Prospects for mathematical modeling in mining system development: accounting for global oscillations and seismic waves

Komolkhan Karimov[1], *Bakhtiyor* Mardonov[2], *Azamat* Akhmedov[1], and *Murtoza* Toirov[2*]

[1]Tashkent State Technical University, Tashkent, Uzbekistan
[2]Navoi State University of Mining and Technologies, Navoi, Uzbekistan

Abstract. The article discusses the potential of mathematical modeling in understanding the impact of vibrations and seismic waves, aiming at enhancing the sustainability of systems within the mining industry. It explores the dynamic response of a tall, elastic structure with a uniform cross-section and a fixed cylindrical fluid reservoir, subject to various complex boundary conditions. The study delves into the vibrational behavior of the structure when exposed to seismic and harmonic forces, calculating frequency, vibration periods, and deriving formulas for stress, tension, deformation, bending moments, and shear forces in different parts of the structure through both theoretical and experimental approaches. Additionally, the article derives the differential equation for the free oscillation of a tall hydraulic structure in pure bending with an incorporated mass load under appropriate boundary conditions, identifying specific vibration frequencies and periods. The forced vibration scenario is also examined, focusing on the structure's foundation movement due to external harmonic forces. Numerical computation technology is utilized to analyze the change laws of principal quantities that describe both free and forced vibrational movements of the hydraulic structure, showcasing the applicability of these models in predicting and mitigating the effects of seismic activities on mining infrastructure.

1 Introduction

Mathematical modeling of the vibrational behavior of complex hydrostructures, integral to the mining, oil, and gas industries, specifically focusing on tall structures with liquid-filled reservoirs under diverse and complicated boundary conditions, represents a critical challenge in the field of solid and fluid mechanics. Additionally, the development of mathematical models to understand the combined vibrational behavior of tall buildings equipped with cylindrical liquid reservoirs addresses several practically significant issues. These models are especially vital for optimizing fluid reservoirs found in a variety of industrial settings, including mining, metallurgy, and liquid processing plants and factories within the oil and gas sectors. This theoretical research work emphasizes the importance of such models by

* Corresponding author: murtoza.toirov@mail.ru

examining both free and forced vibrations of these structures under dynamic loads, thereby contributing to our understanding and management of these complex systems.

In the mining industry, the study focused on an elastic, homogeneous structure with a uniform cross-section and a fixed cylindrical reservoir containing liquid. The research aimed to mathematically model the free and forced vibrational movements of a cylindrical reservoir with liquid and a tall structure with an embodied mass within mining systems. The objective was to determine the roots of frequency equations under various specific conditions, thereby contributing to a deeper understanding of the dynamic behaviors of these structures.

The research aims to enhance the mathematical modeling of both free and forced vibrational movements in tall structures equipped with a fixed cylindrical tank containing liquid, commonly found within the mining industry, especially under the influence of seismic and harmonic forces.

Research tasks:

- Calculating the velocity potential function to examine the wave motion within the liquid inside the cylindrical reservoir;
- Deriving and solving frequency equations under complex boundary conditions for the free transverse vibration of a vertical column with an attached cylindrical liquid reservoir;
- Theoretically determining the dynamic coefficients of interaction between the structure and the ground;
- Establishing the dynamic displacement function for both free and forced vibrations;
- Conducting a thorough analysis of the results, focusing on key kinematic and dynamic parameters.

The research conducted on this topic has been thoroughly analyzed and summarized. It investigates the impact of seismic forces on the transverse vibration of structures. The roots of the frequency equations for free oscillating motion have been determined with high precision. Analytical solutions for the mathematical model of both free and forced vibration movements of the embodied mass in a hydraulic structure, utilizing integral substitutions, have been identified and critically analyzed.

In this study, the focus was on analyzing the frequency equation for the free and transverse vibration of a vertical column equipped with a liquid reservoir, commonly encountered in the mining industry's systems. Interaction coefficients with the soil were determined, and dynamic displacement functions were established. The investigation adopts the assumption of straight sections for the structure's cross-section and utilizes Winkler's hypothesis as the basis for understanding the laws governing the interaction between the structure and the ground.

2 Materials and methods

The text provides a comprehensive review of oscillatory motion, harmonic vibrations, wave motion, and the mathematical modeling of bodies and systems along with their combined oscillatory behaviors. It specifically highlights the results of practical research into the interaction of hydraulic structures with the ground. The study includes experimental determinations of the free vibration frequencies for the 42.8-meter high Kalon tower in Bukhara is 6,34, 25,6, 63,4, 120,5, 204,6 (c^{-1}) and for the 28-meter-high Kaltaminor Tower in Khorezm, listing their respective frequencies in 19,6, 84, 230, 462, 775 (c^{-1}) [1].

The vibrational behavior of vertical columns is significantly influenced by the ground characteristics where these columns are installed. Literature offers extensive insights into soil composition, mechanics, and its effects on structural dynamics. Numerous researchers have delved into boundary condition problems, examining the oscillatory behavior of concrete mass hydrostructures, especially those featuring cylindrical liquid reservoirs. This area has seen a wealth of practical experiments conducted by specialists in the field. Studies have been and continue to be undertaken to understand how the ground conditions affect both the free and forced vibrations of such hydraulic structures. This ongoing research contributes to a deeper comprehension of the interaction between soil properties and structural vibrational responses. [2, 3].

In this field, the scientific works and results obtained by Academician H.A. Rahmatulin and his students are presented in the monograph [4]. These works are worthy of high attention. Professor I.M. Babakov has generally derived the frequency equations for the transverse vibrational motion of beams with embodied mass [5].

It is known that the issues of vibrational motion of structures with embodied mass are of significant importance in engineering fields. Determining the frequencies and periods of vibrational motion under the influence of external seismic and harmonic forces, and calculating the bending moments and shearing forces in various parts of the structure present considerable challenges.

In turn, the vibrational motion issues of hydraulic structures with embodied mass differ in systems connected with liquids. In this context, it is necessary to consider the motion of the liquid while studying the movement of columns. Several works in this area are presented in [6]. The problems of free and forced vibrational motion of a vertical column with a cylindrical reservoir containing liquid, partially submerged in water, have been studied.

The equation for the vibrational motion of a vertical column attached to a cylindrical tank containing liquid, in its most comprehensive form, alongside frequency equations, expressions for shearing force, and twisting-bending moments at various cross-section points, are detailed. This includes the general case of the frequency equation for free oscillating motion and several special instances [7-8].

The study also touches upon hydroelastic systems in motion, focusing on the dynamic interaction between fluid-based structures and the ground. These structures may exhibit stationary or oscillatory non-stationary movements. The analysis necessitates considering the structure's impact on movement, ground interaction, and the physical properties and conditions that frame the system.

Specifically, the vibrational movements of bodies with fixed mass are examined through the lens of hydromechanics, incorporating the pressure and torque exerted by an ideal, incompressible fluid on the cylinder containing it. The hydrodynamic pressure exerted by the liquid is typically assessed based on findings from.

This research represents an extension of these discussions, delving into complex challenges around the free and forced vibration of tall structures with attached cylindrical liquid tanks, particularly under seismic and harmonic forces' influence.

Further, the article encourages readers to explore various practical challenges related to mathematical modeling of intricate systems and technological processes, alongside experimental research outlined in the literature [9, 10], thus providing a comprehensive view into the advanced study of vibrational dynamics in engineering structures

The research draws upon the principles of classical mechanics, the theory of elasticity, and the laws governing the vibration of elastic bodies. To address the partial differential equations involved, Laplace-Carson integral substitutions were employed. The analytical solutions obtained, along with the main kinematic and dynamic quantities, were subjected to both analytical and graphical analysis using the Maple 18 software package.

This study focuses on a cylindrical tank filled with an ideal, incompressible fluid, mounted on a tiltable column that is vertically secured to the ground and has a length l. The investigation centers on the problem of free transverse vibration of the column in conjunction with the liquid-filled reservoir.

We consider the free transverse motion of a vertical column to which a cylindrical tank containing liquid is fixed. The differential equation of free transverse vibration motion for the tilting function $U(z,t)$ of the column with respect to the main axis is as follows [1]:

$$U^{(IV)}(z,t)+c^2\ddot{U}(z,t)=0,\ c^2=\frac{\rho_2 A}{EI}, \quad (1)$$

where ρ_2 - density of column material, A - column cross-sectional surface, E - Young's modulus for the column material, I - moment of inertia.

$U(z,t)$ boundary conditions for: $z=0$

$$EIU''(z,t)=K_\varphi U'(z,t), \quad (2)$$

$$EIU'''(z,t)=-K_z U(z,t), \quad (3)$$

K_z, K_φ - magnitudes that are determined by experience, characterizing the soil.

$$z=l$$

$$EIU'''=(m_r+m_j)\ddot{U}+\frac{2m_j}{h_0}\sum_{n=1}^{\infty}a\ddot{f}_n(t)\varepsilon_n\xi_n th(\xi_n h_0), \quad (4)$$

$$EIU''=-m_r\frac{H}{2}\ddot{U}-\frac{m_j h}{2}\ddot{U}(1+\frac{1}{2h_0})+\frac{2h}{{h_0}^2}m_j\sum_{n=1}^{\infty}a\ddot{f}(t)\varepsilon_n d_{n0}, \quad (5)$$

m_r - reservoir mass, m_j - the mass of liquid in the reservoir, $h_0=\frac{h}{a}$, $\varepsilon_n=\frac{1}{{\xi_n}^2({\xi_n}^2-1)}$.

$z=l$ relative displacement of the column at $U(z,t)$ with $f_n(t)$ function is bound as follows:

$$\ddot{f}_n(t)+\varpi_n^2 f_n(t)=-\frac{1}{a}\ddot{U}|_{z=l},\ (n=1,\ 2,\dots), \quad (6)$$

(1) solution

$$U(z,t)=Z(kz)\cos\omega t \quad (7)$$

look for it.

$$Z(kz)=AS(kz)+BT(kz)+CL(kz)+DV(kz).$$

(7) where ω - the frequency of free oscillation of a vertical column to which a cylindrical reservoir of liquid is fixed is to be determined, S,T,L,V - Krylov functions, A,B,C,D - unchanging.

(7) → (2), (3):

$$BK_\varphi-C\frac{EI}{l}\gamma=0, \quad (8)$$

$$AK_z+DEI\frac{\gamma^3}{l^3}=0 \quad (9)$$

will get. Here $\gamma=kl$, $k^4=\frac{\rho_2 A}{EI}\omega^2$.

(6) solution for $f_n(t)=Q_n\cos\omega t$ can be found,

$$a\ddot{f}_n(t)=\frac{\omega^4}{\omega^2-\varpi_n^2}Z(kl)\cos\omega t \quad (10)$$

(4), (5) from the boundary conditions and (8), (9) of relations C and D we form the following system of homogeneous equations with respect to constants:

$$C\left[V(\gamma)+\frac{EI}{lK_\varphi}\gamma L(\gamma)+\gamma\beta(\gamma)(L(\gamma)+\frac{EI}{lK_\varphi}\gamma T(\gamma))\right]+D\left[S(\gamma)-\frac{EI}{K_z l^3}\gamma^3 T(\gamma)+\right.$$
$$\left.+\gamma\beta(\gamma)(V(\gamma)-\frac{EI}{K_z l^3}\gamma^3 S(\gamma))\right]=0,$$
$$C\left[S(\gamma)+\frac{EI}{lK_\varphi}\gamma V(\gamma)-\frac{\gamma^2}{l}\alpha(\gamma)(L(\gamma)+\frac{EI}{lK_\varphi}\gamma T(\gamma))\right]+D\left[T(\gamma)-\frac{EI}{K_z l^3}\gamma^3 L(\gamma)-\right.$$
$$\left.-\frac{\gamma^2}{l}\alpha(\gamma)(V(\gamma)-\frac{EI}{K_z l^3}\gamma^3 S(\gamma))\right]=0. \quad (11)$$

The frequency equation of the free oscillation motion of a vertical column fixed to a cylindrical tank filled with liquid is derived from the condition of existence of constants in the system of equations (11) formed C, D from the boundary conditions.

(11) C , D the determinant formed from the coefficients in front of the unknowns must be equal to 0.

Here,

$$\left[V(\gamma)+\frac{EI}{lK_\varphi}\gamma L(\gamma)+\gamma\beta(\gamma)(L(\gamma)+\frac{EI}{lK_\varphi}\gamma T(\gamma))\right]\cdot\left[T(\gamma)-\frac{EI}{K_z l^3}\gamma^3 L(\gamma)-\right.$$
$$\left.-\frac{\gamma^2}{l}\alpha(\gamma)(V(\gamma)-\frac{EI}{K_z l^3}\gamma^3 S(\gamma))\right]-\left[S(\gamma)-\frac{EI}{K_z l^3}\gamma^3 T(\gamma)+\gamma\beta(\gamma)(V(\gamma)-\right.$$
$$\left.-\frac{EI}{K_z l^3}\gamma^3 S(\gamma))\right]\cdot\left[S(\gamma)+\frac{EI}{lK_\varphi}\gamma V(\gamma)-\frac{\gamma^2}{l}\alpha(\gamma)(L(\gamma)+\frac{EI}{lK_\varphi}\gamma T(\gamma))\right]=0. \quad (12)$$

follows

$$S(\gamma)=\frac{1}{2}(ch\gamma+\cos\gamma)\quad,\quad T(\gamma)=\frac{1}{2}(sh\gamma+\sin\gamma),$$
$$L(\gamma)=\frac{1}{2}(ch\gamma-\cos\gamma)\quad,\quad V(\gamma)=\frac{1}{2}(sh\gamma-\sin\gamma) \quad (13)$$

If we write in terms of Krylov functions, (12) takes the following form:

$$1+ch\gamma\cos\gamma+\frac{EI}{K_z l^3}\gamma^3\left[ch^2\gamma-\cos^2\gamma+\sin\gamma(\cos\gamma-ch\gamma)\right]-\frac{\alpha(\gamma)}{l}\gamma^2 sh\gamma\sin\gamma-$$
$$-\frac{2EI}{K_\varphi l}\gamma^2\beta(\gamma)sh\gamma\sin\gamma-(ch\gamma\sin\gamma-sh\gamma\cos\gamma)\left[\gamma\beta(\gamma)+\frac{EI}{lK_\varphi}-\frac{EI}{lK_\varphi}\frac{EI}{K_z l^3}\gamma^5\beta(\gamma)\right]-$$
$$-\frac{EI}{lK_\varphi}\gamma^3\frac{\alpha(\gamma)}{l}(ch\gamma\sin\gamma+sh\gamma\cos\gamma)-\frac{EI}{2K_z l^3}\gamma^4\beta(\gamma)(ch\gamma+\cos\gamma)^2+\frac{EI}{K_z l^3}\frac{EI}{lK_\varphi}\gamma^4\cdot$$
$$\cdot(1-ch\gamma\cos\gamma)+\frac{EI}{2lK_\varphi}\gamma^4(ch\gamma-\cos\gamma)^2-\frac{1}{2}\gamma^6\frac{EI}{K_z l^3}\frac{\alpha(\gamma)}{l}(ch^2\gamma+\cos^2\gamma)+$$
$$+\frac{1}{2}\gamma^6\frac{EI}{K_z l^3}\frac{EI}{lK_\varphi}\frac{\alpha(\gamma)}{l}(sh\gamma+\sin\gamma)^2=0\quad, \quad (14)$$

$$\beta(\gamma)=\frac{m_j}{\rho_2 Al}\left[\frac{m_r+m_j}{m_j}+\frac{1}{h_0}\sum_{n=1}^{\infty}\frac{\delta c_n}{\xi_n-\delta cth(\xi_n h_0)}\right],\ \delta=\frac{a\omega^2}{g},$$
$$\alpha(\gamma)=\frac{h}{2}\frac{m_j}{\rho_2 Al}(1+\frac{1}{2h_0{}^2})+\frac{H}{2}\frac{m_r}{\rho_2 Al}-\frac{h}{l}\frac{1}{\rho_2 A}\frac{2m_j}{h_0{}^2}\sum_{n=1}^{\infty}\frac{\varepsilon_n d_{n0}}{\varpi_n^2\frac{\rho_2 A}{EI}(l/\gamma)^4-1},$$
$$c_n=\frac{2}{\xi_n(\xi_n{}^2-1)},\ \omega_k=\frac{\gamma_k^2}{l^2}\sqrt{\frac{EI}{\rho_2 A}}.$$

Special cases:

1) The base of the column is rigidly fixed $U(0,t)=U'(0,t)=0$, (14) the equation becomes:

$$1+ch\gamma\cos\gamma-\gamma\beta(\gamma)(ch\gamma\sin\gamma-sh\gamma\cos\gamma)-\frac{\alpha(\gamma)}{l}\gamma^2 sh\gamma\sin\gamma=0. \quad (15)$$

$z=l$ if there is no torsion of the reservoir, $\alpha(\gamma)=0$ (15) the equation takes on a simpler form:

$$1+ch\gamma\cos\gamma-\gamma\beta(\gamma)(ch\gamma\sin\gamma-sh\gamma\cos\gamma)=0. \quad (16)$$

2) The base of the column should be fixed so that, $z=0$, at the base of the column there is a displacement, but there is no torsion, that is $K_z\prec\infty$, $K_\varphi\to\infty$. This is the case $z=0$

$$U'(z,t)=0,\ U(z,t)=-\frac{EI}{K_z}U^{III}(z,t)$$

comes to the boundary conditions. As a result, the frequency equation (14)

$$1+ch\gamma\cos\gamma+\frac{EI}{K_z l^3}\gamma^3\left[ch^2\gamma-\cos^2\gamma+\sin\gamma(\cos\gamma-ch\gamma)\right]-\frac{\alpha(\gamma)}{l}\gamma^2 sh\gamma\sin\gamma-$$

$$-\gamma\beta(\gamma)(ch\gamma\sin\gamma-sh\gamma\cos\gamma)-\frac{EI}{2K_zl^3}\gamma^4\beta(\gamma)(ch\gamma+\cos\gamma)^2-$$
$$-\frac{1}{2}\gamma^6\frac{EI}{K_zl^3}\frac{\alpha(\gamma)}{l}(ch^2\gamma+\cos^2\gamma)=0 \quad (17).$$

3. The base of the column should be fixed so that, $z=0$, at the base of the column, there should be no displacement, but torsion, that is $K_z\to\infty$, $K_\varphi\prec\infty$. This is the case $z=0$

$$U(z,t)=0,\ U'(z,t)=\frac{EI}{K_\varphi}U^{II}(z,t)$$

comes to the boundary conditions. The frequency equation

$$1+ch\gamma\cos\gamma-\frac{\alpha(\gamma)}{l}\gamma^2sh\gamma\sin\gamma-\frac{2EI}{K_\varphi l}\gamma^2\beta(\gamma)sh\gamma\sin\gamma-(ch\gamma\sin\gamma-sh\gamma\cos\gamma)\cdot$$
$$\cdot\left(\gamma\beta(\gamma)+\frac{EI}{lK_\varphi}\right)-\frac{EI}{lK_\varphi}\gamma^3\frac{\alpha(\gamma)}{l}(ch\gamma\sin\gamma+sh\gamma\cos\gamma)+\frac{EI}{2lK_\varphi}\gamma^4(ch\gamma-\cos\gamma)^2=0 \quad (18)$$

takes over the view.

3 Results and discussion

3.1 Determination of the roots of frequency equations under various complex boundary conditions

We determine the roots of the frequency equations of free oscillation motion derived in the above-mentioned different boundary conditions by numerical methods using the Maple 18 mathematical package and programming languages.

In the first case

$$1+ch\gamma\cos\gamma-\gamma\beta(\gamma)(ch\gamma\sin\gamma-sh\gamma\cos\gamma)-\frac{\alpha(\gamma)}{l}\gamma^2sh\gamma\sin\gamma=0 \quad (19)$$

it is necessary to determine the roots of the equation.

It is known,

$$\beta(\gamma)=\frac{m_j}{\rho_2Al}\left[\frac{m_r+m_j}{m_j}+\frac{1}{h_0}\sum_{n=1}^{\infty}\frac{\delta c_n}{\xi_n-\delta cth(\xi_nh_0)}\right],$$
$$\alpha(\gamma)=\frac{h}{2}\frac{m_j}{\rho_2Al}\left(1+\frac{1}{2h_0{}^2}\right)+\frac{H}{2}\frac{m_r}{\rho_2Al}-\frac{h}{l}\frac{1}{\rho_2A}\frac{2m_j}{h_0{}^2}\sum_{n=1}^{\infty}\frac{\varepsilon_nd_{n0}}{\varpi_n^2\frac{\rho_2A}{EI}(l/\gamma)^4-1},$$

will be $\varepsilon_n=\frac{1}{\xi_n{}^2(\xi_n{}^2-1)}$, $d_{n0}=1-\xi_nh_0th(\xi_nh_0)-\frac{2}{ch(\xi_nh_0)}$.

(15) we determine the root of the frequency equation for the following specific case.

1) $H=5m$, $h=4m$, $a=2m$, $E=2\cdot10^{11}\frac{N}{m^2}$, $I=\frac{4}{3}m^4$, $l=20m$, $\frac{m_r}{m_j}=0{,}1$, $\frac{m_j}{m_s}=0{,}1$.

In this case

$$\beta(\gamma)=0{,}11-0{,}267\left[\frac{0{,}228}{5{,}34\gamma^4-9{,}2}+\frac{0{,}0068}{5{,}34\gamma^4-26{,}65}+\frac{0{,}0016}{5{,}34\gamma^4-42{,}7}\right.$$
$$+\frac{0{,}0006}{5{,}34\gamma^4-58{,}55}+$$
$$\left.+\frac{0{,}0003}{5{,}34\gamma^4-74{,}3}+\frac{0{,}00017}{5{,}34\gamma^4-90}+\frac{0{,}0001}{5{,}34\gamma^4-105{,}8}\right],$$
$$\alpha(\gamma)=0{,}25-1{,}068\gamma^4\left[\frac{0{,}333}{5{,}34\gamma^4-9{,}2}+\frac{0{,}012}{5{,}34\gamma^4-26{,}65}+\frac{0{,}003}{5{,}34\gamma^4-42{,}7}\right.$$
$$+\frac{0{,}001}{5{,}34\gamma^4-58{,}55}+$$
$$\left.+\frac{0{,}0005}{5{,}34\gamma^4-74{,}3}+\frac{0{,}00033}{5{,}34\gamma^4-90}+\frac{0{,}00021}{5{,}34\gamma^4-105{,}8}\right].$$

(19) the roots of the equation

$\gamma_1 = 1{,}7165, \gamma_2 = 4{,}3026, \gamma_3 = 7{,}2494, \gamma_4 = 10{,}2251, \gamma_5 = 13{,}2406, \gamma_6 = 16{,}2857,$ $\gamma_7 = 19{,}3525, \gamma_8 = 22{,}4351, \gamma_9 = 25{,}5293$

Determined γ_k , $\omega_k = \frac{\gamma_k^2}{l^2}\sqrt{\frac{EI}{\rho_2 A}}$ from the expression, the roots of the frequency are determined as one-valued and $U(z,t) = Z(kz)\cos\omega t$ we will be able to determine dynamic displacements according to the expression. The bending moment and shearing force expressions generated in the cross-sections of the structure are formed by taking derivatives of the appropriate order from the expression (7).

Thus, the roots of the frequency equation for the first special case are completely determined. In specific cases, we assumed that 0,8, 0,75, 0,5 and 0,3 parts of the reservoir were filled with liquid, in which cases the column cross-section was considered for rectangular, circular, triangular, and arbitrary cases. In turn, the material of the column was also considered differently, in particular, iron, cast iron and brick.

From the obtained solutions, it became clear that when the liquid is filled to half of the tank, the roots of the frequency equation grow faster than in other cases. The roots of the Chatot equations are almost the same in cases where 0,75 is filled and 0,8 is filled. The length of the column is longer, and the smaller the combined mass of the liquid with the cylindrical tank, the higher the frequencies of free oscillation.

4 Conclusion

The research yielded the following pivotal results:

1. Determined the roots of the frequency equation for free transverse vibrations in tall hydraulic structures commonly found in mining systems, across various specific cases.
2. Addressed the challenge of theoretically determining the dynamic coefficients of interaction between the structure and the soil.
3. Analyzed and determined the roots of the derived frequency equation for different boundary conditions, utilizing programming languages and numerical methods.
4. Employed the Fourier method to seek a general solution, identifying and analyzing individual vibration form functions corresponding to each vibration frequency through Laplace-Carson integral substitutions.
5. Derived the equation for the forced transverse vibration of the hydraulic structure under the influence of an external harmonic force.
6. Investigated the impact of fluid movement within cylindrical reservoirs on the structure's free and forced vibrations.

These investigations are considered to have practical significance. The main findings serve as a reference for addressing related practical problems. Theoretically derived results, based on comprehensive experiments, offer time, energy, and labor savings, contributing to economic efficiency.

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