

Mathematical modeling of unsteady temperature regime of solar pond device

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Abstract. In the article, the thermal regimes of the layers of experimental solar pond with dimensions of 0.7 m and 1.5 m, height of 1.5 m, built in the training ground of Karshi Engineering Economics Institute, were mathematically modeled based on the climatic conditions of Karshi city. In the dynamic mode of the solar pond device, at a concentration of 25% of pond water, the energy collection efficiency of the layers was calculated analytically for the months of May, June, September and October, 2022. A mathematical model of the temperature changes of the layer over time during the absorption of solar radiation through the depth of the solar pond has been developed. The experimental and model calculation results of this solar pond were compared based on Fisher's criterion. Among the monthly comparison results for each layer, the closest difference was observed in the LCZ layer, which was 8.8%.

1 Introduction

In recent years, increasing the energy efficiency of these systems using solar energy and other renewable energy sources in buildings and energy supply systems with a large energy demand, which is the basis of the development of the green economy in our country, has been receiving a lot of attention [1-2]. One of the most widely used renewable energy sources is solar energy, which occupies a leading position in meeting the energy needs of various industries. There are many solar energy devices, and solar ponds can be used to collect and accumulate its energy [3-5].

Solar ponds are widely used in the world. The solar pond is a salt water pool, divided into three layers depending on the density of the water. It collects solar energy in its lower layer and can be used in the production of electricity and technological processes, providing buildings with heat energy [6-8].

The solar pond is divided into upper convective zone (UCZ), middle non-convective zone (NCZ) and lower convective zone (LCZ), which clearly shows the distribution of salt according to its concentration in the water [9].

Many studies have been conducted by scientists in the field to increase the efficiency of solar ponds and apply it to various heating systems. Methods of obtaining electrical energy from the solar pond and using it for technological systems were carried out by Australian

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Ding and other scientists [10]. A small power generation of electricity from a solar pond using 16 thermoelectric generators was studied by Singh and others.[11].

The analysis of conducted scientific-research works and studied literature and scientific articles shows that many studies have been conducted by scientists in the world and in our country to improve the energy efficiency of solar ponds. However, our Republic is one of the countries rich in solar energy sources, and until today, not enough research has been conducted on the application of heating systems based on the solar pond or its combination to heating systems of buildings. In this research work, the unstable temperature regime of solar pond device was mathematically modeled taking into account the climatic conditions of Karshi city in order to apply it to the heating system of swimming pools.

2 Materials and methods

Calculation of the amount of solar energy absorbed by the solar pond and heat loss from the pool is important in determining the efficiency of the solar pond. Energy is extracted from solar radiation and converted into useful heat by the solar pond.

The formula proposed by Bryant and Colbeck can be used to calculate the radiation flux falling on the surface of a solar pond and the radiation flux absorbed by the pond [12]:

$$\frac{Q_x}{Q_o} = \left\{ 0,36 - 0,08 \ln\left(\frac{x}{\cos \alpha_r}\right) \right\} \quad (1)$$

Where, Q_x is x , the radiation flux falling to depth, W/m^2 ; Q_o is insolation on the horizontal surface, W/m^2 ; α_r - the angle of refraction of the radiation falling on the pond.

The refractive index of solar radiation for the water surface is as follows [13]:

$$\frac{\sin \alpha_i}{\sin \alpha_r} = \frac{n_r}{n} = 1,333 \quad (2)$$

Where, n - air refractive index, n_r - water refractive index, α_i - angle of direct incidence of radiation to the normal horizontal plane.

The energy efficiency of a solar pond depends on its thermal energy storage capacity, the concentration of salt water in it, structural aspects and construction costs. In the accurate analysis of its heat balance, it is necessary to take into account the above factors. The heat balance of the solar pond is solved by developing a dynamic mathematical model for the pool zones (Figure 1).

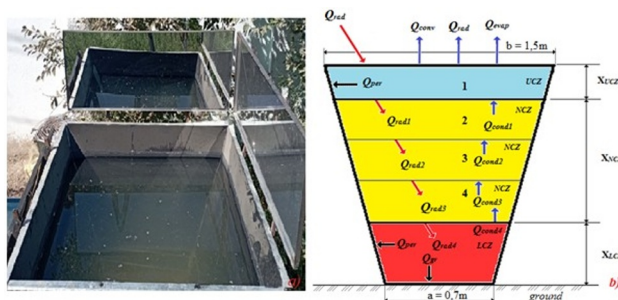


Fig. 1. Solar pond device (a) and its heat balance diagram (b).

The amount of energy falling on the layers of the pond and the radiant energy absorbed by the layer is equal to the loss of energy from the layer and the accumulation of energy in the layer over time [12-19]. The energy balance for each time period is created for each, UCZ, NCZ and LCZ layers of the solar pond dynamic model.

Heat losses from the surface and wall of the solar pond significantly affect the energy collection performance of solar pond. Heat losses are observed as a result of evaporation, radiation, conduction and convection processes. Although the solar pond is a source of heat, its surface temperature is 3-5% lower than the ambient temperature during research [6].

In solar pond zones, heat is absorbed by solar radiation, but some of this heat is lost through the 4 processes mentioned above. The heat balance equation of solar pond layers can be expressed mathematically as:

$$\begin{cases} Q_{UCZ} = Q_{rad1} + Q_{cond1} - Q_{total} - Q_{per_{UCZ}} \\ Q_{NCZ} = Q_{rad2} + Q_{cond3} - Q_{cond1} - Q_{per_{NCZ}} \\ Q_{LCZ} = Q_{rad3} - Q_{cond3} - Q_{per_{LCZ}} - Q_{gr} \end{cases} \quad (3)$$

Here: Q_{rad1} , Q_{rad2} , Q_{rad3} - absorbed heat of solar radiation in solar pond zones, W; Q_{cond1} - heat transferred from the middle layer to the upper layer, W; Q_{cond3} - heat transferred from the lower layer to the middle layer, W; Q_{total} - total heat loss from the surface of the solar pond water to the environment, W; $Q_{per_{UCZ}}$, $Q_{per_{NCZ}}$, $Q_{per_{LCZ}}$ - heat loss from the side wall of the pool (by pool zones), W; Q_{gr} - heat lost from the pond to the ground, W.

The absorbed heat of solar radiation falling on the solar pond zones can be calculated using the following formulas:

$$Q_{rad1} = \alpha\tau F_{UCZ} q_{rad} \quad (4)$$

$$Q_{rad2} = \alpha\tau F_{NCZ} q_{rad} \quad (5)$$

$$Q_{rad3} = \alpha\tau F_{LCZ} q_{rad} \quad (6)$$

Here: α, τ - coefficients of solar pond water transmittance and absorption; F_{UCZ} ; F_{NCZ} ; F_{LCZ} - the surface of the upper, middle and lower layers of the solar pond, m²; q_{rad} - solar radiation intensity, W/m².

The amount of heat given by the solar pond from the middle layer to the upper layer and from the lower layer to the middle layer can be calculated using formulas (7) and (8):

$$Q_{cond1} = \frac{\lambda \cdot F_{NCZ}}{L_{UCZ}} (t_{NCZ} - t_{UCZ}) \quad (7)$$

$$Q_{cond3} = \frac{\lambda \cdot F_{LCZ}}{L_{NCZ}} (t_{LCZ} - t_{NCZ}) \quad (8)$$

Where, t_{NCZ} - the temperature of the middle layer, $^{\circ}C$; t_{UCZ} - upper layer temperature, $^{\circ}C$; t_{LCZ} - lower layer temperature, $^{\circ}C$; λ - heat transfer coefficient of solar pond water, $W / m \cdot K$; L_{UCZ} , L_{NCZ} - upper and middle layer thickness, m .

The total heat loss from the upper water surface of the solar pond to the environment by radiation, convective and evaporation can be calculated from the following formula:

$$Q_{total} = Q_{conv} + Q_{rad} + Q_{evop} \quad (9)$$

This formula can be simplified as follows:

$$Q_{total} = (\alpha_{conv} + \alpha_{rad}) F_{UCZ} \cdot (t_{UCZ} - t_a) \quad (10)$$

Where, α_{conv} - convective heat transfer coefficient, $W / m \cdot K$; α_{rad} - heat transfer coefficient of water in solar radiation, $W / m \cdot K$; t_a - ambient temperature, $^{\circ}C$.

The convective heat transfer coefficient is calculated as follows [14-16]:

$$\alpha_{conv} = 2,8 + 3,0 w_{air} \quad (11)$$

w_{air} is the speed of air movement above the water level of the solar pond, m / s .

When calculating the energy balance of solar pond layers, it is necessary to calculate the heat losses from the side walls of each layer.

It can be calculated using the following formulas:

$$Q_{per_{UCZ}} = \frac{F_{wall_{UCZ}}}{R} (t_{UCZ} - t_a) \quad (12)$$

$$Q_{per_{NCZ}} = \frac{F_{wall_{NCZ}}}{R} (t_{NCZ} - t_a) \quad (13)$$

$$Q_{per_{LCZ}} = \frac{F_{wall_{LCZ}}}{R} (t_{LCZ} - t_a) \quad (14)$$

Where, $F_{wall_{UCZ}}$, $F_{wall_{NCZ}}$, $F_{wall_{LCZ}}$ - the surface of the side walls of the upper, middle, lower layer of the solar pond, m^2 ; R - Thermal resistance of the solar pond wall, $m^2 \cdot ^{\circ}C$.

The amount of heat lost from the base of the solar pond to the ground can be calculated as follows:

$$Q_{gr} = \frac{F_{basis}}{R} (t_{LCZ} - t_{gr}) \quad (15)$$

Where, F_{basis} - the surface of the base of the solar pond, m^2 ; R - thermal resistance of the base of the solar pond, $m^2 \cdot ^{\circ}C$; t_{gr} - ground temperature, $^{\circ}C$.

When calculating the thermal resistance of the wall of the solar pond, its insulation layer was also taken into account. The insulation layer consists of 10 cm thick synthetic fiber (glass wool). Since the water in the solar pond is still, the formula for calculating the coefficient of thermal resistance for a two-layer wall without taking into account heat transfer is as follows:

$$R = \frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} \quad (16)$$

Here: λ_1 - the thermal conductivity coefficient of the metal wall; δ_1 - wall thickness; λ_2 - thermal conductivity coefficient of the insulation layer, δ_2 - insulation thickness.

Using the calculations given above, we make the system of equations (3) as the following form:

$$\begin{cases} \frac{dt_{UCZ}}{d\tau} = - \left(\frac{\alpha_{conv}F_{UCZ} + \alpha_{rad}F_{UCZ} + \frac{F_{wallUCZ}}{R}}{\rho_w C_w V_{UCZ}} \right) t_{юкз} + \frac{\alpha_{conv}F_{UCZ} + \alpha_{rad}F_{UCZ}}{\rho_w C_w V_{UCZ}} t_a + \frac{\alpha T F_{UCZ} q_{rad}}{\rho_w C_w V_{UCZ}} + \frac{5,4\lambda F_{NCZ}}{\rho_w C_w V_{UCZ} L_{UCZ}} \\ \frac{dt_{NCZ}}{d\tau} = - \frac{F_{wallNCZ}}{R\rho_w C_w V_{NCZ}} t_{NCZ} + \frac{F_{wallNCZ}}{R\rho_w C_w V_{NCZ}} t_a + \frac{\alpha T F_{NCZ} q_{rad}}{\rho_w C_w V_{NCZ}} + \frac{11,57\lambda \cdot F_{LCZ}}{L_{NCZ}\rho_w C_w V_{NCZ}} - \frac{5,4\lambda \cdot F_{юкз}}{L_{UCZ}\rho_w C_w V_{NCZ}} \\ \frac{dt_{LCZ}}{d\tau} = - \left(\frac{F_{wallLCZ} + F_{basis}}{R\rho_w C_w V_{LCZ}} \right) t_{LCZ} + \frac{F_{wallLCZ}}{R\rho_w C_w V_{LCZ}} t_a + \frac{F_{basis}}{R\rho_w C_w V_{LCZ}} t_{gr} + \frac{\alpha T F_{LCZ} q_{rad}}{\rho_w C_w V_{LCZ}} - \frac{11,57\lambda \cdot F_{LCZ}}{L_{NCZ}\rho_w C_w V_{LCZ}} \end{cases} \quad (17)$$

3 Materials and methods

Dynamic state block diagram in MATLAB/Simulink package for calculating temperature changes of solar pond device zones (Figure 2).

Based on this block scheme, the result in the dynamic state of block diagram in Figure 2 is shown in Figure 3 according to the data of the GlobaSolarMap system as in the dynamic mode, the hourly change of outdoor air temperature, outside air speed and solar radiation on May 25, June 25, September 25, and October 25, 2022 in the climatic conditions of Karshi city. When the solar pond water concentration is 25%, graphs of the temperature of the solar pond zones over time were obtained.

During the research, the temperature change indicators of the pond zones for each day were determined. The graphs showed that the temperature of each zone can be observed rising until the hour 16:00. A decrease in zone temperatures was observed between 16:00 and 17:00. The lower energy collection zone of the solar pond achieved maximum energy collection at 17:00. The reason for this is the high energy storage capacity of salt water at the given concentration and density, and the fact that the upper and middle layers have performed the function of a thermos.

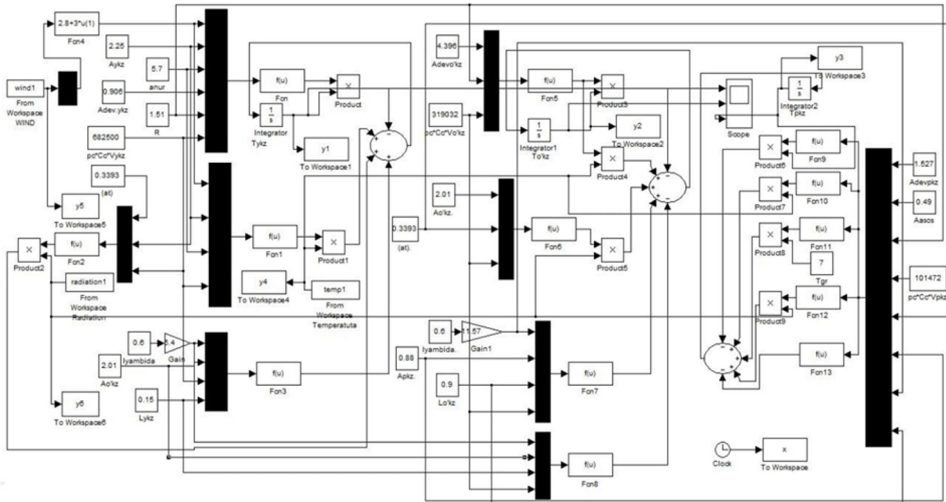


Fig. 2. Block diagram of the dynamic state mathematical model of the solar pond device in the MATLAB/Simulink program.

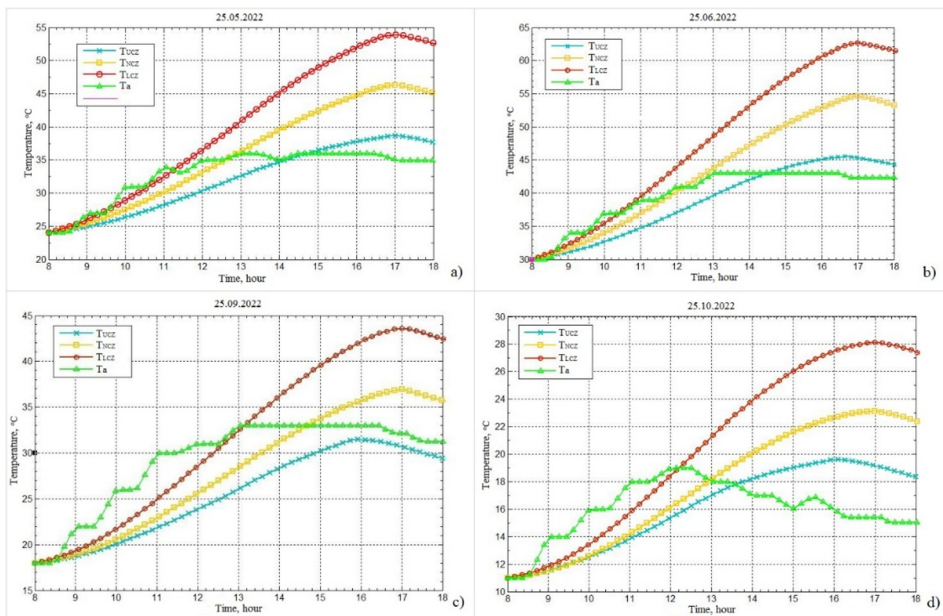


Fig. 3. Graphs of changes in the temperature of the solar pond zones with respect to time.

In order to verify the temperature change process of the solar pond device, the average meteorological data obtained from GlobaSolarMap for the 25th days of May, June, September and October in Karshi city are presented in Table 1. The heat balance calculation parameters of the experimental solar pond are given in Table 2.

The results of the research conducted in different months showed that factors such as the high outdoor temperature and solar radiation value, as well as the required salt water concentration, have an effect to increase the temperature of the layers of the solar pond between 9:00 a.m. and 4:00 p.m.

Table 1. Meteorological data of the conditions of Karshi city.

No.	Month	Average solar radiation (W/m ²)	Average wind speed (m/s)	Average air temperature (°C)
1	May	929	4.1	35
2	June	989	6.3	40
3	September	789	4	31
4	October	701	7	19

Table 2. Calculation table of heat balance of solar pond experimental device.

No.	Parameters	Assignment	Unit of measurement	Value
1	Water density	ρ_w	kg/m ³	1000
2	Heat capacity of water	C_w	J/(kg K)	4200
3	Upper convective zone capacity of solar pond	V_{UCZ}	m ³	0.325
4	Middle non-convective zone capacity of solar pond	V_{NCZ}	m ³	1.266
5	Lower convective zone capacity of solar pond	V_{LCZ}	m ³	0.302
6	UCZ temperature of solar pond	t_{UCZ}	°C	calculates
7	NCZ temperature of solar pond	t_{NCZ}	°C	calculates
8	LCZ temperature of solar pond	t_{ncz}	°C	calculates
9	Ambient temperature	t_a	°C	measures
10	Ground temperature under the solar pond	t_{gr}	°C	measures
11	The surface of the UCZ water of solar pond	F_{UCZ}	m ²	2.25
12	The upper surface of the NCZ of solar pond	F_{NCZ}	m ²	2.01
13	The upper surface of the LCZ of solar pond	F_{LCZ}	m ²	0.88
14	The surface of the side walls of the UCZ of solar pond	$F_{wall_{ucz}}$	m ²	0.906
15	The surface of the side walls of the NCZ of solar pond	$F_{wall_{ncz}}$	m ²	4.396
16	The surface of the side walls of the LCZ of solar pond	$F_{wall_{lcz}}$	m ²	1.527
17	The surface of the base of the solar pond	F_{basis}	m ²	0.49
18	Solar radiation	q_{rad}	W/m ²	0 ÷ 989
19	Absorption coefficient of solar pond water	τ	m ⁻¹	0.377
20	Transmittance coefficient of solar pond water	α	-	0.9
21	Convective heat transfer coefficient	α_{conv}	W/(m ² K)	2.8 ÷ 24
22	Radiation heat transfer coefficient of water in solar radiation	α_{rad}	W/(m ² K)	0 ÷ 5.7
23	Average heat transfer coefficient of water	λ	W/(m K)	0.6
24	Solar pond UCZ thickness	L_{UCZ}	m	0.15
25	Solar pond NCZ thickness	L_{NCZ}	m	0.9
26	Solar pond LCZ thickness	L_{LCZ}	m	0.45
27	Thermal conductivity coefficient of metal wall	λ_1	W/(m K)	45.4
28	The thickness of the pond wall	δ_1	m	0.0009
29	Thermal conductivity coefficient of insulation layer	λ_2	W/(m K)	0.41
30	Insulation thickness	δ_2	m	0.02

The results of comparing the experimental and modeling results of the solar pond are shown in Figure 4. The difference (adequacy) between experimental and calculated results was compared based on Fisher's test.

A comparison of the difference between the experimental and calculated results was carried out for the results of the study carried out on a monthly basis. According to the results of the comparison, the difference between the results of the small experiment and modeling was 11.74% for UCZ, 10.8% for NCZ, and 9.1% for LCZ. In June, it was 10.11% for UCZ, 9.95% for NCZ, and 9.02% for LCZ. In September, 9.86% for UCZ, 9.8% for NCZ, 9.15% for LCZ. In October, it was 9.65% for UCZ, 9.52% for NCZ, and 8.8% for LCZ.

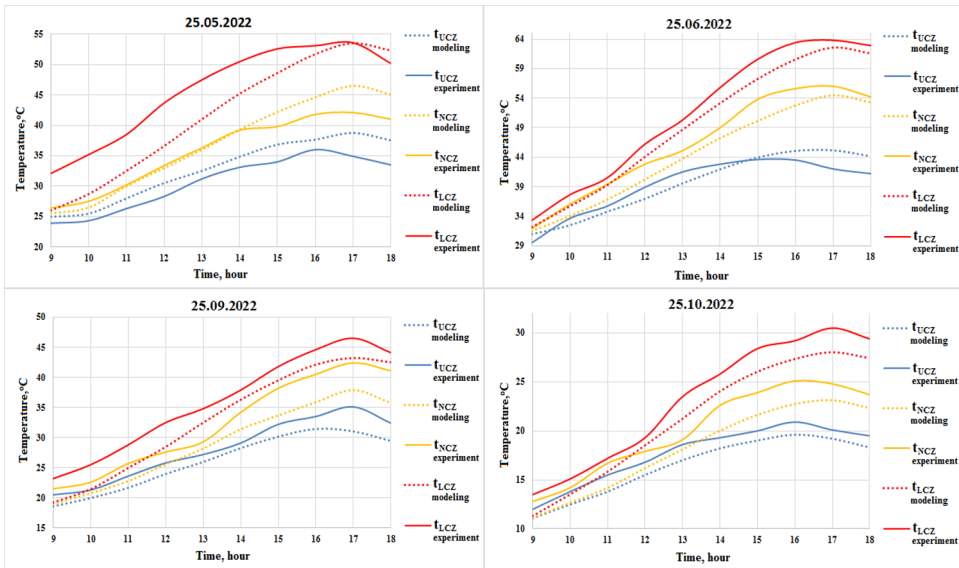


Fig. 4. Graphs of changes in solar pond temperatures over time calculated based on the experiment and modeling.

4 Conclusions

The heat balance of the solar pond was studied based on the results of mathematical modeling of the solar pond based on the equations and thermodynamic laws for calculating the heat balance. Based on the analysis of the research results, the following conclusions were made:

By applying these solar ponds to heating systems of buildings, it is possible to increase their energy efficiency and save traditional fuel energy resources.

It has been scientifically proven that the lower energy collection zone of the solar pond device achieves maximum energy collection at 17:00 at a water concentration at a value 25-30%.

When the difference between the experimental and calculated results of the temperatures of the solar pond layers is compared with Fisher's criterion, it was found that the difference between the experimental and calculated results of the lower heat storage zone is closer to 8.8% compared to the rest of the layers.

The fact that solar pond energy collection efficiency changes depending on solar radiation, ambient temperature, salt water concentration and volume was scientifically substantiated based on research results.

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