

# Sine-cosine rotating transformers in zenith angle converters

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**Abstract.** The article proposes a principle for constructing contactless primary zenith angle converters based on sine-cosine rotating transformers. This principle involves isolating signals caused by changes in the mutual induction coefficient between the primary and secondary windings and suggests using a reference signal to establish the required functional relationship. The study revealed that errors in displaying the sine and cosine functions stem from the identical maximum inductive resistances of mutual induction, active and inductive components of the secondary load circuits, and the asymmetry of the zero points, characterized by a non-perpendicular electrical angle. It has been determined that to enhance measurement accuracy, it is essential to eliminate the reciprocity of signal phases from the parameters of signal circuits.

## 1 Introduction

Sine-cosine rotating transformers (SCRT) are widely used in modern automation and control systems, as elements of computers [1,2,3], analog and analog-to-digital converters [4,6,7]. SCRT s provide high measurement accuracy thanks to carefully developed serial production technology that minimizes instrumental errors. Issues of design and error analysis of SCRT are described in sufficient detail in the literature. Therefore, we will pay attention only to the basic properties and errors of SCRT, which are most often encountered in practice.

Let's consider the mathematical model of SCRT, for which we turn to the equivalent circuit shown in Figure 1. Let's determine the output voltages of the SCRT  $\dot{U}_s = R_{ns}\dot{I}_s$  and  $\dot{U}_c = R_{nc}\dot{I}_c$ . Neglecting losses in the steel of the magnetic core and the unevenness of the air gap, solving the following system of equations, we find the values of the currents  $I_s$  and  $I_c$  (1):

$$\begin{cases} (R_r + r_b + jx_b)\dot{I}_b + jx_{m1}\sin\theta\dot{I}_s + jx_{m2}\cos(\theta + \theta_{np})\dot{I}_c = \dot{E}_b, \\ jx_{m1}\sin\theta\dot{I}_b + (R_{ns} + r_s + jx_s)\dot{I}_s + jx_{m3}\sin\theta_{np}\dot{I}_c = 0, \\ jx_{m2}\cos(\theta + \theta_{np})\dot{I}_b + jx_{m3}\sin\theta\dot{I}_s + (R_r + r_c + jx_c)\dot{I}_c = 0, \end{cases} \quad (1)$$

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$$\begin{cases} (R_r + r_b + jx_b)I_b + jx_{m1}\sin\theta I_s + jx_{m2}\cos(\theta + \theta_{np})I_c = \dot{E}_b, \\ jx_{m1}\sin\theta I_b + (R_{ns} + r_s + jx_s)I_s + jx_{m3}\sin\theta_{np}I_c = 0, \\ jx_{m2}\cos(\theta + \theta_{np})I_b + jx_{m3}\sin\theta I_s + (R_r + r_c + jx_c)I_c = 0, \end{cases}$$

where  $r_b, r_s, r_c$ ;  $x_b, x_s, x_c$  - respectively active and inductive resistance of the SCRT windings;  $x_{m1}, x_{m2}, x_{m3}$  - maximum inductive resistances of mutual induction, respectively, between the sine winding and the excitation winding, the cosine winding and the excitation winding, the sine and cosine windings;  $R_r$  - internal resistance of the SCRT power supply;  $\theta_{np}$  is the angle characterizing the non-perpendicularity of the sine and cosine windings. We define the required voltages in the following form (2):

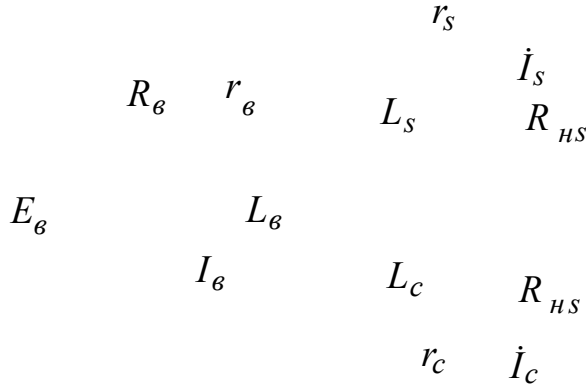
$$\dot{U}_s = \frac{R_{ns}\dot{E}_b}{\Delta} [x_{m1}x_c\sin\theta - x_{m3}\sin\theta_{np}\cos(\theta + \theta_{np}) - jx_{m1}R_c\sin\theta], \quad (2)$$

$$\dot{U}_s = \frac{R_{ns}\dot{E}_b}{\Delta} [x_{m1}x_c\sin\theta - x_{m3}\sin\theta_{np}\cos(\theta + \theta_{np}) - jx_{m1}R_c\sin\theta],$$

$$\dot{U}_c = \frac{R_{ns}\dot{E}_b}{\Delta} [x_{m2}x_s\cos(\theta + \theta_{np}) - x_{m1}x_{m3}\sin\theta_{np}\sin\theta - jx_{m2}R_c\cos(\theta + \theta_{np})], \quad (3)$$

where

$$\begin{aligned} R_s &= R_{ns} + r_s, R_c = R_{nc} + r_c \text{ and } \Delta = [R_bR_sR_c - x_bx_sR_c - x_bx_cR_s - x_sx_cR_b + \\ &R_sx_{m3}^2\sin^2\theta_{np} + R_cx_{m1}^2\sin^2\theta + R_cx_{m1}^2\sin^2\theta + R_sx_{m2}^2\cos^2(\theta + \theta_{np})] + \\ &+ j[R_sx_{m2}^2\sin\theta_{np} - 2x_{m1}x_{m2}x_{m3}\sin\theta\cos(\theta + \theta_{np})\sin\theta_{np} + x_cx_{m1}^2\sin^2\theta + \\ &+ x_sx_{m1}^2\cos^2(\theta + \theta_{np}) + R_bR_sR_c + R_bR_cx_c + R_sR_cx_b - x_bx_sx_c]. \end{aligned}$$



**Fig. 1.** SCRT equivalent circuit.

Moving on to instantaneous values, we get the following:

$$u_s = R_{ns}E_{bm}\sqrt{\frac{c_s^2+d_s^2}{a^2+b^2}}\sin\left(\omega t - \arctg\frac{ad_s+bc_s}{ac_s-bd_s}\right), \quad (4)$$

$$u_c = R_{nc}E_{bm}\sqrt{\frac{c_c^2+d_c^2}{a^2+\beta^2}}\sin\left(\omega t - \arctg\frac{ad_c+bc_c}{ac_c-bd_c}\right), \quad (5)$$

Where;

$$\begin{aligned} a &= R_bR_sR_c - x_bx_sR_c - x_bx_cR_s - x_sx_cR_b + R_bx_{m3}^2\sin^2\theta_{np} + R_cx_{m1}^2\sin^2\theta + \\ &+ R_sx_{m2}^2\cos^2(\theta + \theta_{np}); b = x_bx_{m3}^2\sin^2\theta_{np} - 2x_{m1}x_{m2}x_{m3}\sin\theta\cos(\theta + \theta_{np}) \cdot \\ &\cdot \sin\theta_{np} + x_cx_{m1}^2\sin^2\theta + x_sx_{m2}^2\cos^2(\theta + \theta_{np}) + R_bR_sx_c + R_bR_cx_s + \end{aligned}$$

$$s + R_s R_c x_B - x_B x_s x_c; d_s = x_{m1} R_c \sin \theta; d_c = x_{m2} R_s \cos(\theta + \theta_{np});$$

$$c_s = x_{m1} x_c \sin \theta - x_{m2} x_{m3} \sin \theta_{np} \cos(\theta + \theta_{np});$$

$$c_c = x_{m2} x_s \cos(\theta + \theta_{np}) - x_{m1} x_{m3} \sin \theta_{np} \sin \theta.$$

Measuring the angle of rotation of the SCRT rotor is accompanied by errors due to inaccuracy in the design of the transformer design. These include the error in displaying harmonic functions, asymmetry of zero points, residual EMF at zero points and the difference in transformation ratios. The error in displaying harmonic (sine and cosine) functions appears as a result of the identity of the maximum inductive resistances of mutual induction  $x_{m1}, x_{m2}$ , active  $R_s, R_c$  and inductive  $X_s, X_c$  components of the secondary load circuits, as well as the asymmetry of the zero points, characterized by the non-perpendicular electrical angle.  $\theta_{np}$

Due to the interconnection of the electrical circuits of the SCRT, the indicated errors are also interrelated. Thus, the difference in transformation coefficients and the asymmetry of the zero points lead to an error in the display of the harmonic function. The residual EMF at the zero points appears mainly due to the asymmetry of the zero points and determines the measurement error in the region of angles that are multiples of  $90^0$  (Figure 2).

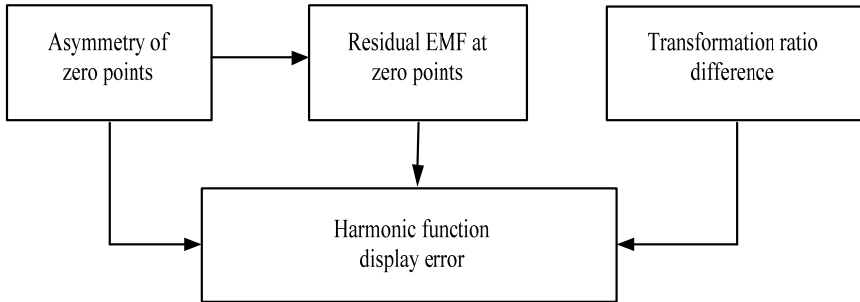


Fig. 2. Structure of the relationship between the main errors in angle measurement using SCRT.

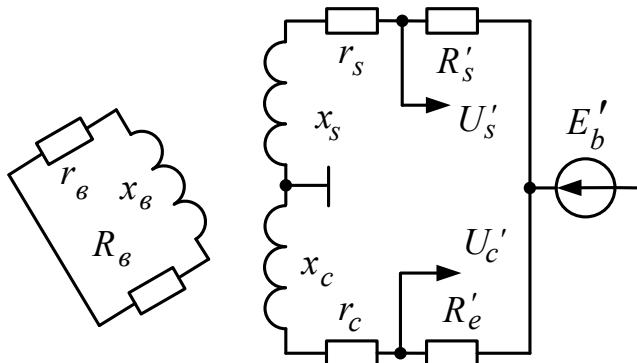


Fig. 3. Equivalent circuit for contactless switching on of SKVT.

## 2 Materials and methods

Let us analyze the operating errors of the SCRT by presenting its output signals (4) and (5) in the following form:

$$u_s = U_{sm} \sin(\omega t + \varphi_s), \tag{6}$$

$$u_c = U_{cm} \sin(\omega t + \varphi_c). \tag{7}$$

Let us assume that the input voltages of the SCVT are shifted by  $90^\circ$  through some ideal phase shifter and summed. Then the phase of the following resulting signal will carry information about the measured angle:

$$\begin{aligned} u_p &= U_{ms} \sin(\omega t + \varphi_s - 90^\circ) + U_{mc} \sin(\omega t + \varphi_c) = \\ &= M \sin \left[ \omega t + \arctg \frac{U_{sm} \sin \varphi_s + U_{mc} \sin \varphi_c}{U_{cm} \cos \varphi_c - U_{ms} \sin \varphi_s} \right] = M \sin(\omega t + \varphi_p). \end{aligned} \quad (8)$$

To analyze errors, we will use the expansion of the function describing the phase of the resulting signal into a Taylor series. Taking into account the first two terms of the expansion, we obtain the following expression:

$$\varphi_p = \varphi_p(p_i = p_{0i}) + \sum_i \left( \frac{\partial \varphi_p}{\partial p_i} \right) p_{0i} (p_i - p_{0i}), \quad (9)$$

where  $p_{0i}$  and  $p_i$  are the values of the parameters characterizing the ideal and real converter, respectively. From (10) it is clear that  $\varphi_p$  has two components, one of which describes the operation of an ideal SCRT, the second - measurement errors caused by the presence of factors  $\Delta p_i = p_i - p_{ci}$ . For definiteness, here and below, we assume that all angle parameters characterize the ideal SCRT and the factors  $\Delta p_i$  are determined by the following relative spread of parameters related to the cosines of the angle measurement:

$$\Delta p_i = p_{ci} - p_{si}. \quad (10)$$

Taking this into account, for the considered SCRT scheme, the factors  $\Delta p_i$  have the following expressions:

$$\Delta x = x_c - x_s; \Delta x_m = x_{m2} - x_{m1}; \Delta r = r_c - r_s, \quad (11)$$

$$\Delta R_n = R_{nc} - R_{ns}; \Delta \theta_{np} = (\theta - \theta_{np}) - \theta. \quad (12)$$

Thus, for an ideal SCRT, the following can be stored:

$$\varphi_p(\Delta p_i = 0) = \theta + \varphi, \quad (13)$$

Where  $\varphi_p(\Delta p_i = 0) = \varphi_p(\Delta p_i = 0) =$

$$= \arctg \frac{(R_s^2 - x_s^2)(R_b R_s - x_b x_s) + x_{m1}^2 (R_s^2 + x_s^2) + 2x_s (x_s^2 R_b - x_b R_s^2)}{(R_s^2 - x_s^2)(R_b x_s - x_b R_s) + 2R_s x_s (R_b x_b - x_b x_s)}.$$

Then the error in measuring the zenith angle can be represented in the following form:

$$\begin{aligned} \Delta \theta &= \frac{1}{U_m} \sum_i \left[ - \left( \frac{\partial U_{cm}}{\partial p_i} \right)_{\Delta p_i=0} \sin \theta + \left( \frac{\partial U_{cm}}{\partial p_i} \right)_{\Delta p_i=0} \cos \theta \right] \Delta p_i + \\ &+ \sum_i \left[ \left( \frac{\partial \varphi_s}{\partial p_i} \right)_{\Delta p_i=0} \sin \theta + \left( \frac{\partial \varphi_c}{\partial p_i} \right)_{\Delta p_i=0} \sin \theta \right] \Delta p_i. \end{aligned} \quad (14)$$

Analyzing the resulting expression, we can come to the conclusion that in order to increase the measurement accuracy, it is necessary to exclude the reciprocity of the phases of the signals from the parameters of the signal circuits, because It is the scatter of the latter that causes the error. The specified condition is met only in the idle mode of the SCRT, when the phases of the signals are determined only by the parameters of the excitation circuit. Therefore, this operating mode must be considered the main one when using SCRT in zenith angle converters.

In idle mode, the SCRT signals take the following form:

$$u_s = - \frac{x_{m1} E_{bm} \sin \theta}{\sqrt{R_b^2 + x_b^2}} \sin \left( \omega t + \arctg \frac{R_b}{x_b} \right), \quad (15)$$

$$u_s = - \frac{x_{m1} E_{bm} \cos(\theta - \theta_m)}{\sqrt{R_b^2 + x_b^2}} \sin \left( \omega t + \arctg \frac{R_b}{x_b} \right). \quad (16)$$

The error is determined only by the following two factors: the electrical asymmetry of the signal windings and the spread of the mutual induction coefficients is defined as:

$$\Delta \theta = 0,5(1 - \cos 2\theta) \theta_m - 0,5 \frac{\Delta x_m}{x_{m1}} \sin 2\theta. \quad (17)$$

As can be seen from (18), the temperature error is determined only by the correspondence of the drift to the angle of electrical perpendicularity, since the relative

spread of the mutual induction coefficients is practically independent of temperature changes. The latter is a distinctive advantage of sine-cosine converters.

When the SCRT is powered from quadrature sources  $E_{Bm} \sin(\omega t - 90^\circ)$  and  $E_{Bm} \sin(\omega t)$ , connected respectively to the excitation winding and the quadrature winding (index "k"), its operation is described by the expressions:

$$\begin{aligned} R_B \rightarrow R_S, R_S \rightarrow R_B, X_B \rightarrow X_S, X_S \rightarrow X_B, X_{m3} \rightarrow X_{m4}, \\ X_{m2} \rightarrow X_{m5}, R_C \rightarrow R_k, X_S \rightarrow X_k, \theta_{np} \rightarrow \theta_{np1}, \end{aligned} \quad (18)$$

where  $R_k, X_k$  - are the active and inductive resistance of the quadrature winding circuit, respectively;  $X_{m4}$  and  $X_{m5}$  are the coefficients of mutual induction, respectively, between the excitation and quadrature windings, as well as the sine and quadrature windings;  $\theta_{np1}$  - angle characterizing the electrical non-perpendicularity between the excitation and quadrature windings.

Measurement errors are also described by similar expressions given above, but are determined by the following factors:

$$\Delta x_{m1} = x_{m5} - x_{m4}; \Delta x = x_k - x_B; \Delta r = r_k - r_B; \Delta R_B \rightarrow \Delta R_S = R_{rk} - R_{rB}; \Delta \theta_{np1} = (\theta + \theta_{np1}) - \theta, \quad (19)$$

where  $R_{ck}$  and  $R_{rB}$  are the internal resistances of the SCRT power supplies. In this case, the measurement accuracy is increased due to the relative increase in the internal resistance of the SCRT power supplies, i.e., when using current sources. If the excitation currents are equal to each other  $I_{Bm} = I_{km} = I_{nm}$ , then the voltage on the sinus winding of the SCRT and the measurement error will be respectively equal:

$$u_s = -\frac{R_S X_{m1} I_{nm}}{\sqrt{R_S^2 + X_S^2}} \sin \left[ \omega t + \arctg \frac{R_S}{X_S} + \theta + \Delta \theta \right], \quad (20)$$

$$\Delta \theta = 0,5(1 - \cos 2\theta) \theta_m - \frac{\Delta X_m}{2 X_{m1}} \sin 2\theta. \quad (21)$$

For both options for switching on the SCRT, the criterion for the greatest accuracy is the independence of the excitation currents from the angle of rotation of the rotor, when there are no reactions of signal circuits to the excitation circuits that distort the functional dependencies of the SCRT signals. In other words, the greatest accuracy is achieved when unloading those pairs of windings that are simultaneously used on the stator or rotor side of the SCRT. The scatter of SCRT parameters is normalized and, using well-known tables, a preliminary assessment of the accuracy of angle measurement can be made depending on the accuracy class of the SCRT used. In this case, the relative spread of mutual induction resistances can be represented as the spread of SCRT transformation coefficients. This can be verified by referring to the following expressions for the mutual induction resistances and transformation coefficients of the SCRT:

$$x_{m1} = \lambda w'_B w'_S; x_{m2} = \lambda w'_B w'_C; k_{T1} = \frac{w'_S}{w'_B}; k_{T2} = \frac{w'_C}{w'_B}, \quad (22)$$

where  $w'_B, w'_S$  and  $w'_C$  are the effective numbers of SCRT turns [6];  $\lambda$  - magnetic conductivity (does not depend on the angle of rotation of the rotor due to the uniformity of the air gap), from which it is clear that the relative spreads of the effective numbers of turns of the measuring windings, transformation coefficients and mutual induction resistances are equal to each other, i.e.:

$$\frac{w'_C - w'_S}{w'_S} = \frac{k_{T2} - k_{T1}}{k_{T1}} = \frac{x_{m2} - x_{m1}}{x_{m1}}. \quad (23)$$

Thus, a quantitative assessment of the error is carried out using the normalized values of the spread of transformation coefficients and the angle of electrical perpendicularity. For example, for SKT-220-1D, class. 0.35 these values are 0.2% and 10 respectively and the angle measurement error is

$$\Delta \theta = (1 - \cos 2\theta) 0,089^\circ - 0,067^\circ \sin 2\theta. \quad (24)$$

The main disadvantage of zenith angle converters based on SCRT is the presence of sliding contacts in the latter. This is the reason for the low reliability of the converters and, due to friction in the sliding contacts, reduces the installation accuracy of the sensitive element (pendulum) of the converter. However, the use of industrial contactless SCRT does not solve the issue because they have significant dimensions and relatively low accuracy.

In this regard, a principle for constructing contactless primary zenith angle converters based on SCRT is proposed, based on the selection of signals corresponding to changes in the mutual induction coefficients between the SCRT windings. Let's assume that one of the SCRT rotor windings is shunted by resistor  $R_{sh}$  and the stator windings are connected to the power source through resistors  $R'_s = R'_c = R$  (Figure 3). Let us also assume that the SCRT is ideal, i.e.  $r_s = r_c = r_1$ ;  $x_s = x_c = x_1$ ;  $x_{m1} = x_{m2} = x_m$ ,  $\theta_{np} = 0$ .

We determine the currents  $I_s$  and  $I_c$  from the following system of equations compiled for SCRT circuits:

$$\begin{cases} (R_1 + jx_1)I_s + jx_m I_B \sin\theta = \dot{E}_B, \\ (R_1 + jx_1)I_c + jx_m I_B \cos\theta = \dot{E}_B, \\ jx_m \sin\theta I_s + jx_m I_c \cos\theta + (R_B + jx_B)I_B = 0, \end{cases} \quad (25)$$

Where  $R_1 = r + R$ ;  $R_B = r_B + R_{sh}$ . On the basis of the above, an algorithm for finding  $T_i$  is constructed, provided that  $C_1, C_2, C_3$  and  $C_4$  are known.

### 3 Results and discussion

Based on the currents  $I_s$  and  $I_c$  we will find the voltages removed from the stator windings, as  $U'_c = \dot{E}_B - R I'_s$  and  $U'_B = \dot{E}_B - R I'_c$ :

$$\dot{U}'_B = \frac{\dot{E}_B}{\Delta} [(R_B + jx_B)(R_1 + jx_1)^2 + x_m^2(R_1 + jx_1) - R(R_1 + jx_1)(R_B + jx_B) - Rx_m^2 \cos^2\theta + Rx_m^2 \sin\theta \cos\theta], \quad (26)$$

$$\dot{U}'_c = \frac{\dot{E}_B}{\Delta} [(R_B + jx_B)(R_1 + jx_1)^2 + x_m^2(R_1 + jx_1) - R(R_1 + jx_1)(R_B + jx_B) - Rx_m^2 \cos^2\theta + Rx_m^2 \sin\theta \cos\theta], \quad (27)$$

Where  $\Delta = (R_B + jx_B)(R_1 + jx_1)^2 + x_m^2(R_1 + jx_1)$ .

Expressions (27) and (28) show that the voltages  $U'_c$  and  $U'_s$  each have two components, some of which depend on the angle being measured, others are independent, constant and equal to each other. If from  $U'_c$  and  $U'_s$  we subtract the reference voltage  $U'_{op}$  obtained in some way and equal to the indicated constant components, then the resulting signals will take the following form:

$$\dot{U}'_c = U'_s - U'_{op} = \frac{Rx_m^2 E_B}{\Delta} \cos\theta (\sin\theta - \cos\theta), \quad (28)$$

$$\dot{U}'_s = U'_c - U'_{op} = -\frac{Rx_m^2 E_B}{\Delta} \cos\theta (\sin\theta - \cos\theta). \quad (29)$$

The reference voltage is removed from some equivalent resistance  $Z_e$  connected through resistance  $R$  to the power source. The value of  $Z_e$  is determined from the condition  $\dot{U}'_s = \dot{U}'_c = \dot{U}'_{op}$  at  $\theta = 45^\circ$  by the following expression:

$$Z_e = \left( r_1 + \frac{R_B x_m^2}{R_B^2 + R^2} \right) + j \left( x_1 - \frac{R_B x_m^2}{R_B^2 + R^2} \right) = R_e + jx_e. \quad (30)$$

In addition, the resulting signals can be obtained by adding and subtracting voltages

$$U_s = \dot{U}'_s + U'_c = \frac{\dot{E}_s}{\Delta} [2(R_B + jx_B)(R_1 + jx_1)^2 + 2x_m^2(R_1 + jx_1) - 2R(R_1 + jx_1)(R_B + jx_B) - Rx_m^2 + Rx_m^2 \sin 2\theta], \quad (31)$$

$$\dot{U}'_s = \dot{U}'_s - U'_c = -\frac{\dot{E}_B}{\Delta} [Rx_m^2 \cos 2\theta]. \quad (32)$$

In this case, the reference voltage is removed from the following equivalent resistance:

$$Z_e = R \left[ \frac{\Delta}{2R(R_1 + jx_1)(R_b + jx_b) + Rx_m^2 - \Delta} - 1 \right]. \quad (33)$$

Any *LC* and *RC* - serial or parallel chains - can be used as  $Z_e$ . However, the main criterion for choosing equivalent resistances should be considered the identity of the temperature drifts of voltages  $\dot{U}'_s$ ,  $\dot{U}'_c$  and  $\dot{U}'_{op}$ . Therefore, to obtain  $\dot{U}'_{op}$ , it is advisable to use additional SCRT with rigidly fixed rotors in positions  $\theta = 45^\circ$  0, in the first case, and  $\theta = 0$ , in the second case, processing signals (22) and (23).

## 4 Conclusion

Experimental studies of a non-contact zenith angle transducer determined not only its performance, but also showed that the angle measurement error does not exceed the limits specified by the accuracy class of the used SCRT.

Note that contactless converters based on the principle considered are devices in which reference signals are used to form the necessary functional dependence of the signals.

Thus, the article proposes a principle for constructing non-contact primary zenith angle converters based on sine-cosine rotating transformers, which consists in isolating signals caused by a change in the mutual induction coefficient between the primary and secondary windings, and proposes the use of a reference signal to form the required functional relationship.

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