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Harmonic analysis of the torque of autotractor engines by speed and load modes of operation

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Abstract. When assessing the cause of torsional vibrations on the crankshaft of tractor engines, forced vibrations are taken into account, which are provoked by torques arising from each cylinder and transmitted to the connecting rod necks, taking into account the order of operation of the cylinders. Researchers have only calculated or experimental values of the torque for different angular positions of the crankshaft with a certain sampling step. The algorithm for solving the torsional vibration problem involves the decomposition of discrete values of the torque of one cylinder into a harmonic series in order to determine its amplitudes and phase shifts. The adequacy of the resulting decomposition is determined by the number of harmonics taken into account and, as a rule, includes 10-12 harmonics. At the same time, it is necessary to perform such decompositions for each speed and load mode, which increases the amount of calculations. An algorithm is proposed that considers these modes separately, and depending on the values of the gas and inertial forces, allows using the characteristics of the nominal operating mode and the decomposition with a smaller number of terms. In this case, the results of the expansions have a satisfactory convergence. The proposed method for determining the parameters of motor harmonics of exciting moments under changing speed and load modes of the internal combustion engine, can be useful in the design of new and fine-tuning dampers of torsional vibrations of crankshafts.

1 Introduction

Calculating the torsional vibrations of the crankshafts of autotractor engines is a rather time-consuming task. This is especially true in the presence of low frequencies of natural oscillations and several values of the main and strong harmonics. In this case, it is necessary to check the crankshaft in the entire range of speed characteristics of the engine. Let us consider the possibility of simplifying these calculations [1, 2].

2 Status of the issue

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When analyzing the torsional vibrations of the crankshaft, the torque on each of the cranks is represented as a trigonometric polynomial

$$M_{kp}(t) = f(\omega t) = M_{cp} + \sum_{k=1}^{\infty} M_k \sin\left(k2\pi \frac{t}{T} + \varphi_k\right), \tag{1}$$

where M_{cp} - is the average torque acting on the knee;

k =1, 2, 3… ∞ - the order of a harmonically changing moment;

$\frac{2\pi}{T} = \omega$ - circular torque frequency;

T - torque change period, for 4-stroke engine $T = \frac{120}{n}$ sec., for 2-stroke engine $T = \frac{60}{n}$ sec. (n – crankshaft speed, rpm).

φ_k - initial phases of harmonically varying moments;

t – countdown time;

The term M_{cp}, as a constant value, does not affect the oscillations in the linear torsional system. The values M_k and φ_k for each harmonic are determined by the Cauchy integrals:

$$M_k = \sqrt{A_k^2 + B_k^2}\,; \tag{2}$$

$$\varphi_k = arctg\left(\frac{A_k}{B_k}\right), \tag{3}$$

where

$$A_k = \frac{2}{T}\int_0^T M_{kp}(t)\cos\left(k2\pi\frac{t}{T}\right)dt\,;\ B_k = \frac{2}{T}\int_0^T M_{kp}(t)\sin\left(k2\pi\frac{t}{T}\right)dt. \tag{4}$$

For the analytical determination of the Fourier coefficients, it is necessary to have a functional dependence $M_{kp} = f(t)$. The existing methods of calculating the internal combustion engine do not allow us to obtain equations describing changes M_{cp} in time. Therefore, in practice, graphical dependencies $M_{kp} = f(t)$ or their discrete values are used to calculate these coefficients.

For this purpose, the segment of the abscissa axis corresponding to the period of change in the torque is conditionally assumed, regardless of its true value, to be equal 2π. Dividing this segment into equal N parts of length $\frac{2\pi}{N}$ each, we then determine by direct measurement the ordinates of the given torque curve corresponding to the points of division [3, 4].

Denote the coordinates of the points of the torque curve by $x_0, y_0, x_1, y_1, x_2, y_2, ..., x_i, y_i, ..., x_N, y_N$ be equal to and the abscissae of these points will

$$x_0 = 0, x_1 = \frac{2\pi}{N}, x_2 = 2\frac{2\pi}{N}, x_3 = 3\frac{2\pi}{N}, ..., x_i = i\frac{2\pi}{N}, ..., x_N = N\frac{2\pi}{N} = 2\pi.$$

Denoting also $\omega t = x$, $M_{kp} = f(\omega t) = f(x) = y$, $\Delta x = \frac{2\pi}{N}$, is calculated for each value and for each harmonic order of the product $y_i \sin(k\, x_i), y_i \cos(k\, x_i)$. Summing up for a given order the harmonics obtained for all points of the product, multiply them by $\frac{2\pi}{N}$ and divide by π, resulting in the following formulas for determining the Fourier coefficients and the average torque M_{cp}. In this case A_k, B_k, M_{cp} are defined approximately. The general algorithm for their calculation consists in replacing Cauchy integrals with finite sums:

$$A_k \approx \frac{2}{N}\sum_{i=0}^{N} M_{кр.i} \cos\left(k\frac{2\pi}{N}i\right) = \frac{2}{N}\sum_{i=0}^{N} y_i \cos(kx_i); \tag{5}$$

$$B_k \approx \frac{2}{N}\sum_{i=0}^{N} M_{кр.i} \sin\left(k\frac{2\pi}{N}i\right) = \frac{2}{N}\sum_{i=0}^{N} y_i \sin(kx_i) \tag{6}$$

$$M_{cp} = \frac{1}{2\pi}\sum_{i=0}^{N} M_{кр.i}\frac{2\pi}{N} = \frac{1}{N}\sum_{i=0}^{N} y_i \tag{7}$$

or in the expanded form

$$\begin{aligned} A_k \approx \frac{2}{N}(y_0 \cos(k\frac{2\pi}{N}*0) + y_1 \cos(k\frac{2\pi}{N}*1) + y_2 \cos(k\frac{2\pi}{N}*2) + \\ y_3 \cos(k\frac{2\pi}{N}*3) + ... + y_i \cos(k\frac{2\pi}{N}*i) + ... + y_N \cos(k\frac{2\pi}{N}*N); \end{aligned} \tag{8}$$

$$\begin{aligned} B_k \approx \frac{2}{N}(y_0 \cos(k\frac{2\pi}{N}*0) + y_1 \sin(k\frac{2\pi}{N}*1) + y_2 \sin(k\frac{2\pi}{N}*2) + \\ y_3 \sin(k\frac{2\pi}{N}*3) + ... + y_i \sin(k\frac{2\pi}{N}*i) + ... + y_N \sin(k\frac{2\pi}{N}*N); \end{aligned} \tag{9}$$

$$M_{\text{cp}} \approx \frac{1}{N}(y_0 + y_1 + y_2 + y_3 + ... + y_i + ... + y_N). \tag{10}$$

As the number of points N increases , the accuracy of the calculations increases.
Motor harmonics are calculated

$$M_{\text{kp}}(t) = M_{\text{cp}} + \sum_{k=0.5}^{15} M_k \sin\left(k_m\varphi + \varphi_{k_m}\right), \tag{11}$$

where $M_{cp} = \frac{1}{N}\sum_{i=0}^{N} y_i$— average torque acting on the knee;
$k_м$=0,5k=0,5;1;1,5;…;15— motor harmonic index.

3 Proposed solution

The torque can be represented as the sum of the gas component and the inertial component [5]:

$$M_{kp} = M_{kp}{}^{г} + M_{kp}{}^{j}. \tag{12}$$

Let's return to the tangential force T directed tangentially to the circle of the crank radius:

$$T = \frac{P_\Sigma \, sin(\varphi+\beta)}{cos\beta}. \tag{13}$$

The total forces acting in the crank mechanism are determined by the algebraic addition of the gas pressure forces and the forces of reciprocating moving masses:

$$P_\Sigma = P_g + P_j. \tag{14}$$

Accordingly, the tangential force is decomposed into 2 components, gas and inertial

$$T = \frac{(P_g + P_j)\, sin(\varphi+\beta)}{cos\beta} = \frac{P_g \, sin(\varphi+\beta)}{cos\beta} + \frac{P_j \, sin(\varphi+\beta)}{cos\beta}. \tag{15}$$

We substitute the resulting expression into the torque equation

$$M_{kp} = TR = \left(\frac{P_g \, sin(\varphi+\beta)}{cos\beta} + \frac{P_j \, sin(\varphi+\beta)}{cos\beta}\right) \cdot R = \frac{P_g \, sin(\varphi+\beta)}{cos\beta} R + \frac{P_j \, sin(\varphi+\beta)}{cos\beta} R. \tag{16}$$

The first term of this equation is the gas component of the torque

$$M_{kp}{}^{g} = \frac{P_g \, sin(\varphi+\beta)}{cos\beta} R. \tag{17}$$

The second term of this equation is the inertial component of the torque

$$M_{kp}{}^{j} = \frac{P_j \sin(\varphi+\beta)}{\cos\beta} R. \tag{18}$$

Accordingly, we obtain the desired equation for finding the torque in the form of the sum of two components [6, 7]:

$$M_{kp} = M_{kp}{}^{g} + M_{kp}{}^{j}. \tag{19}$$

A harmonic analysis is performed for the gas component of the torque, the Cauchy integrals and the values are found $M^{j}{}_{k}$ and $\varphi^{j}{}_{k}$:

$$A_k{}^{g} = \frac{2}{N}\int_0^N y_i{}^{g} \cos(k \cdot x_i), \tag{20}$$

$$B_k{}^{g} = \frac{2}{N}\int_0^N y_i{}^{g} \sin(k \cdot x_i), \tag{21}$$

$$M_k{}^{g} = \sqrt{(A_k^g)^2 + (B_k^g)^2}, \tag{22}$$

$$\varphi_k{}^{g} = arctg\left(\frac{A_k{}^{g}}{B_k{}^{g}}\right). \tag{23}$$

Motor harmonics are calculated

$$M_{kp}{}^{g}(t) = M_{cp}{}^{g} + \sum_{k=0.5}^{15} M_k{}^{g} \cdot \sin\left(k_m \cdot \varphi + \varphi_{k_m}{}^{g}\right). \tag{24}$$

A harmonic analysis is performed for the inertial component of the torque, the Cauchy integrals and the values are found $M_{\mathrm{k}}^{\mathrm{j}}$ and $\varphi_{\mathrm{k}}^{\mathrm{j}}$:

$$A_k{}^{j} = \frac{2}{N}\int_0^N y_i{}^{j} \cos(k \cdot x_i), \tag{25}$$

$$B_k{}^{j} = \frac{2}{N}\int_0^N y_i{}^{j} \sin(k \cdot x_i), \tag{26}$$

$$M_k{}^{j} = \sqrt{(A_k^j)^2 + (B_k^j)^2}, \tag{27}$$

$$\varphi_k{}^{j} = arctg\left(\frac{A_k{}^{j}}{B_k{}^{j}}\right). \tag{28}$$

Motor harmonics are calculated

$$M_{kp}{}^{j}(t) = M_{cp}{}^{j} + \sum_{k=0.5}^{15} M_k{}^{j} \cdot \sin\left(k_m \cdot \varphi + \varphi_{k_m}{}^{j}\right). \tag{29}$$

We determine the amplitudes of the torques from the gas forces and inertia forces for other high-speed modes of the engine [8, 9].

Because

$$N_e = \frac{p_e \cdot i \cdot n \cdot V_h}{30\tau}, \tag{30}$$

where p_e - the average effective gas pressure in the cylinder;

V_h - working volume of one cylinder;

i - number of cylinders;

n - engine crankshaft speed;

τ - tact factor.

But on the other hand:

$$N_e = N_{e\ nom} \frac{n}{n_{nom}} \left(a + b\frac{n}{n_{nom}} + c\left(\frac{n}{n_{nom}}\right)^2 \right), \tag{31}$$

where a, b and c - coefficients;

$N_{e\ nom}$ - rated engine power;

n_{nom} - rated speed of rotation.

Where from

$$p_e = \frac{30\ \tau}{V_h\ i} \frac{N_{enom}}{n_{nom}} \left(a + b\frac{n}{n_{nom}} + c\left(\frac{n}{n_{nom}}\right)^2 \right). \tag{31}$$

Then the amplitudes of the harmonics of the torque from the gas forces $(M_{\mathrm{k}}^{\mathrm{g}})_n$ for an arbitrary speed of rotation of the crankshaft n are determined by the formula [10, 11]:

$$(M_k^{\mathrm{g}})_n = (M_k^{\mathrm{g}})_{n\ nom} \frac{p_{e\ n}}{p_{e\ n\ nom}}, \tag{32}$$

where $(M_k^{\mathrm{g}})_n$ - the amplitude of the k-th harmonic of the torque from the gas forces for the speed of rotation n;

$(M_k^{\mathrm{g}})_{n\ nom}$ - the amplitude of the k-th harmonic of the torque from the gas forces for the rated speed n_{nom};

$p_{e\ n}$ - average effective gas pressure in the cylinder at the speed of rotation n;

$p_{e\ n\ nom}$ - the average effective gas pressure in the cylinder at the rated speed n_{nom}.

For the amplitudes of the torque harmonics from the inertia forces, we can write [12]:

$$(M_k^{\mathrm{j}})_n = (M_k^{\mathrm{j}})_{n\ nom} \left(\frac{n}{n_{nom}}\right)^2, \tag{33}$$

where $(M_k^{\mathrm{j}})_n$ - the amplitude of the k -th harmonic of the torque from the forces of inertia for the speed of rotation n ;

$(M_k^{\mathrm{j}})_{n\ nom}$ - the amplitude of the k -th harmonic of the torque from the forces of inertia for the rated speed n_{nom} .

Below are the results of calculations for an O6 opposed gasoline engine with a power of N_e= 170 kW at a rated speed of n = 5800 rpm.

The results of the values A_k , B_k, M_k and φ_k are presented in Table 1.

Table 1. The results of the values Ak , Bk, Mk and φk.

k	A_k^{j}	B_k^{j}	M_k^{j}	φ_k^{j}	k	A_k^{j}	B_k^{j}	M_k^{j}	φ_k^{j}
1	-82.863	-57.248	100.715	55.360	16	-7.073	3.359	7.830	-64.595
2	48.629	125.322	134.427	21.208	17	7.243	-2.409	7.633	-71.603
3	-10.995	-95.504	96.134	6.567	18	-5.594	1.706	5.848	-73.039
4	9.373	-165.810	166.074	-3.236	19	5.937	-1.096	6.037	-79.540
5	10.108	-64.273	65.062	-8.937	20	-4.418	0.692	4.472	-81.094
6	-9.445	-46.893	47.834	11.388	21	4.874	-0.353	4.887	-85.855
7	15.508	-41.563	44.362	-20.462	22	-3.517	0.128	3.520	-87.923
8	-14.784	23.591	27.841	-32.075	23	4.028	0.094	4.029	88.658
9	15.685	-25.313	29.778	-31.784	24	-2.746	-0.118	2.749	87.540
10	-13.556	19.321	23.602	-35.054	25	3.431	0.227	3.438	86.223
11	12.987	-14.687	19.605	-41.485	26	-2.229	-0.258	2.444	83.394
12	-11.208	11.564	15.979	-43.640	27	2.963	0.281	2.976	84.585
13	11.079	-8.576	14.011	-52.259	28	-1.845	-0.330	1.875	79.863
14	-9.025	6.397	11.063	-54.670	29	2.532	0.306	2.550	83.110
15	9.034	-4.624	10.149	-62.894	30	-1.517	-0.207	1.531	82.218

Graphs of the change in torque and motor harmonics from the angle of rotation of the crankshaft are shown in Fig. 1.

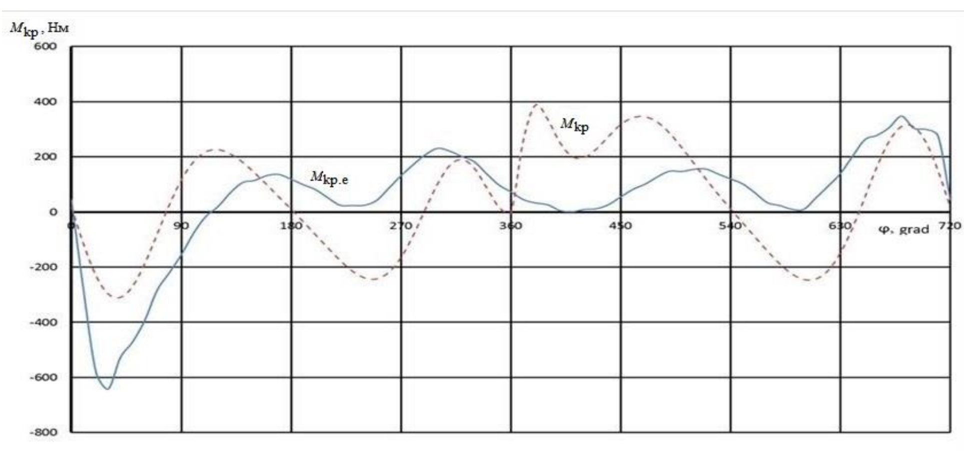

Fig. 1. Decomposition of a function M_{kp} into a Fourier series.

We determine the parameters of the harmonics of the torque from the gas forces. The results are presented in Table 2.

Table 2. The results of the parameters determination of the harmonics of the torque from the gas forces.

k	A_k^j	B_k^j	M_k^j	φ_k^j	k	A_k^j	B_k^j	M_k^j	φ_k^j
1	-83.398	-57.126	101.087	55.589	16	-7.616	3.345	8.319	-66.289
2	47.333	92.844	104.214	27.013	17	6.684	-2.414	7.295	-70.680
3	-11.440	-95.521	96.203	6.830	18	-5.972	1.694	6.208	-74.160
4	-4.321	79.174	79.291	-3.124	19	5.418	-1.099	5.528	-78.537
5	9.675	-64.298	65.022	-8.557	20	-4.929	0.662	4.973	-82.347
6	-12.688	52.791	54.295	-13.514	21	4.425	-0.333	4.438	-85.699
7	15.067	-41.660	44.301	-19.884	22	-4.029	0.143	4.031	-87.969
8	-15.097	32.715	36.031	-24.771	23	3.509	0.078	3.510	88.734
9	15.178	-25.278	29.485	-30.982	24	-3.181	-0.072	3.181	88.700
10	-13.966	19.012	23.590	-36.299	25	2.954	0.282	2.968	84.552
11	12.493	-14.584	19.204	-40.583	26	-2.759	-0.192	2.766	86.070
12	-11.458	11.523	16.250	-44.839	27	2.489	0.333	2.511	82.380
13	10.667	-8.579	13.689	-51.190	28	-2.334	-0.425	2.372	79.689
14	-9.539	6.374	11.473	-56.248	29	2.146	0.341	2.173	80.970
15	8.548	-4.598	9.706	-62.894	30	-2.007	-0.180	2.015	84.863

A graph of the change in the gas component of the torque and motor harmonics from the angle of rotation of the crankshaft is shown in Fig. 2.

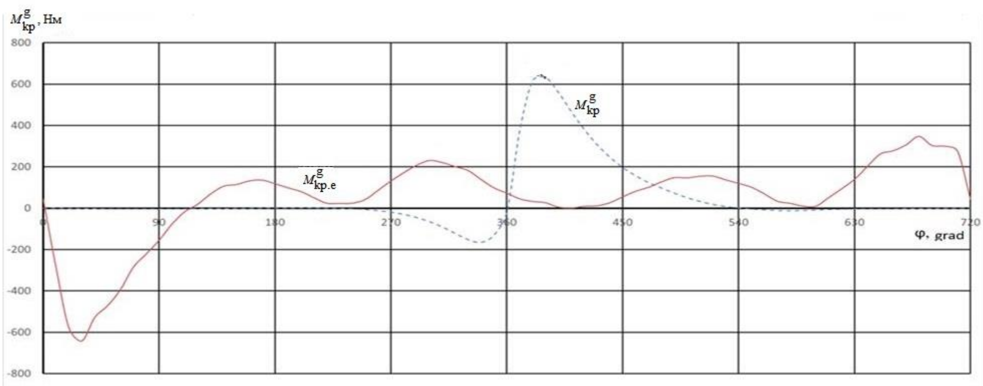

Fig. 2. Decomposition of a function M_{kp}^{g} into a Fourier series.

We determine the parameters of the harmonics of the torque from the forces of inertia. The results are presented in Table 3.

Table 3. The results of the parameters determination of the harmonics of the torque from the forces of inertia.

k	A_k^j	B_k^j	M_k^j	φ_k^j	k	A_k^j	B_k^j	M_k^j	φ_k^j
1	0.472	0	0.472	-90	11	0.472	0	0.472	90
2	1.273	32.39	32.421	2.234	12	0.611	0.144	0.628	76.711
3	0.472	0	0.472	-90	13	0.472	0	0.472	-90
4	13.696	-243.01	243.397	-3.199	14	0.534	-0.051	0.537	-84.593
5	0.472	0.000	0.472	90	15	0.472	0	0.472	-90
6	3.123	-99.63	99.679	-1.797	16	0.397	0.013	0.398	88.063
7	0.472	0	0.472	-90	17	0.472	0	0.472	90
8	0.332	-9.23	9.233	-2.038	18	0.528	0	0.528	90
9	0.472	0	0.472	90	19	0.472	0	0.472	-90
10	0.350	0.27	0.444	52.035	20	0.456	0.059	0.460	82.682

Graphs of changes in the inertial component of the torque and motor harmonics from the angle of rotation of the crankshaft are shown in Fig. 3.

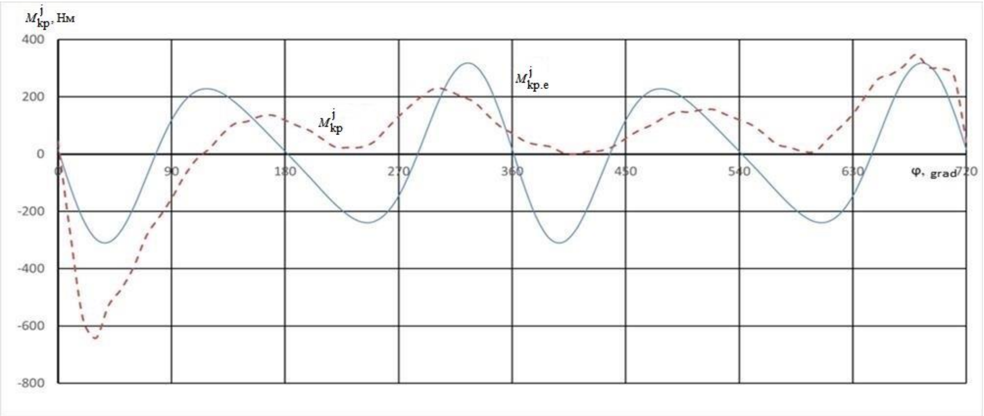

Fig. 3. Decomposition of a function M^{j}_{kp} into a Fourier series.

4 Conclusions

The results obtained allow us to use the harmonic decomposition of the torque in the nominal operating mode over the entire range of changes in speed and load modes with satisfactory accuracy. The algorithm can be useful in the design and calculation of crankshafts of autotractor engines for torsional vibrations.

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