shallow tunnels often arises. This problem is typical both for existing sections of the metro and in places where the construction of metro lines is only planned. To confirm the possibility of constructing a building in an area of high vibration levels, it is necessary to predict the expected vibration levels of building structures and, if necessary, provide vibration protection measures. In addition to carrying out full-scale vibration measurements at the construction site from existing sources, it is possible to predict the vibrations expected from promising newly constructed metro lines. To carry out such a forecast, you can use various finite element software packages, both general purpose, for example, Ansys, Nastran, Ls-Dyne, Pro/Engeneer, Scad, Lira, Plaxis, Midas, which require the construction of a finite element mesh to carry out the forecast, taking into account the geometric features of the location of subway tunnels, the designed building structures, and the location of soil layers. The finite element modeling process is very labor-intensive, especially in the presence of

*Corresponding author: newfff@mail.ru

extended sections of new construction located along the designed metro lines. In these cases, it is necessary to carry out repeated reconstruction of the finite element mesh for some characteristic sections of geological data and the relative position of the designed objects. To solve the problem of wave propagation in a soil mass from two vibrating tunnels, a method based on finite-difference grid modeling of a soil mass and the tunnels located in it requires significantly less labor input. In fact, this method allows you to turn the problem of calculating the wave field into a problem with some specified parameters, which are: the depth of the tunnels, the distance between the metro tunnels, as well as the marks of individual soil layers surrounding the tunnels. The finite difference method is the simplest method [1] that allows solving partial differential equations in spaces of various dimensions. There are a large number of works [2-6] devoted to this problem. The most famous are explicit and implicit solution algorithms [1]. In the case of using an explicit algorithm (the simplest version of which is the Euler method), to solve the problem it is necessary to iteratively calculate the values of the unknown unknown values at the nodes of the finite-difference grid based on previously calculated values of the unknown function, but located on earlier layers. On the two initial time layers of the space-time grid, the values of vibration displacements are considered known. This algorithm faces a number of difficulties, one of which is quite strict restrictions on the time sampling step. The use of implicit schemes allows one to avoid strict restrictions on the time step, but requires a system of equations at each step, which somewhat complicates the algorithm for solving the wave problem. There are also “intermediate” schemes that make it possible to initially obtain solutions at each time step using the Euler method, and then carry out its refinements, the so-called “predicator-corrector” schemes. In the article, the most simple to implement Euler method was chosen as the main one, but, to improve the stability of the calculation, values from not one, but two previous time layers were taken as the “previous” values of the required values of the components of the displacement vector.

2 Statement of the problem and simulation method

2.1 Basic provisions

Due to the fact that the metro tunnel is an extended structure, and the nature of the distribution of individual soil layers does not change abruptly, a possible approach to solving the problem of modeling wave propagation in a soil mass is to consider it as a plane deformation problem, when the movements of points of the soil mass in a plane perpendicular to axis of the tunnel are considered independent of the coordinate measured along the axis directed along the tunnel. This assumption leads to the vanishing of two tangential stresses and to independence of the coordinate measured along the tunnel axis for the remaining stress components.

Despite the fact that in reality soils are, in general, subject to nonlinear dependencies, but in the case of vibration effects that occur during the movement of subway trains, the vibration amplitudes usually do not exceed 1 micron, which allows, as a first approximation, to consider the process of wave propagation in the soil, as oscillations of an elastic half-space with two vibrating ring inclusions.

Linear wave phenomena are characterized by a sequence of process development over time, that is, the newly arrived state of the system is determined by previous states, and it is this property that is reflected when using numerical solution procedures. Most often, iterative algorithms are based on the sequential consideration of individual states, where several time layers preceding the current one are connected by equations. Depending on the order of the finite-difference approximation of the derivatives, the number of connected space-time layers in the problem may differ, depending on the required accuracy of the solution. However, the number of simultaneously taken into account time layers cannot be less than three, which is determined by the order of the time derivative of the function included in the wave equation. Two of which are the initial ones for obtaining a solution on the current (third) time layer.

The possibility of generalizing the algorithm presented in the article to the three-dimensional case, taking into account the presence of a third coordinate, should not cause additional difficulties, in contrast, for example, to the methods of “alternating directions” for solving two-dimensional equations, in which the parity or oddness of the dimension of the space under consideration is important.

In addition to the algorithm presented in the article, a number of researchers solved wave problems using analytical methods [7-10], however, taking into account arbitrary bedding of soils when using the analytical approach is difficult to achieve, as well as generalizing the problem to three dimensions. Works [11-14] are devoted to the issues of numerical solution of wave problems.

One of the important points when solving wave problems in a semi-infinite half-space is modeling the conditions for wave attenuation at the boundary, which can be done using the attenuation coefficient in front of the speed of the corresponding component of the displacement vector. The second crucial point is the use of individual fragments of the finite-difference grid, located closer to the boundary with terms of the equations that take into account the damping of oscillations at infinity. The presence of boundaries that take into account the damping of oscillations at infinity makes it possible to prevent the reflection of oscillations from the boundaries of the considered fragment of the finite-difference grid. A simpler way to model “non-reflective boundaries” is to consider a sufficiently large section of the soil mass, to the boundaries of which the wave will not have time to propagate during the modeling.

The vibrations of the tunnel walls that arise at the initial moment of time successively propagate over time, capture more and more new areas of the soil, reach the surface of the soil, and, having been reflected from the surface, again propagate inside the computational area.

2.2 Basic relations describing the propagation of linear waves in a soil mass

As is known, Hooke’s law for the case of plane deformation has the form:

$$e_x=(1-nu^2)/E\ \{sigma_x-[nu/(1-nu)]\ sigma_y\} \quad (1)$$
$$e_y=(1-nu^2)/E\ \{sigma_y-[nu/(1-nu)]\ sigma_x\} \quad (2)$$
$$gamma_{xy}=2\ (1-nu)\ tau_{xy} \quad (3)$$

e_x, e_y, $gamma_{xy}$ - deformation components, E, nu - modulus of elasticity and Poisson's ratio, $sigma_x$, $sigma_y$, tau_{xy}- corresponding components of the stress tensor.

The Cauchy relations for the case of plane deformation have the form:

$$e_x=diff[u(x,y,t),x] \quad (4)$$
$$e_y=diff[v(x,y,t),y] \quad (5)$$
$$gamma_{xy}= diff[u(x,y,t),y]+diff[v(x,y,t),x] \quad (6)$$

$diff[u(x,y,t),x]$- partial derivative of the displacement vector component along the x axis with respect to the x coordinate, $u(x,y,t)$, $v(x,y,t)$ - components of the displacement vector along the x and y axes, respectively.

The equations of motion taking into account the action of the own weight of the soil layers have the form:

$$diff(sigma_x, x)+diff(tau_{xy}, y)=r_o\, diff\{ diff[u(x,y,t),x], x\} \quad (7)$$
$$diff(tau_{xy}, x)+diff(sigma_y, y)+r_o\, g=r_o\, diff\{ diff[v(x,y,t),y], y\} \quad (8)$$

diff{ diff[u(x,y,t),x], x}- second partial derivative of the displacement vector, r_o- soil density at the current point, g - acceleration of gravity.

Substituting stresses from Hooke's law into the equations of motion expressed in terms of displacements, we finally obtain the equations of motion written in terms of displacements:

$$E/(2\, nu^2+nu-1)\, (diff\{diff[u(x,y,t),x], x\}\, nu-(diff\{diff[u(x,y,t),x],x\}+ diff\{diff[v(x,y,t),x], y\}\, nu)) +E/2/(1+nu)(diff\{diff[u(x,y,t),y],y\}+ diff\{diff[v(x,y,t),x], y\})= r_o\, diff\{diff[u(x,y,t),t], t\} \quad (9)$$
$$E/(2\, nu^2+nu-1)\, (diff\{diff[v(x,y,t),y], y\}\, nu-(diff\{diff[v(x,y,t),y],y\}+ diff\{diff[u(x,y,t),x], y\}\, nu)) +E/2/(1+nu)(diff\{diff[v(x,y,t),x],x\}+ diff\{diff[u(x,y,t),x], y\})+ r_o g=r_o\, diff\{diff[u(x,y,t),t], t\} \quad (10)$$

Replacing the derivatives in (9), (10) with finite differences, we introduce the following notation:

$$k_1=E_{ij}/(2\, nu_{ij}\, nu_{ij}+nu_{ij}-1) \quad , k_2=E_{ij}/2/(1+nu_{ij}) \quad (11)$$
$$q_{x2}=1/2/h_x^2\, \{(q_{i+1,j,k}-2q_{i,j,k}+q_{i-1.j.k})+(q_{i+1,j,k+1}-2q_{i,j,k+1}+q_{i-1.j.k+1})\} \quad (12)$$
$$q_{y2}=1/2/h_y^2\, \{(q_{i,j+1,k}-2q_{i,j,k}+q_{i,j-1.k})+(q_{i,j+1,k+1}-2q_{i,j,k+1}+q_{i.j+1.k+1})\} \quad (13)$$
$$q_{xy}=1/8/h_xh_y \quad \{(q_{i+1,j+1,k}-q_{i-1,j+1,k}-q_{i+1,j-1,k}+ q_{i-1,j-1,k})+ (q_{i+1,j+1,k+1}-q_{i-1,j+1,k+1}-q_{i+1,j-1,k+1}+ q_{i-1,j-1,k+1})\} \quad (14)$$

q - it is either a component of the displacement vector or . The values characterize the number of the grid node along the axis and, respectively, and the index characterizes the current time layer. h_x , h_y - grid steps along x and y coordinates respectively.

Taking into account the introduced notation (11), (12), (13), (14) the finite-difference analogue of the equations of motion (9), (10) will take the form:

$$\{k_1[u_{x2}nu_{i,j}- (u_{x2}+v_{xy}nu_{i,j})]+k_2(u_{y2}+v_{xy})\}h_t^2/r_{oi,j}+2u_{i,j,k}-u_{i,j,k-1}=u_{i,j,k+1} \quad (15)$$
$$\{k_1[v_{y2}nu_{i,j}- (v_{y2}+u_{xy}nu_{i,j})]+k_2(v_{x2}+u_{xy})\}h_t^2/r_{oi,j}+2v_{i,j,k}-v_{i,j,k-1}=v_{i,j,k+1} \quad (16)$$

h_t - time step, $nu_{i,j}$, $r_{oi,j}$- value of density and Poisson's ratio at i, j point.

These expressions allow you to perform successive iterations and determine the values of displacements on new time layers, based on the data of the previous two time layers (Fig. 1).

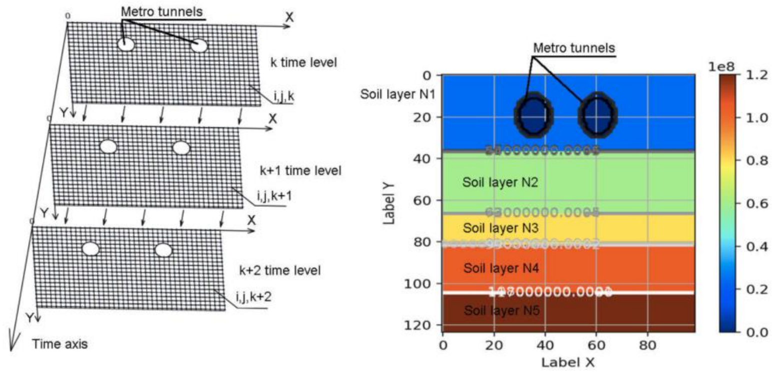

Fig. 1. Application of a time step procedure to solve the wave problem, dividing the area into separate spatial sections depending on the type of soil conditions

The vertical component of vibration acting along the Y axis is taken as the active vibration effect (Fig. 2).

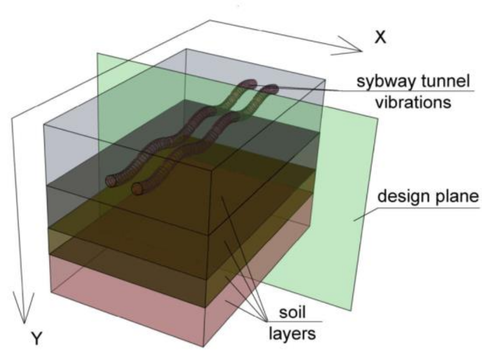

Fig. 2. Vibration effects from subway tunnels

As is known from the results of field measurements of vibration levels propagating from the movement of subway trains, the main impact is observed in the octave bands of 8, 16 and 31.5 and 63 Hz. The article separately considers the vibration impact in the form of a sinusoidal load with vibration acceleration amplitudes equal to 0.008 m/s2 at a frequency of 8 Hz, 0.010 m/s2 at a frequency of 16 Hz and 0.077 at a frequency of 31.5 Hz, as well as 0.128 m/s2 at a frequency 63 Hz.

Modeling of tunnel walls is carried out by specifying in the computational domain individual elements of a finite-difference mesh with values of the elastic modulus and Poisson's ratio corresponding to the known values for reinforced concrete structures. To increase the stability of the computational process, the region of space located inside the tunnel structures is also modeled using a finite-difference mesh with the same step, but with elastic modulus values several orders of magnitude smaller than the elastic modulus of the soil surrounding the tunnel. The value of the density of the "material" of the grid function points located inside the tunnels is also assumed to be small (relative to other specified densities of the layers of the soil massif(Fig. 3)).

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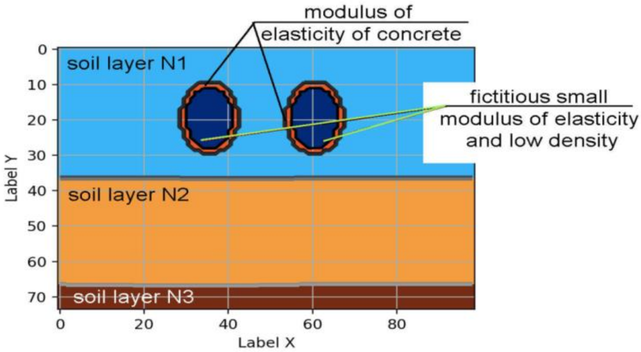

Fig. 3. Fragments of a finite-difference mesh located in the subway tunnel area (different elastic moduli are highlighted in different colors).

To ensure automatic generation of rectangular grid areas near areas with a circular outline, logical conditions of the form are applied:

$$Boolean\_Flag=(x_{i,j}-x_{i,j}^{0})\,(x_{i,j}-x_{i,j}^{0})-(y_{i,j}-y_{i,j}^{0})\,(y_{i,j}-y_{i,j}^{0})>=R^{2}\ AND$$
$$(x_{i,j}-x_{i,j}^{0})\,(x_{i,j}-x_{i,j}^{0})-(y_{i,j}-y_{i,j}^{0})\,(y_{i,j}-y_{i,j}^{0})<=(R+h)^{2} \quad (17)$$

$x_{i,j}$, $y_{i,j}$ - coordinates i, j of a point in the computational domain, $x_{i,j}^0$, $y_{i,j}^0$- coordinates of the centers of the metro tunnels, R- tunnel radius, h- tunnel wall thickness.

3 Results

The resulting solution graphs at various times are presented in the following figures (Fig. 4-11).

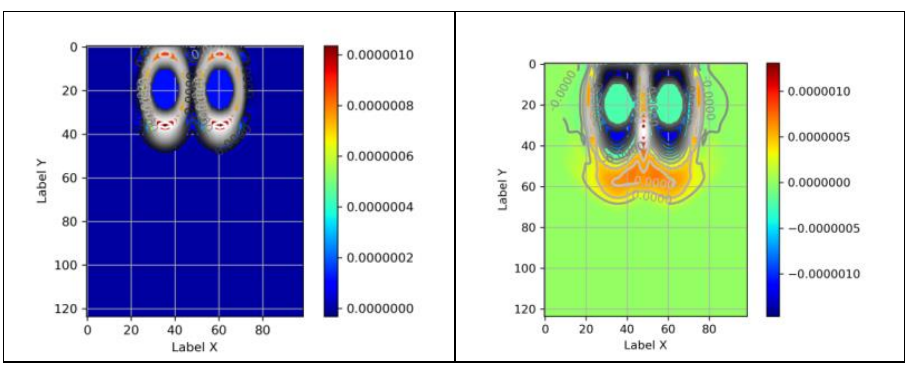

Fig. 4. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 8 Hz. (for times 0.06 (left) and 0.12 (right) seconds from the start of movement)

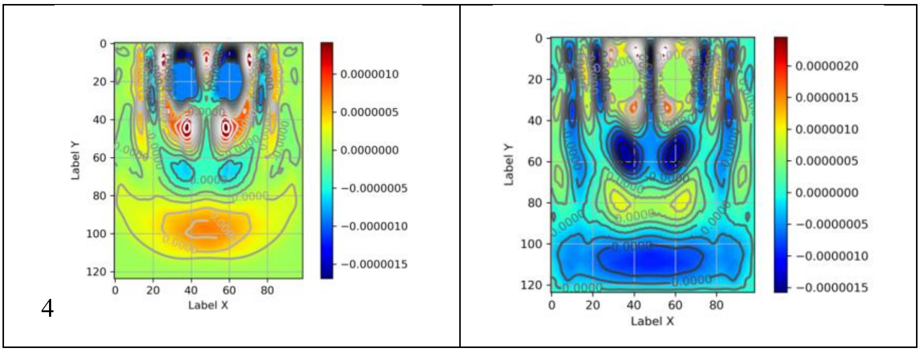

Fig. 5. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 8 Hz. (for time points 0.21 (left) and 0.3 (right) seconds from the start of movement)

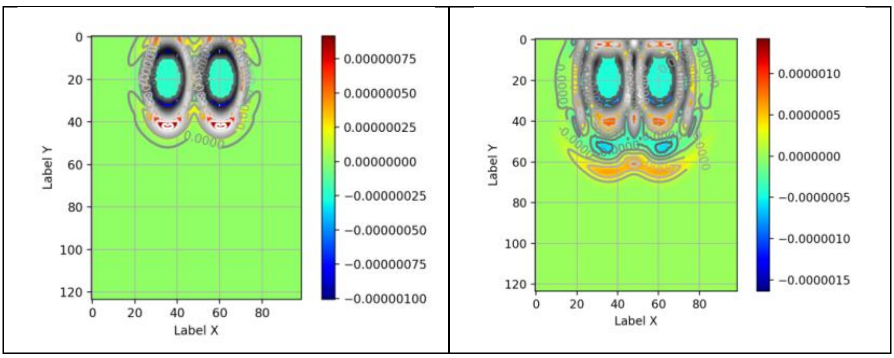

Fig. 6. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 16 Hz. (for time points 0.06 (left) and 0.12 (right) seconds from the start of movement)

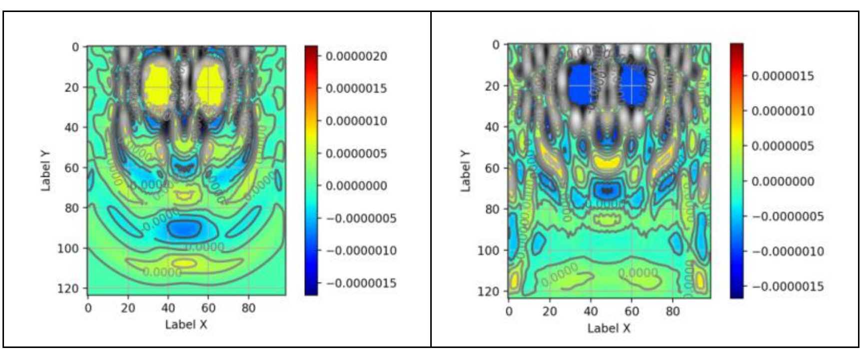

Fig. 7. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 16 Hz. (for time points 0.21 (left) and 0.3 (right) seconds from the start of movement)

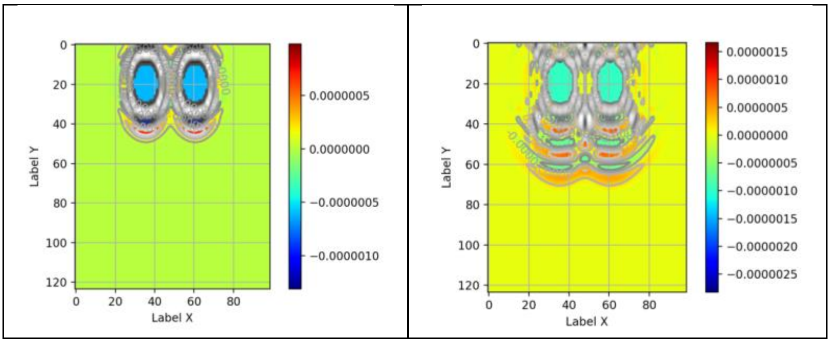

Fig. 8. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 31.5 Hz. (for time points 0.06 (left) and 0.12 (right) seconds from the start of movement)

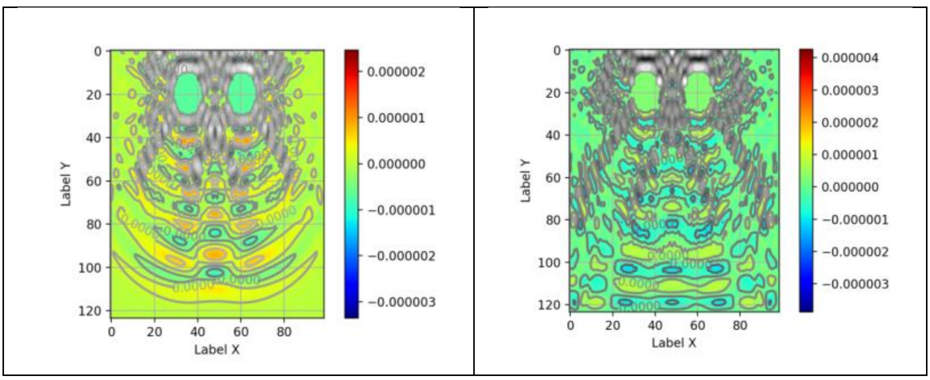

Fig. 9. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 31.5 Hz. (for time points 0.21 (left) and 0.3 (right) seconds from the start of movement)

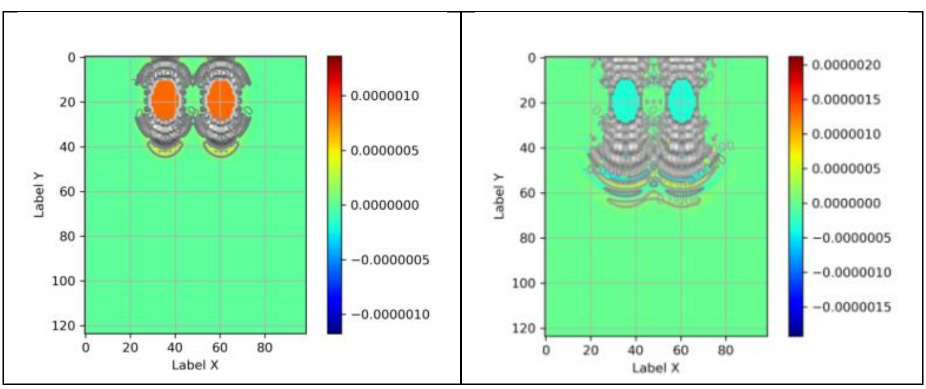

Fig. 10. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 63.0 Hz. (for time points 0.06 (left) and 0.12 (right) seconds from the start of movement)

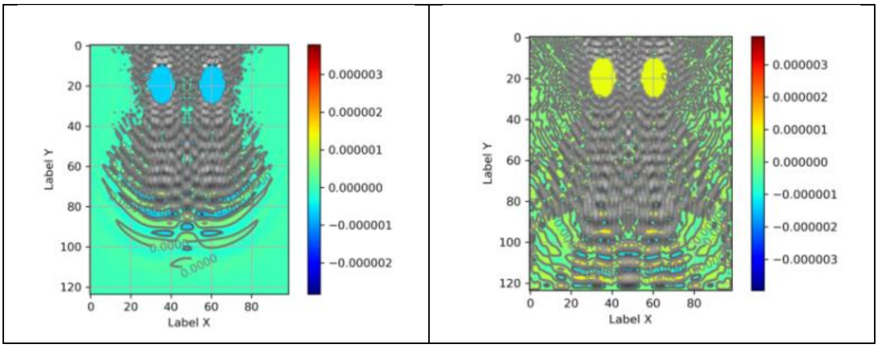

Fig. 11. Graphs of the vibration displacement component v(x,y), m on the next time layer at an impact frequency of 63.0 Hz. (for time points 0.21(left) and 0.3 (right) seconds from the start of movement)

When obtaining a solution, it was taken into account that the initial conditions correspond to zero values of vibration displacements, and the value of vibration velocity at the initial moment of time, determined based on the accepted sinusoidal law of oscillations of the tunnel walls in the vertical plane, determines the magnitude of vibration displacements at the next (second) time step from the beginning of the observation process. As another assumption made in the calculation, we can note the absence of terms that take into account the presence of pre-compression stresses characteristic of the soil foundation under study. As shown in a number of works, ignoring pre-acting stresses can have an additional impact on the nature of the propagation of the wave process in the medium.

The presented solution graphs illustrate the possibility of obtaining a solution to the problem using the proposed finite-difference approximation of the differential equations of motion. The resulting isofields in the components of the displacement vector illustrate the fact that with increasing frequency of exposure, the distance over which the wave propagates in a given time interval decreases.

Of course, the proposed procedure can be extended to solve a three-dimensional problem, but in this case the overall complexity of obtaining a solution will increase significantly. However, a new opportunity arises for mathematical modeling of the process of passage of a subway train along the section of the subway tunnel under consideration. To overcome the computational difficulties and reduce the inevitable rounding errors for computing solutions in the 3D case, it may be necessary to use a larger number of “preceding time layers” as initial conditions for the next time step [14].

4 Conclusion

An algorithm has been implemented for the numerical solution of the wave problem for the case of plane deformation, when the required displacements do not depend on one of the coordinates corresponding to the direction of the coordinate axis along the tunnel axis.
The solution was obtained using the explicit method of integrating the equations of motion. The solutions obtained illustrate the possibility of a fairly simple accounting of the bedding of soils with different physical and mechanical properties and the location of metro tunnels, including taking into account the full or partial location within one or several layers of soil.
To increase the stability of the numerical solution, data on the desired components of the displacement vector from two previous time layers was used to obtain a solution at the current step.

The implementation of the numerical algorithm was carried out using the Python programming language with the Matplotlib graphic library to obtain illustrations of the propagation of waves in the ground at individual points in time.

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