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Algorithms for adaptive control with limited disturbance compensation

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Abstract. This paper proposes an adaptive control scheme for systems affected by parametric uncertainty and limited external disturbances, without requiring measurements of the controlled variable's derivatives. The control approach provides compensation for these disturbances, addressing control problems with a reference model. Adaptive control is a technique where a system's parameters are adjusted based on external conditions or internal changes. In the context of constrained disturbance compensation, control algorithms must account for limitations on available resources or system capacity to counteract disturbances. Such algorithms can be beneficial in situations where the system has limited resources or where disturbances cannot be fully eliminated. The proposed adaptive control scheme aims to effectively manage systems subject to parametric uncertainty and bounded external disturbances, without the need for derivative measurements of the controlled variable. The control approach compensates for these disturbances, making it suitable for applications where complete disturbance rejection is not feasible due to resource or capacity constraints.

1 Introduction

The problem of compensating uncontrolled disturbances and parametric uncertainty is a long-standing challenge in control theory. Despite a vast body of scientific literature describing various methods and approaches to address this complex problem, the search for innovative control system methods and principles continues to evolve. In recent years, several promising studies [1-5] have explored various control system strategies capable of effectively compensating for the impact of uncontrolled disturbances on the system's output with high precision.

One approach [2] involves using state observer techniques applied to a problem-oriented model of the controlled system. An algorithm for control is developed through an iterative "backstepping" procedure, which can compensate for the effects of constant external disturbances on the controlled variable.

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Another approach, as presented in works [3, 4], is based on the use of an internal model. This approach proposes the synthesis of observers for external deterministic disturbances, using a model to estimate the external influences. This method enables the development of a control system capable of effectively compensating for these influences.

Most similar in methodology to the presented article is the study in reference [6], in which the original model of the controlled system is transformed by introducing a new control signal in the first-order equation with a known gain coefficient. In this case, the disturbances consist of unknown parameters and an external signal. An observer is used to estimate the disturbance and its derivatives, and a special signal is generated for this purpose, which allows obtaining the corresponding estimates. A similar problem is also solved in [7-11], but with known parameters of the controlled system.

However, the main challenge in synthesizing control systems for disturbance compensation lies in developing a signal that carries information about them and can be used to obtain the required estimates. This article presents one possible structure of an adaptive control system capable of compensating for parametric uncertainty and limited external disturbances with high precision. For this purpose, a signal formation scheme is proposed, through which disturbance estimation is performed, and then a control signal is generated to ensure the necessary dynamic accuracy and transient time.

It is important to note that in this work, all parameters of the controlled system are considered unknown, and the implementation of the method is simplified as it does not require the formation of a regression vector. This represents a significant achievement in the field of control systems with parametric uncertainty and uncontrolled disturbances.

2 Problem statement

In this case, the equations $\partial R / \partial C_j = 0$ make an appearance:

Suppose that the dynamic processes in the control object are described by the equation.

$$Q(p)y(t) = kR(p)u(t) + f(t) \tag{1}$$

Where $y(t)$ - adjustable value, $u(t)$ - controlling influence, and $f(t)$ - disturbing effect. In this study, normalized linear differential operators are considered $Q(p), R(p)$ of orders n and m are considered, where the symbol $p = d/dt$ represents a differential operator. It is important to note that the coefficients in these operators remain unknown parameters. The parameter k, which should be positive ($k > 0$), adds an additional aspect to this problem.

Let the reference model be given in the form of the equation:

$$Q_m(p)y_m(t) = k_m R_m(p)g(t) \tag{2}$$

here $g(t)$ - limited setting effect, $k_m > 0$, $Q_m(p), R_m(p)$ - is a normalized linear differential operator of order m, $y_m(t)$ - reference signal.

The control objective in this context is to achieve a specified relationship

$$\lim_{t\to\infty}|e(t)| = \lim_{t\to\infty}|y(t) - y_m(t)| < \delta \tag{3}$$

using an appropriate control law. However, within the framework of adaptive control systems with a reference model and compensation for parametric uncertainty and uncontrolled disturbances, several important assumptions and conditions are typically considered when developing such systems.

- The coefficients of the operators $Q(p)$ and $R(p)$, and the number $k > 0$, are constant unknown values.
- The polynomials $Q_m(\lambda), R_m(\lambda), R(\lambda)$ - are Gurwitz polynomials, where λ - represents a complex variable in Laplace transformations.

- The orders $n, m, \gamma = n - m > 2$ are known.
- The reference input $f(t)$, $g(t)$ - is a bounded function.
- Signal derivatives cannot be used in the control system $y(t), u(t), g(t)$.

3 Results and discussion

In this research work, they adhere to the principle that all signals in a closed system should be limited in magnitude, which contributes to ensuring the safety and stability of the system.

To lead to the main result of the proposed article, attention is focused on an exceptional methodological approach [12-20] based on the analysis of derivatives of input and output signals that are available for measurement. This method represents a unique and innovative way that provides compelling evidence of the effectiveness and feasibility of our new control algorithm.

During the development of the control law, a decisive decision is made to choose one that provides the optimal solution for this task. This approach takes into account all contextual features and unique characteristics of this research.

For further analysis, a control law is selected, which has the following form:

$$u(t) = T(p)v(t) \tag{4}$$

where $v(t)$ - is an auxiliary control input, adding an additional layer of complexity to the task. The orders of the differential operators $T(p)$ and $n - m - 1$ are equal, which represents an interesting mathematical aspect. Furthermore, we choose the coefficients $Q_m(p)$ and $R_m(p)$ in such a way as to ensure the specified equality.

$$\frac{R_m(p)T(p)}{Q_m(p)} = \frac{1}{(p+a)}, \ a > 0 \tag{5}$$

Let us decompose operators $Q(p), R(p)$ into two terms, $R(p) = R_m(p) + \Delta R(p)$, $Q(p) = Q_m(p) + \Delta Q(p)$. Operators $\Delta R(p)$ and $\Delta Q(p)$, with unknown coefficients, have orders $m - 1$ and $n - 1$, respectively.

Given this system of equations and equation (1), there is a fundamental opportunity for transforming this problem. Equation (1), in conjunction with (4), undergoes the following change in form:

$$Q_m(p)y(t) = kR_m(p)T(p)v(t) - \Delta Q(p)y(t) + k\Delta R(p)T(p)v(t) + f(t) \tag{6}$$

Using (5), express (6) in the following form:

$$y(t) = \frac{kR_m(p)}{Q_m(p)}\left(u(t) + \frac{\Delta R(p)}{R_m(p)}u(t) - \frac{\Delta Q(p)}{kR_m(p)}y(t) + \frac{f(t)}{kR_m(p)}\right) \tag{7}$$

$$y_m(t) = \frac{k_m R_m(p)}{Q_m(p)}g(t) \tag{8}$$

By subtracting equations (7) and (8), we obtain equation (9).

$$y(t) - y_m(t) = \frac{kR_m(p)}{Q_m(p)}\left(u(t) + \frac{\Delta R(p)}{R_m(p)}u(t) - \frac{\Delta Q(p)}{kR_m(p)}y(t) - \frac{k_m}{k}g(t) + \frac{f(t)}{kR_m(p)}\right) \tag{9}$$

Let's formulate an equation for the error $e(t) = y(t) - y_m(t)$, taking into account (4) and (9):

$$e(t) = \frac{kR_m(p)T(p)}{Q_m(p)}\left(v(t) + \frac{\Delta R(p)}{R_m(p)}v(t) - \frac{\Delta Q(p)}{kR_m(p)T(p)}y(t) - \frac{k_m}{kT(p)}g(t) + \frac{\varepsilon(t)}{k}\right) \tag{10}$$

where $\varepsilon(t) = \dfrac{f(t)}{R_m(p)T(p)}$

Let's introduce filters:

$$\begin{aligned} &\dot V_y(t) = \Phi_1 V_y(t) + by(t),\ \ V_y(0) = 0 \\ &\dot V_u(t) = \Phi_1 V_v(t) + bu(t),\ \ V_v(0) = 0 \\ &\dot V_g(t) = \Phi_2 V_g(t) + bg(t),\ \ g(t) = LV_g(t),\ \ V_g(0) = 0 \end{aligned} \tag{11}$$

In this research work, equations (11) are used. Here, $V_y(t) \in R^{n-1}, V_g(t) \in R^{\gamma-1}, V_v(t) \in R^m$ represents state vectors of filters, and Φ_1, Φ_2 - are numerical matrices formed in Frobenius form, with characteristic polynomials $R_m(\lambda)T(\lambda)$ and $T(\lambda)$. Additionally, a matrix $b = [0,...,0,1]^T, L = [1,0,...,0]$ is introduced. In the context of the study, matrices b and L have corresponding dimensions for each of the considered equations, which is a significant aspect in this work. These matrices can be used to determine the characteristics of the system and control.

Taking into account (11), equations (5), (7) and (8) are transformed into the form:

$$y_m(t) = \frac{k_m}{p+a} g_r(t),\ \ y(t) = \frac{k}{p+a}\left(v(t) - c_{01}y(t) - c_{02}^T V_y - c_{03}^T V_v + \frac{f(t)}{kR_m(p)T(p)} \right) \tag{12}$$

where $g_r(t) = \frac{g(t)}{T(p)}, \frac{\varepsilon(t)}{k} = \frac{f(t)}{kR_m(p)T(p)}$, and c_{01} - coefficients obtained when extracting the integer part of the expression $\frac{\Delta Q(p)}{kR_m(p)T(p)} = c_{01} + \frac{\Delta \overline{Q}(p)}{kR_m(p)T(p)}$.

In this unique mathematical context, the vector c_{02} - represents a set of coefficients that make up the polynomial $\Delta\overline{Q}(p)$, and they are divided by the coefficient k, adding further complexity to this system. The vector c_{03}, in turn, contains coefficients of the polynomial $\Delta R(p)$, but with opposite signs.

Let's introduce vectors:

$$c_0^T = \left[c_{01}, c_{02}^T, c_{03}^T, \frac{k_m}{k}, \frac{1}{k} \right], \omega^T = \left[y(t), V_y^T(t), V_v^T(t), g_r(t), \varepsilon(t) \right] \tag{13}$$

Then, from (9), we obtain the error equation.

$$\dot e(t) = -ae(t) + k(v(t) - c_0^T \omega(t))$$

Choosing the control algorithm in the form:

$$v(t) = c^T(t)\omega(t), \quad \dot c(t) = -\rho e(t)\omega(t) - \alpha e^2(t)c(t) \tag{14}$$

Where $\rho > 0$ $\alpha > 0$, we obtain the equations of the closed-loop system:

$$\dot e(t) = -ae(t) + k(c(t) - c_0)^T \omega(t) \tag{15}$$

Within the framework of this unique system of equations (11) and (15), the research goal is to demonstrate the dissipativity of the system and determine the number ρ_0 that satisfies the condition $\lim_{t\to\infty} e(t) = 0$ under the given $\rho \ge \rho_0$. Let's take a Lyapunov function

$$V_1 = he^2(t) + \frac{k}{\rho_1}\bar c^T(t)\bar c(t),$$

where $h > 0, \rho = h\rho_1, \bar c(t) = c(t) - c_0$, and calculate the total derivative according to (11) and the fourth equation (15):

$$\dot V_1 = 2ahe^2(t) - 2\frac{\alpha k}{\rho_1}\bar c^T(t)e^2(t)c(t) \tag{16}$$

Using identity $-2\bar{c}^T(t)c(t) = -\bar{c}^T(t)\bar{c}(t) - c^T(t)c(t) + |c_0|^2$, then from (11), we obtain:

$$\dot{V}_1 \leq -2ahe^2(t) - \frac{\alpha k}{\rho_1}\bar{c}^T(t)e^2(t)\bar{c}(t) - \frac{\alpha k}{\rho_1}e^2c^T(t)c(t) + \frac{\alpha k}{\rho_1}e^2|c_0|^2 \tag{17}$$

from which it follows:

$$\dot{V}_1 < -e^2\left(2ah - \frac{\alpha k}{\rho_1}e^2|c_0|^2\right) \tag{18}$$

If we choose the numbers ρ_1, h in the condition $2ah - \frac{\alpha k}{\rho_1}e^2|c_0|^2 > 0$, then.

To illustrate the application possibilities of the proposed approach, let's consider the results of a specific model research [7-11].

The system is described by a third-order equation $n = 3$:

$$(P^3 + a_1P^2 + a_2P + a_3)y(t) = b_0u(t) + f(t)$$

And the reference model is given in the form:

$$(P^3 + 6P^2 + 11P + 6)y^*(t) = g(t)$$

The operator $T(p) = P^2 + 2P + 1$, then the number "a" in equation (15) is equal to 1, and the filters take the following form:

$$\dot{V}_{y1} = V_{y2},\ \dot{V}_{y2}(t) = -V_{y1} - 2V_{y2} + y(t)$$

$$\dot{V}_{g1} = V_{g2},\ \dot{V}_{g2}(t) = -V_{g1} - 2V_{g2} + g(t)$$

One filter is excluded because $\deg R_m(p) = 0$. The regression vector:

$$\omega^T = \left[y(t), V_y^T(t), V_v^T(t), g_r(t), \varepsilon(t)\right]$$

The filter is in the form [9]:

$$\dot{\xi}_1 = \xi_2 - \frac{6}{\mu}(\xi_1 - v(t)),\ \ \dot{\xi}_2 = -\frac{12}{\mu^2}(\xi_1 - v(t)),$$

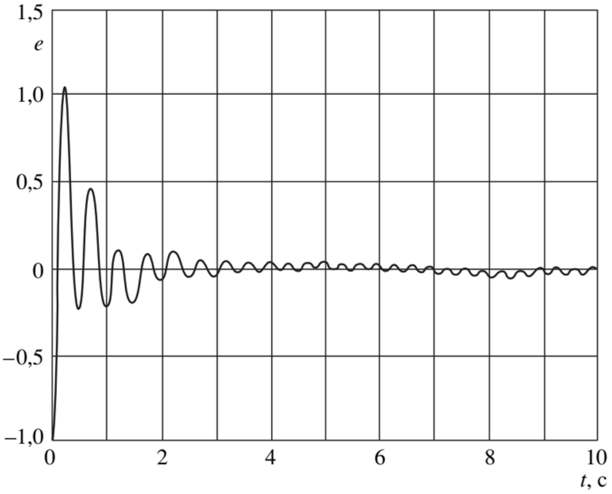

Fig. 1. Schedule change deviation.

The control law is formulated as:

$$u(t) = \xi_1 + 2\xi_2 + \dot{\xi}_2,\ \ v(t) = c^T(t)\omega(t)$$

$$\dot{c}(t) = -\rho e(t)\omega(t) - \alpha e^2(t)c(t)$$

Figure 1 depicts a plot of the error $e(t) = y(t) - y_m(t)$ as a function of the following data: $y(0) = \dot{y}(0) = \ddot{y}(0) = 1$, with all other initial conditions in the system being zero; $k = 2,\ a_1 = a_2 = a_3 = -5;\ g(t) = 1 + \sin t,\ \mu = 0{,}01,\ \rho = 20,\ \alpha = 5$, etc.

4 Conclusion

This work considers the classical problem of adaptive control with a reference model for a linear system characterized by unknown parameters. A distinctive feature of this research is the lack of access to derivatives of both input and output signals, which makes the problem more complex and necessitates innovative solutions.

Within this work, a modified high-order adaptive algorithm is proposed and rigorously justified. In this algorithm, the number of adjustable parameters precisely corresponds to the number of unknown parameters in the controlled system, ensuring a high degree of adaptability. The exceptional feature of this algorithm is that, unlike known high-order adaptation algorithms, it does not use filters to process all components of the regression vector.

This significantly reduces the order of the closed-loop control system and contributes to the algorithm's more effective operation. This innovative research approach demonstrates the potential for solving complex adaptive control tasks in conditions of limited information and may have significant practical implications in the field of automatic control.

References

1. T. Botirov, N. Abduazizov, B. Sodiqov, Z. Khusanov, E3S Web of Conferences **390** 04012 (2023)
2. A.M. Tsykunov, Automation and Remote Control **7** 103-115 (2007)
3. A.M. Tsykunov, Automation and Remote Control **8** 143-153 (2006)
4. I.B. Furtat, A.M. Tsykunov, Automation and Remote Control **6** 109-118 (2010)
5. E. Saribaev, A. Abrorov, B. Otaboev, N. Sariboev, A. Daliev, Journal of Physics **2388** 012174 (2022)
6. Sh.R. Khurramov, J. Izv.Vyss. Ucheb. Zav.Tech. Tekstil.Prom **4(394)** (2021)
7. Djaloliddin Mukhitdinov et al., E3S Web of Conferences **486** 03021 (2024)
8. Yorkin Kadirov et al., Journal of Physics: Conference Series **2697** 012040 (2024)
9. I. Kalandarov, *Algorithm for the problem of loading production capacities in production systems*, In book: XIV International Scientific Conference "INTERAGROMASH 2021", pp.887-896 (2022). https://www.doi.org/10.1007/978-3-030-81619-3_99
10. O.A. Jumaev et al., J. Phys.: Conf. Ser. **1679(5)** 052018 (2020)
11. T. Botirov, Sh. Latipov, Z. Khusanov, E3S Web of Conferences **417** 05015 (2023)
12. Sh.R. Khurramov, S. Bektoshev, K. Jonkobilov, J. E3S Web of Conferences **443** 05008 (2023)
13. O.A. Jumaev, J.T. Nazarov, G.B. Mahmudov, M.T. Ismoilov and M.F. Shermuradova, J. Phys.: Conf. Ser. **2094(2)** 022030 (2021)
14. A. Amanov, Sh.R. Khurramov, G.A. Bahadirov, A. Abdukarimov, T.Y. Amanov, Journal of Leather Scince and Engineering **3** 14 (2021)
15. A.N. Atassi, H.K. Khalil, IEEE Trans. Automat. Control. **44 (9)**, 1672- 1687 (1999)

16. X.Z. Igamberdiev, T.V. Botirov, S.B. Latipov, Journal of Physics: Conference Series, **2388(1)** 012051 (2022)
17. J. Sevinov, O. Boeva, AIP Conference Proceedings **2647** 030007 (2022)
18. Y. Kadirov, O. Boeva, R. Eshqobilov, S. Toshmurodova, D. Abdullaeva, E3S Web of Conferences **414** 05003 (2023)
19. Olim Sattarov, E3S Web of Conferences **390** 03012 (2023)
20. A. Vakhromeev, A. Boboev, N. Namozov, Sh. Khushvaqtov, E3S Web of Conferences **417** 04007 (2023)