Study of complex motion of a plane point at rotation of a supporting body

Ekaterina Oparina[1*]

[1]Emperor Alexander I St. Petersburg State Transport University, Moskovsky ave., 9, St. Petersburg, 190031, Russia

Abstract. The paper studies the kinematic characteristics of the complex motion of a point under the rotation of a supporting body. The trajectory of the point performing complex motion is obtained, and the graphs of changes in absolute velocity, absolute acceleration of the point and their components in time are obtained. The solution of the problem was carried out in Mathcad package, for which the problem formulation and solution was carried out by the matrix method. The use of the mathematical package allowed us to take chronograms of the processes of relative motion of the point, translational motion of the body and absolute motion of the point.

1 Introduction

As is known, any body with mass, whose dimensions can be neglected under given conditions, can be taken as a material point. A complex motion of a body is a motion in which the body participates in two or more motions. Let there is a body of finite mass, the dimensions of which can be neglected and taken further as a material point M [1-3]. The material point M moves along the plate surface along an elliptical trajectory (Fig. 1). The local coordinates x_{2M} and y_{2M} of the point change according to Eqs:

$$\begin{cases} x_{2M}(t) = 7\,cos(2t) \\ y_{2M}(t) = 4\,cos(2t) \end{cases}$$

$$(1)$$

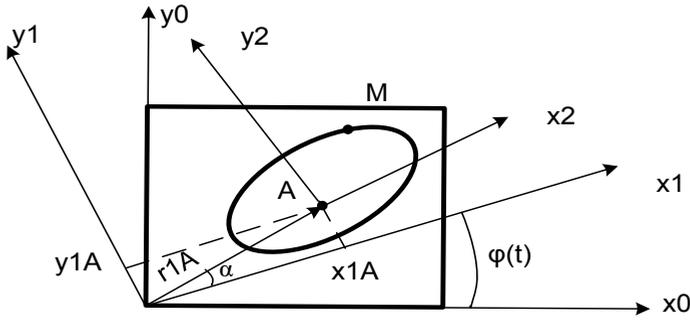[*] Corresponding author: sirayaekaterina@mail.ru

Fig. 1. Motion of a point on the plate surface along an elliptical trajectory.

The relative motion of the point and the translational motion of the carrier plate is described by three coordinate systems: the global coordinate system Ox_0y_0 and two local coordinate systems Ox_1y_1 and Ax_2y_2.

The system of local coordinates Ax_2y_2, in which the equations of relative motion $x_2(t)$ and $y_2(t)$ (1) are defined, is rotated with respect to the local coordinates Ox_1y_1, rigidly connected with the plate, by the angle α (anti-clockwise) and has its origin at the point A with coordinates x_{1A}, y_{1A} (Fig. 1). The plate rotates around the axis Oz_0, perpendicular to its plane, according to the law:

$$\varphi(t) = 1,5t^2 + 2t . \tag{2}$$

2 Methods

Let us represent the radius-vector of the point A in the local coordinate system Ox_1y_1 by the following matrix:

$$r_{1A} = \begin{pmatrix} x_{1A} \\ y_{1A} \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \end{pmatrix}. \tag{3}$$

The radius-vector of the point M in the local coordinate system Ax_2y_2 is described by a matrix:

$$r_{2M} = \begin{pmatrix} x_{2M}(t) \\ y_{2M}(t) \end{pmatrix} = \begin{pmatrix} 7\cos(2t) \\ 4\cos(2t) \end{pmatrix} . \tag{4}$$

We determine the position of the point M on the plate in the axes Ox_1y_1 by parallel transfer along the radius vector r_{1A} (Fig. 1) and transforming the rotation by the angle α :

$$r_{1M}(t) = r_{1A} + H_{z1}(-\alpha) \cdot r_{2M}(t) \tag{5}$$

$$H_{z1}(-\alpha) = \begin{pmatrix} \boldsymbol{cos\,\alpha} & \boldsymbol{sin\,\alpha} \\ -\boldsymbol{sin\,\alpha} & \boldsymbol{cos\,\alpha} \end{pmatrix}$$ - matrix of rotation of the coordinate system Ax_2y_2

around the axis z_1 by the angle α until alignment with the coordinate system Ox_1y_1 .

Let's make an equation of motion of the point M in the global coordinate system Ox_0y_0 . The position of the local coordinate system Ox_1y_1 with respect to the global coordinate system Ox_0y_0 is determined by one clockwise rotation transformation of the angle $\varphi(t)$ around the axis z_0 :

$$r_{0M}(t) = H_{z0}(-\varphi(t)) \cdot r_{1M}(t) \tag{6}$$

For the time instant $t_1 = 2$ s, the radius-vector matrix $r_{0M}(t)$ takes values:

$$r_{0M}(t_1) = \begin{bmatrix} -8,5 \\ -8,8 \end{bmatrix} \text{see.}$$

Let's plot the trajectory of the point M in the time interval from 0 to $t_2 = 10$ s (Fig. 2).
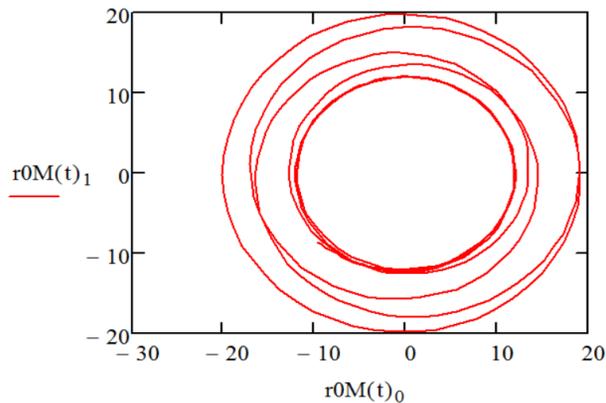

Fig. 2. Plot of trajectory of point M in time interval $\left[0 \le t \le t_2\right]$.

Let us calculate the absolute velocity of the point M at time t_1. The velocity matrix of the point M with respect to the global coordinate axes is calculated as the first time derivative of the global coordinate matrix (5),(6) of the point M [4-7]:

$$v_{0M}(t) = \frac{d}{dt}(H_{z0}(-\varphi(t))(r_{1A} + H_{z1}(-\alpha) \cdot r_{2M}(t))) =$$

$$\frac{d}{dt}H_{z0}(-\varphi(t) \cdot v_{1M}(t) + \frac{d}{dt}H_{z1}(-\alpha) \cdot v_{2M}(t) \qquad , \qquad (7)$$

where $v_{0M}(t) = \frac{d}{dt}(r_{0M}(t))$ is the absolute velocity matrix of the point M in the global coordinate system;

$v_{1M}(t) = \frac{d}{dt}(r_{1M}(t))$ - relative velocity matrix of the point M in the local coordinate system Ox_1y_1;

$v_{2M}(t) = \frac{d}{dt}(r_{2M}(t))$ - velocity matrix of the point M in the local coordinate system Ax_2y_2.

The time derivative of the rotation matrix $H_z(-\varphi(t))$ is expressed as follows:

$$\frac{d}{dt}H_z(-\varphi(t)) = \frac{d}{dt}\varphi \cdot H_z(-\frac{\pi}{2}) \cdot H_z(-\varphi(t)) \qquad , \qquad (8)$$

then the expression of the absolute velocity after transformations takes the form:

$$v_{0M}(t) = \omega_e(t) \cdot H_z(-\frac{\pi}{2}) \cdot r_{0M}(t) + H_{z0}(-\varphi(t)) \cdot v_{1M}(t) \qquad , (9)$$

where $v_{0Me}(t) = \omega_e(t) \cdot H_z(-\frac{\pi}{2}) \cdot r_{0M}(t)$ is the matrix of transport velocity of the point M in global coordinates;

$v_{0Mr}(t) = H_{z0}(-\varphi(t)) \cdot v_{1M}(t)$ - matrix of relative velocity of the point M in global coordinates,

$\omega_e(t) = \frac{d}{dt}\varphi(t)$ - angular velocity of the transfer rotation.

At time t_1 the angle of rotation and angular velocity of the plate, relative, translational and absolute velocities of the point M will take values:

$$\varphi_e(t_1) = 10 \text{ rad}, \ \omega_e(t_1) = 8 \quad \text{with} \quad ^{-1} v_{0Mr}(t_1) = \begin{pmatrix} -6,5 \\ -10,2 \end{pmatrix} \text{ cm/s},$$

$$v_{0Me}(t_1) = \begin{pmatrix} 70 \\ -68 \end{pmatrix} \text{ cm/s}, \quad v_{0M}(t_1) = \begin{pmatrix} 77 \\ -78 \end{pmatrix} \text{ cm/s}.$$

For the moment of time t_1 it is possible to construct an instantaneous scheme of location of vectors (Fig. 3) of velocities of point M and their projections in the global coordinate system:
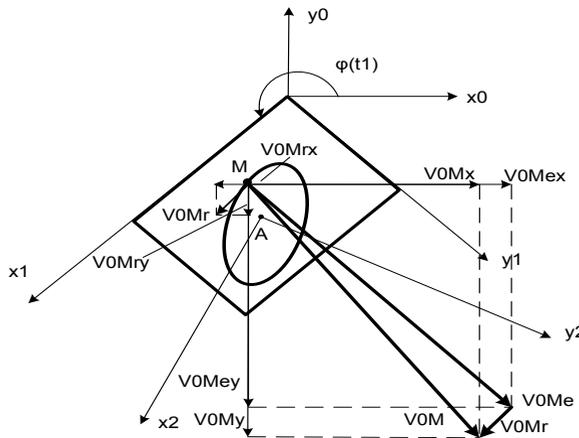

Fig. 3. Position of the plate and the point M moment of time t_1, location of vectors of relative, transport and absolute velocities of the point M.

Differentiating expression (9) by time, we obtain the absolute acceleration of the point M:

$$a_{0M}(t) = a_{0Me}(t) + a_{0Mr}(t) + a_{0MC}(t), \tag{10}$$

where $a_{0Me}(t) = \left[\varepsilon_e(t) \cdot H_z(-\frac{\pi}{2}) + (\omega_e(t) \cdot H_z(-\frac{\pi}{2}))^2 \right] \cdot r_{0M}(t)$ is the matrix of transport acceleration of the point M in global coordinates;

$\varepsilon_e(t) = \dfrac{d}{dt} \omega_e(t)$ - is the translational angular acceleration of the plate;

$a_{0Mr}(t) = H_{z0}(-\varphi(t)) \cdot a_{1M}(t)$ - defines matrix of relative acceleration of the point M in global coordinates;

$a_{1M}(t) = \dfrac{d}{dt} v_{1M}(t)$ - defines matrix of relative acceleration of the point M in local coordinates Ox_1y_1;

$$a_{0MC}(t) = 2 \cdot \omega_e(t) \cdot H_z(-\frac{\pi}{2}) \cdot H_{z0}(-\varphi(t)) \cdot v_{1M}(t) = 2 \cdot \omega_e(t) \cdot v_{0Mr}(t) \qquad -$$

Coriolis acceleration matrix of the point M in global coordinates.

For the moment of time t_1 the angular acceleration of the plate, transfer acceleration, relative acceleration and Coriolis acceleration of the point M will take the values:

$$\varepsilon_e(t_1) = 3_{s-2}, \quad a_{0Me}(t_1) = \begin{pmatrix} 94 \\ 44 \end{pmatrix}_{s/m2},$$

$$a_{0Mr}(t_1) = \begin{pmatrix} -9 \\ -18 \end{pmatrix}_{c/m,^2} \quad a_{0MC}(t_1) = \begin{pmatrix} 164 \\ 105 \end{pmatrix}_{c/m,^2} \quad a_{0M}(t_1) = \begin{pmatrix} 280 \\ 221 \end{pmatrix}_{c/m^2}.$$

According to the obtained values for the moment of time t_1 it is possible to construct an instantaneous scheme of the plate, point and acceleration vectors and their projections (Fig. 4) [8-10]:

Fig. 4. Position of the plate at point M, and its absolute acceleration at the moment of time t_1.

3 Results

On the basis of the obtained matrices (6), (7), (9) let's construct graphs of change of projections of relative velocity of the point M in time, and also graphs of change of modules of transfer, relative and absolute velocities of the point M (Fig. 5 and 6).
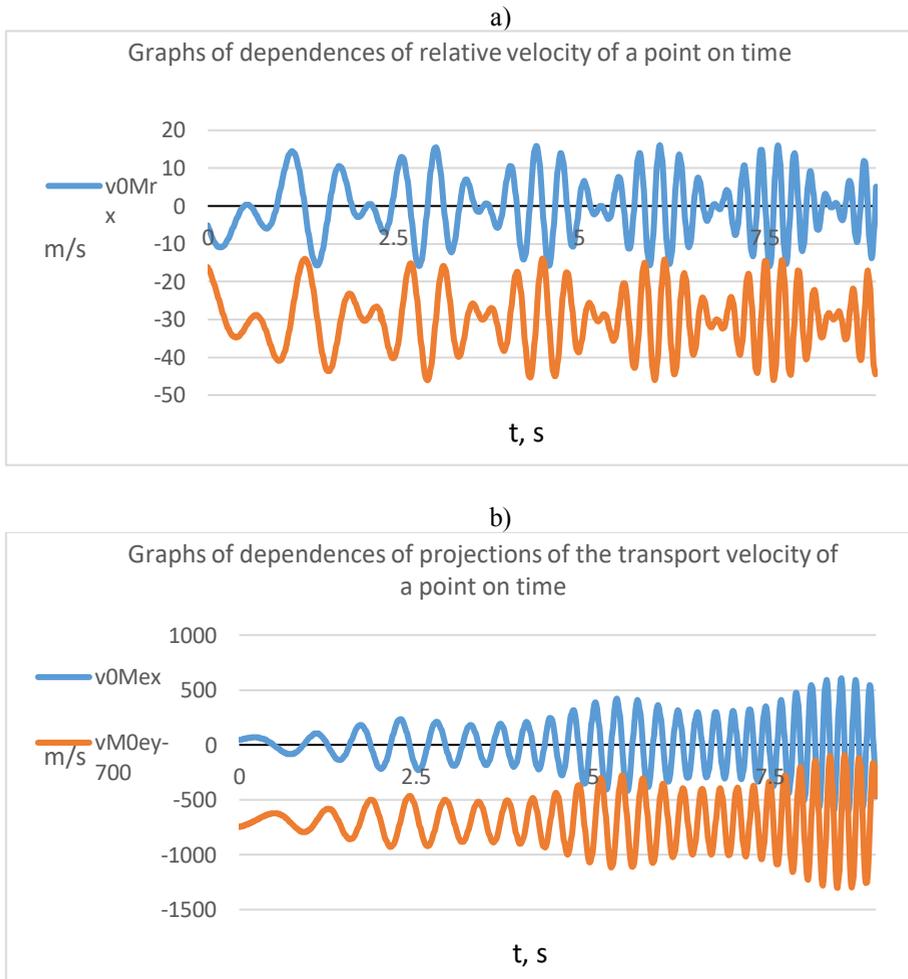
a)

b)

Fig. 5. a) Graphs of dependences of projections of the relative velocity of the point M on time. b) Graph of dependences of projections of the transfer velocity of the point M on time in the global coordinate system.
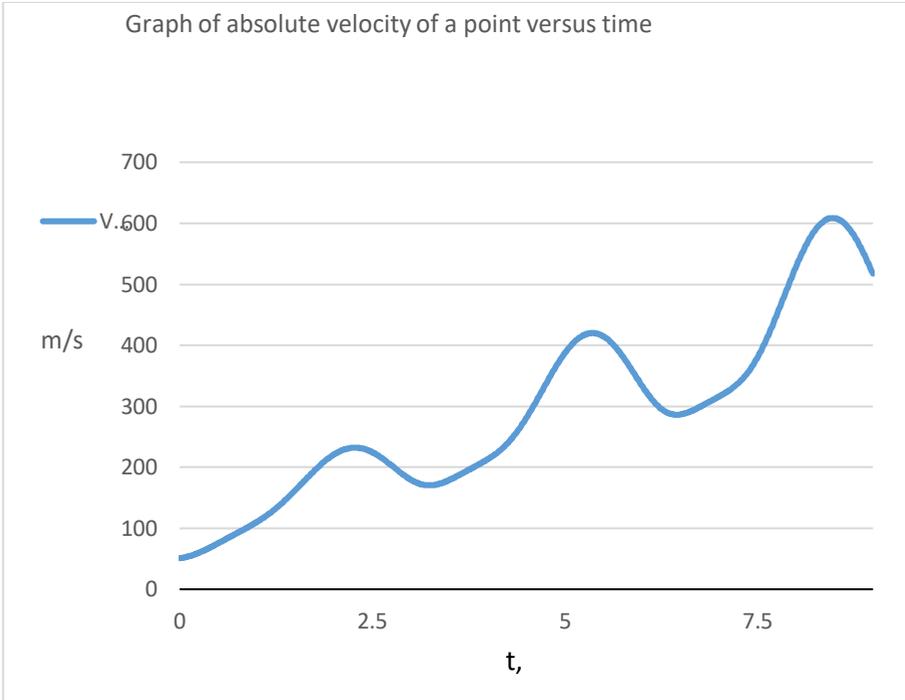
Fig. 6. (a) Graph of dependence of the modulus of absolute velocity of the point M on time in the global coordinate system.

On the basis of the obtained expression (10) and its components, we plot the plots of the change in the projections of the transfer acceleration in time, as well as the plots of the change in the moduli of the relative, absolute and Coriolis acceleration of the point M in time (Fig. 7, 8).
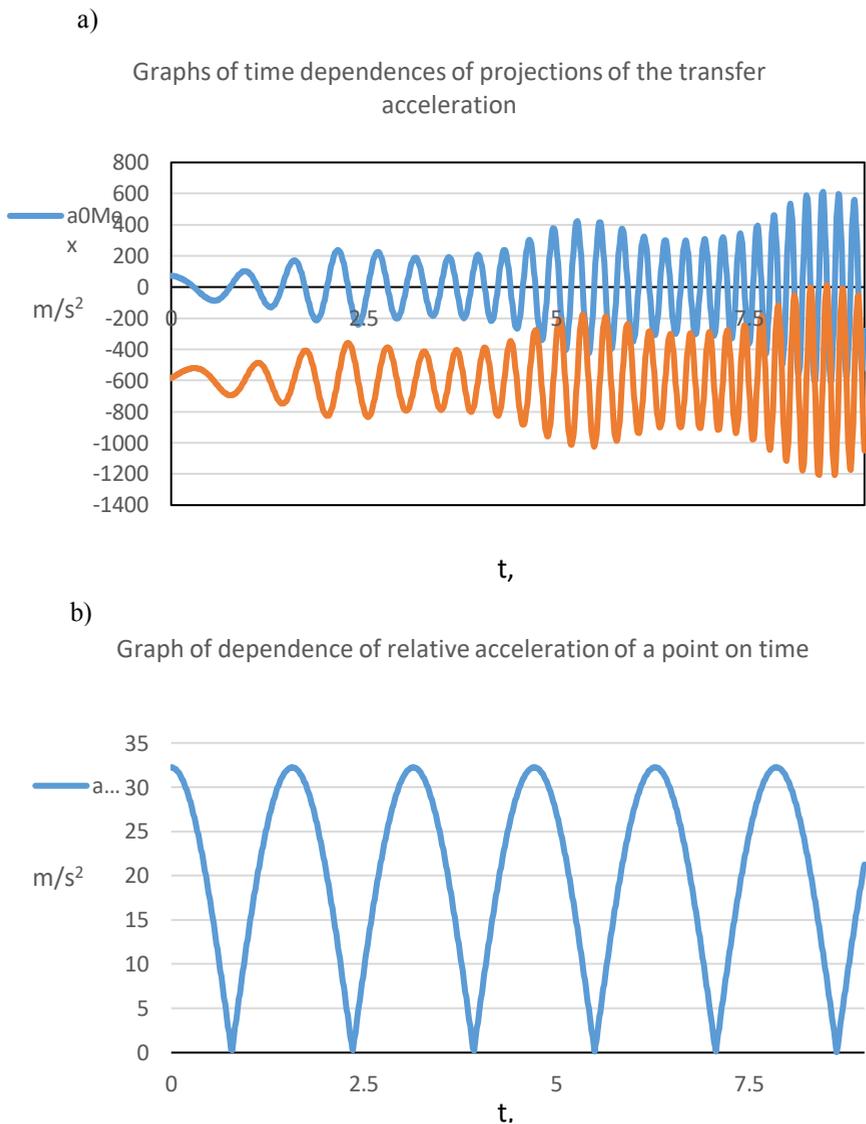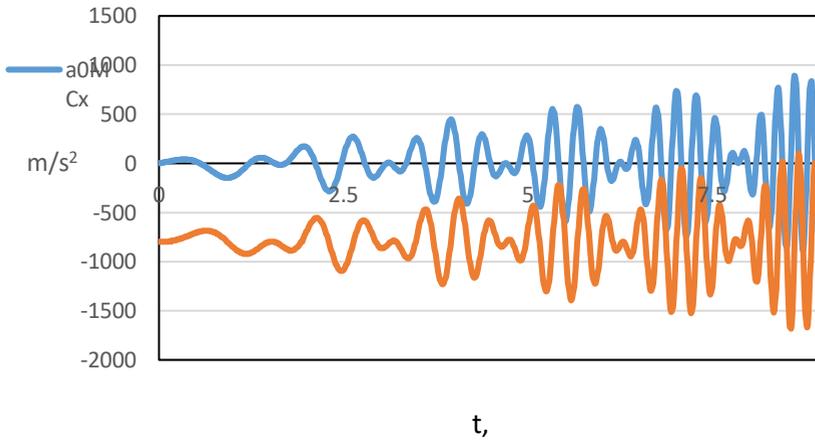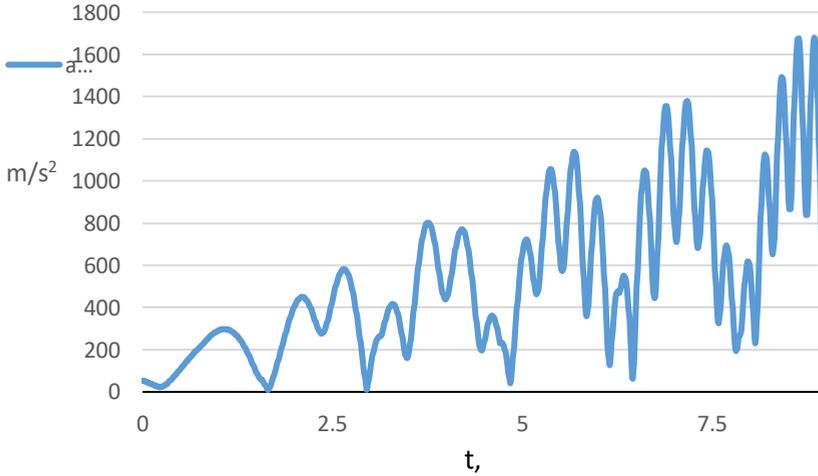
a)

Graphs of time dependences of projections of the transfer acceleration

t,

b)

Graph of dependence of relative acceleration of a point on time

t.

Fig. 7. a) Graphs of dependences of projections of the transfer acceleration of the point M on time on the axes of the global coordinate system b) Graph of dependence of the relative acceleration of the point M on time in the global coordinate system.

Graphs of time dependences of Coriolis acceleration projections

a)

Graph of absolute acceleration of a point versus time

b)

Fig. 8. a) Graphs of dependences of Coriolis acceleration projections of the point M on the axes of the global coordinate system b) Graph of dependence of absolute acceleration of the point M on time.

4 Conclusion

The paper studies the kinematic characteristics of the complex motion of a point under the rotation of a supporting body. The trajectory of the point performing complex motion is obtained, the graphs of changes in absolute velocity, absolute acceleration of the point and their components in time are obtained. The solution of the problem was carried out in Mathcad package, for which the problem formulation and solution was carried out by matrix method. The use of the mathematical package allowed us to take chronograms of the processes of relative motion of the point, translational motion of the body and absolute motion of the point.

References

1. E. Oparina, Lecture Notes in Networks and Systems **509**, 1035-1044 (2023)
2. J. J. Craig, Introduction to Robotics Mechanics and Control, 3rd edn. Pearson Education, Inc. (2005)
3. S. Qinghua, L. Juntao, Advances in Intelligent Systems and Computing **639** (2017)
4. C. Urrea, D. Saa, Applied. Sciences **10(19)**, 6770 (2020)
5. M. A. Al Akkad, .Complexity Reduction for two Human-like Ambidextrous Robotic Hands Control, International Siberian Conference on Control and Communications SIBCON, IEEE, pp. 1-7
6. E. Oparina, E3S Web of Conferences **381**, 02033 (2023)
7. O. Arsirii, M. Yadrova, S, Kondratyev, D. Stelmakh, Herald of Advanced Information Technology **3(2)**, 42-51 (2020) DOI: 10.15276/hait.02.2020.4
8. Zh. Zh. Baigunchekov, A. N. Kassinov, Journal of Mathematics, Mechanics and Computer Science 108(4), 38-46 (2020) DOI: 10.26577/JMMCS.2020.v108.i4.04
9. A. V. Lopota, B. A. Spasskiy, Robotics and Technical Cybernetics **8(1)**, 5-17 (2020) DOI: 10.31776/RTCJ.8101
10. V. A. Leontiev, A. S. Smirnov, B. A. Smolnikov, Robotics and Technical Cybernetics **8(1)**, 53-60 (2020) DOI: 10.31776/RTCJ.8106