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Investigation of the stress state of an elastic hollow ball during filtration of liquid through its wall

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Abstract. When determining the stress state of an elastic hollow ball when filtering liquid through its wall, it is necessary to solve the stress problem for the case of liquid filtration to the center of the ball with a decrease in pressure in its cavity. This case represents for us an element of the general problem of the stressed state of the annular filter behind the casing during well operation. First, the problem of liquid filtration is solved - pressure changes in the body under study are determined during liquid filtration. Then the equilibrium equation with respect to radial deformation is solved. A change in the sign of the filtration potential leads to a change in the tangential stresses on the well wall to a value equal to a tripled depression of reservoir pressure (with radial stresses equal to zero). This explains the negative effect of well shutdowns, and even more so the change in the direction of the filtration flow in the downhole part of the formation on the stability of the walls of wells, the operation of which is complicated by sand formation. The maximum difference of the main normal stresses is observed on the well wall, therefore, in order to prevent formation destruction near the bottom, a necessary condition is that the strength properties of rocks correspond to the stresses acting in this zone. When operating wells prone to plugging, it is necessary to limit the depression of reservoir pressure to the maximum permissible value when the material of the filter zone is in an elastic state throughout the volume.

1 Introduction

Due to the development of modern technologies and the emergence of new materials, the issue of analyzing the stability of the equilibrium of deformable bodies, taking into account various surface phenomena, is becoming more relevant. For example, the nature of deformation of bodies at micro- and nanoscale sizes often differs significantly from their behavior at macro-sizes, which can be explained by surface effects. In addition, these effects can play a significant role in the mechanics of bodies on the surface of which a coating has been applied, or some surface treatment has been performed that changes its properties. In recent decades, the theory of elasticity with surface stresses has been

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developed for modeling surface phenomena, especially in nanomechanics. Within the framework of this theory, in addition to the usual stresses distributed in the volume, independent surface stresses at the boundary of the body or its part are also taken into account, which generalize the scalar surface tension known in hydromechanics to the case of solids. The introduction of surface stresses makes it possible, in particular, to describe the dimensional effect characteristic of nanomaterials. The purpose of this study is to study the equilibrium bifurcation of nonlinear elastic balls with surface stresses. To account for the influence of the latter, the Gertin–Murdoch model is used, which from a mechanical point of view is equivalent to a deformable body with an elastic membrane glued to its surface [1-9].

When determining the stress state of an elastic hollow ball when filtering liquid through its wall, it is necessary to solve the stress problem for the case of liquid filtration to the center of the ball with a decrease in pressure in its cavity ($\chi = -1$). This case represents for us an element of the general problem of the stressed state of the annular filter behind the casing during well operation. The task is solved in the following sequence.

2 Methods and Materials

First, the problem of liquid filtration is solved - pressure changes in the body under study are determined during liquid filtration. Then the equilibrium equation with respect to radial deformation is solved [10]:

$$(\lambda + 2\mu)\frac{d}{dr}\left(\frac{du}{dr} + 2\frac{u}{r}\right) = \chi(1 - \alpha\beta)\frac{dP}{dr}, \tag{1}$$

Where $\mu = \frac{E}{2(1+v)}$, $\lambda = \frac{v \cdot E}{(1+v)\cdot(1-2v)}$, $\beta = \frac{E}{1-2v}$; α – coefficient of linear change of the medium material; P – pressure of the filtered liquid, MPa; r – the outer radius of the production column, m; v – Poisson's ratio; E – Young's modulus, MPa.

After that, substituting the values of this function and its derivatives into equation [11] and using boundary conditions $r = a$, $\sigma_r = 0$, $P = P_a$; $r = b$, $\sigma_r = \sigma_{rs}$, $P = P_B$ (where σ_r – normal radial voltage, MPa), we find the constant integrations of equation (1), which are included in the main dependencies, which essentially ends the solution of the problem.

Let's find the law of pressure distribution in the wall of a hollow ball during filtration of a liquid caused by pressure in its cavity ($\chi = -1$). The liquid flow can be schematically represented as two flows - a radial flow with flow through the reservoir to the filter behind the casing $Q_ж$, which is divided at the filter wall into n independent spherical (hemispherical) flows - according to the number of perforations in the column - with a flow rate of $q = Q_ж/n$ and the total pressure of the liquid at the outlet (in the well). Taking these assumptions and solving the equation of the unsteady flow of liquid through the wall of half the surface of a hollow ball [12]:

$$2\pi r^2 \cdot \frac{K_1}{\mu_ж} \cdot \frac{dP}{dr} = const, \tag{2}$$

Under boundary conditions $r = a$ (the initial radius of the hollow ball, m); $P = P_a$; $r = b$ (the final radius of the hollow ball, m); $P = P_B$, we obtain the law of distribution of liquid pressure during filtration:

$$P = P_B + \frac{a}{b-a}\cdot\left(\frac{b}{r} - 1\right)\cdot(P_a - P_b) = P_a + \frac{b}{b-a}\cdot\left(\frac{a}{r} - 1\right)\cdot(P_a - P_b), \tag{3}$$

$$q = \frac{Q_ж}{n} = \frac{2\pi abK_1}{\mu_ж(b-a)} \cdot (P_a - P_b), \tag{4}$$

Where K_1 – permeability (effective) of oil before coking the rock, m^2. Substituting the value of the pressure drop in the Rv formation, which falls on the thickness of the depression funnel (minus the thickness of the filter behind the column), from the Dupuy formula [13]:

$$P_b = \frac{Q_ж \mu_ж}{2\pi hK_2} \cdot ln \frac{R_K}{r_c \cdot +b} \tag{5}$$

Where h - effective collector thickness, m; R_K – the radius of the supply circuit (depression funnel), that is, the distance from the well to the formation zone, where the pressure is assumed to be constant and equal to the current reservoir pressure (about half the distance between the wells), m; K_2 – permeability (effective) of oil before coking the rock, m^2; r_c – the radius of the well, m.

We will get it finally:

$$P_b = \frac{Q_ж \mu_ж}{2\pi hK_1} \cdot (\frac{b}{r} - 1 + \frac{nbK_1}{hK_2} \cdot ln \frac{R_K}{r_c + b}). \tag{6}$$

From the formula (5) by $r = a$ we find the value of the total pressure drop ΔP on the formation and filter:

$$\Delta P = P_a = \frac{Q_ж \mu_ж}{2\pi hbK_1} \cdot (\frac{b}{a} - 1 + \frac{nbK_1}{hK_2} \cdot ln \frac{R_K}{r_c + b}). \tag{7}$$

Taking into account (7), formula (6) can be represented as:

$$P = \left[1 - \left(1 - \frac{a}{r}\right) \cdot A\right] \cdot P_a, \tag{8}$$

Where $A = \left(1 - \frac{a}{b} + \frac{naK_1}{hK_2} \cdot ln \frac{R_K}{r_c + b}\right)^{-1}$.

The function of the average fluid pressure in the wall of a hollow ball within a radius can be represented by:

$$\varphi = \frac{1}{r^3} \cdot \int_0^r r^2 \, P dr \, . \tag{9}$$

Substituting pressure P from (3), we get:

$$\varphi = \frac{1}{3} P_b + \frac{1}{2} \cdot \frac{b}{b-a} \cdot (\frac{a}{r} - \frac{2}{3} \cdot \frac{a}{b}) \cdot (P_a - P_b) \tag{10}$$

When assessing the condition of the near-filter zone of the formation, the "skin effect" parameter is used, which is the relative value of the decrease in the productivity coefficient of the well in case of contamination of the near-filter zone [14]:

$$S = \left(\frac{\eta_2}{\eta_1} - 1\right) \cdot ln \frac{R_K}{r_c + b}, \tag{11}$$

Where η_2 - well productivity coefficient before coking; η_1- the productivity coefficient of the well after coking.

Substituting values$\eta_2 = \frac{Q_ж}{P_b}$ from equation (5) and $\eta_1 = \frac{Q_ж}{P_a}$ from equation (7), we obtain a formula for determining the “skin effect” due to the presence of a filter behind the column:

$$S = \frac{hK_2}{nbK_1} \cdot \frac{b-a}{a} \tag{12}$$

By $\frac{b}{a} \gg 1$ expression (12) is simplified to the form:

$$S = \frac{hK_2}{naK_1}. \tag{13}$$

Now let's solve the problem of stresses in the wall of a hollow sphere. The integral of the equilibrium equation of an elastic body (1) has the form [15]:

$$E^{\prime} \cdot u = \frac{1}{3} \cdot cr + \frac{D}{r^2} + \frac{1}{r^2} \cdot \int_0^r r^2\, Pdr, \qquad (13)$$

Where $E^{\prime} = \frac{\lambda+2\mu}{\chi(1-\alpha\beta)}$, c=a + b.

From equation (14) we find the values$\frac{du}{dr}$, $\frac{u}{r}$, Δ:

$$E^{\prime} \frac{du}{dr} = \frac{c}{3} - 2\frac{D}{r^3} - 2\varphi + P, \tag{14}$$

$$E^{\prime} \frac{u}{r} = \frac{c}{3} + \frac{D}{r^3} + \varphi, \tag{15}$$

$$E^{\prime}\Delta = \mathrm{c} + P, \tag{16}$$

Where *D* – the diameter of the hole in the production column, m.
Substituting the found value of deformations into equation (16), we obtain the expression of the components of the main normal stresses in the wall of a hollow ball:

$$E^{\prime}\sigma_r = \frac{\beta c}{3} + \left(\lambda + 2\mu + \omega E^{\prime}\right) \cdot P - 4\mu\frac{D}{r^3}, \tag{17}$$

$$E^{\prime}\sigma_\theta = \frac{\beta c}{3} + \left(\lambda + \omega E^{\prime}\right) \cdot P + 2\mu\varphi + 2\mu\frac{D}{r^3}, \tag{18}$$

Or

$$E^{\prime}\sigma_\theta = E^{\prime}\sigma_r - 2\mu(P - 3\varphi) + 6\mu\frac{D}{r^3}\ , \omega = \chi\alpha\beta \tag{19}$$

Where σ_θ – normal tangential voltage, MPa.
Using boundary conditions $r = a,\ P = P_a,\ \sigma_r = 0;\ r = b,\ P = P_B,\ \sigma_r = \sigma_{rb}$ let's find the constant integrations of C and D included in (14):

$$\begin{gathered} 4\mu D = \frac{a^3 b^3}{b^3-a^3} \cdot [E^{\prime}\sigma_{rb} + \left(\lambda + \omega E^{\prime}\right) \cdot (P_a - P_b)], \\ \frac{\beta C}{3} = 4\mu\frac{D}{a^3} - \frac{\beta P_a}{3} + \frac{2}{3} \cdot \frac{\mu_b}{b-a} \cdot (P_a - P_b). \end{gathered} \tag{20}$$

Substituting the found expressions of constants into formulas (14) and (16), we obtain finally:

$$\sigma_r = \frac{b^3}{b^3-a^3}\left(1-\frac{a^3}{r^3}\right)\sigma_{rb} + \frac{\lambda+\omega E'}{E'}[\frac{b^3}{b^3-a^3}\cdot\left(1-\frac{a^3}{r^3}\right)-$$
$$-\frac{b}{b-a}\left(1-\frac{a}{r}\right)]\cdot(P_a-P_b),$$
$$\sigma_\theta = \frac{b^3}{b^3-a^3}\left(1+\frac{a^3}{2r^3}\right)\sigma_{rb} + \{\frac{\lambda+\omega E'}{E'}[\frac{b^3}{b^3-a^3}\cdot\left(1-\frac{a^3}{2r^3}\right)-$$
$$-\frac{b}{b-a}[\omega+\frac{\lambda}{E'}-(\frac{\lambda+\mu}{E'}+\varpi)\frac{a}{r}]\}\cdot(P_a-P_b)$$

$$u = \frac{\sigma_{rb}}{4\mu}\cdot\frac{b^3}{b^3-a^3}\left(\frac{4\mu}{\beta}+\frac{a^3}{r^3}\right)\cdot r + \frac{\omega E'}{\beta}P_a r + \tag{21}$$
$$+[\frac{\lambda+\omega E'}{4\mu E'}\cdot\frac{b^3}{b^3-a^3}\left(\frac{4\mu}{\beta}+\frac{a^3}{r^3}\right)\cdot r + \frac{ab}{2(b-a)E'}\left(1-2\frac{\lambda}{\beta}\cdot\frac{a}{r}\right)]\cdot(P_a-P_b)$$

$$\frac{\lambda+\omega E'}{E'} = \frac{\chi[v+(1-2v)\alpha\beta]}{1-v},\ \frac{\lambda+\mu}{E'} = \chi\frac{1-\alpha\beta}{2(1-v)},$$
$$\frac{\omega E'}{\beta} = \frac{\alpha\beta}{1-\alpha\beta}\cdot\frac{1-v}{1+v},$$
$$\frac{\lambda+\omega E'}{4\mu E'} = \chi\frac{1+v}{2\cdot(1-v)E}\cdot[v+(1-2v)\alpha\beta],\ \frac{4\mu}{\beta} = 2\frac{1-2v}{1+v},\ \frac{\lambda}{\beta} = \frac{v}{1+v}$$

Analyzing the dependencies (20), it can be noted that the sign of the increment of the main normal stresses caused by the action of filtration pressure in a porous medium depends on the sign of the change in the static pressure of the liquid saturating the porous medium [16]. Moreover, in terms of absolute magnitude, the stresses caused by a decrease in the static pressure of the liquid are equal to the stresses caused by an increase in the pressure of the liquid by the same amount, that is (Figure 1):

$$\sigma_{r\,|\chi=+1} = -\sigma_{r\,|\chi=-1},\ \sigma_{\theta\,|\chi=+1} = -\sigma_{\theta\,|\chi=-1} \tag{22}$$

Figure 2 shows the dependences of the change in the main normal voltages σ_r и σ_θ according to the wall thickness of the hollow ball, free from external load ($\sigma_{rb}=0$) when creating excessive pressure in the cavity of the ball ($\chi=+1$) and with depression ($\chi=-1$, $\Delta P=-P_a$, $P_b=0$).

3 Results and Discussion

Curves σ_r and σ_θ constants for $\alpha\beta=0$, $\alpha\beta=0.5$ and $\alpha\beta=1$. It should be noted that for the first time the problem of stresses in an elastic porous medium during liquid filtration through the wall of a cylindrical hole (well) in an oil reservoir was solved by Yu.P. Zheltov and S.A. Khristianovich, who developed the theory of hydraulic fracturing at the value of the parameter $\alpha\beta=0$ on the right side of equation (1) [17]. Soon the same problem of elasticity theory was solved in [8] (by $\alpha\beta=1$), where the influence of the compressibility effect of rock-forming minerals on the stress state of the porous medium during liquid filtration is taken into account.

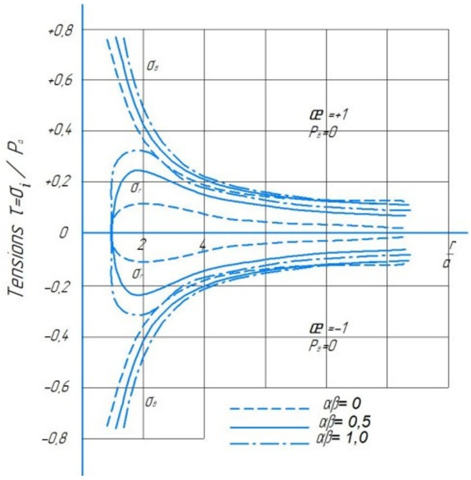

Fig. 1. Change in the main normal voltages σ_r and σ_θ according to the wall thickness of the hollow ball, free from external load ($\sigma_{rb} = 0$).

The data in Table 1 [18] show that the solutions of the problem given in [19] (by $\alpha\beta = 0$ and $\alpha\beta = 1$, $\chi = +1$) they are solutions, since the true value of the parameter $\alpha\beta$ It is found for real rocks at moderate loads in the range of 0.3-0.6.

The analysis of the obtained dependencies (20) shows that the change in the sign of the filtration potential ($\Delta P = \pm P_a$) leads to a change in voltage σ_θ within the range of -1.5 P_a before + 1.5 P_a, *so* $\Delta\sigma_\theta = 3 \cdot P_a$. This can explain the negative effect of well shutdowns, and even more so the change in the direction of the filtration flow in the near-filter zone of the formation, on the stability of the walls of the well, the operation of which is complicated by sand phenomena. With a rapid application of load to the reservoir (when the well is put into operation), the stress state of the filter zone will depend on the parameter $\alpha\beta$, its magnitude. In the future, as plastic deformations develop under conditions of volume conservation, the compressibility effect will play a subordinate role and tend to zero ($\alpha\beta \to 0$). In this case, the Poisson's ratio ν it will tend to 0.5, and the Young's modulus will change over time and determine the relationship between stresses and strain rate [20-23].

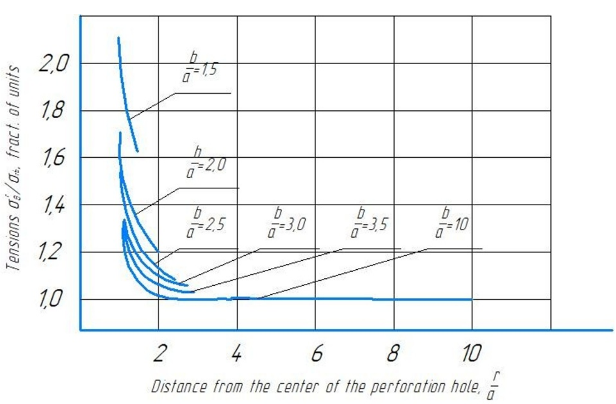

Fig. 2. Distribution of tangential stresses σ_θ^r in a fixed zone, depending on its radius.

Let's pay attention to a very interesting fact. According to dependence (20), all other things being equal, stresses in a porous medium with parameters $\alpha\beta = 1$ and ν arbitrarily equal to the stresses in a porous medium with parameters $\alpha\beta = 0$ and $v = 0.5$. This leads to an important conclusion that the stress state of an elastic porous medium with instantaneous application of a load is preserved during the transition of the medium from an elastic state to a plastic flow with volume conservation when the Poisson's ratio is 0.5. The nature of the Young's modulus can be estimated by direct measurements of the deformation of the walls of a hollow ball or well (Figure 3).

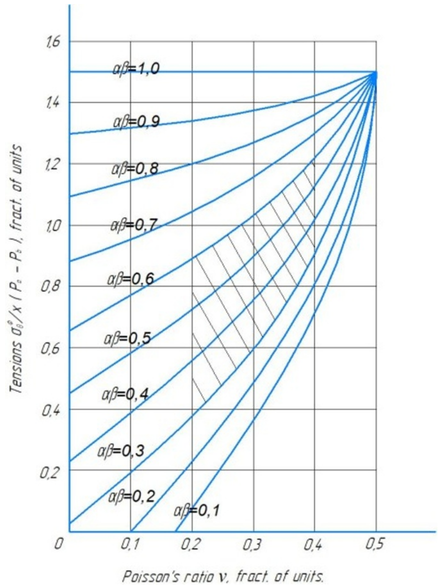

Fig. 3. Changes in filtration tangential stresses on the well wall (by $r = a$) depending on the Poisson's ratio at $\frac{b}{a} = 10$.

A change in the sign of the filtration potential leads to a change in the tangential stresses on the well wall to a value equal to a tripled depression of reservoir pressure (with radial stresses equal to zero). This explains the negative effect of well shutdowns, and even more so the change in the direction of the filtration flow in the downhole part of the formation on the stability of the walls of wells, the operation of which is complicated by sand formation.

4 Conclusion

Thus, the maximum difference of the main normal stresses is observed on the wall of the well, therefore, in order to prevent the destruction of the formation near the bottom, a necessary condition is that the strength properties of rocks correspond to the stresses acting in this zone.

When operating wells prone to plugging, it is necessary to limit the depression of reservoir pressure to the maximum permissible value when the material of the filter zone is in an elastic state throughout the volume.

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