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Algorithm for synthesis of plane gears according to the constant curvature criterion

Nikolay Barbashov*, and *Gennady* Timofeev

Moscow Bauman State Technical University (BMSTU), Moscow, Russia

Abstract. This paper presents an algorithm for synthesizing plane gears based on the constant curvature criterion. The derived formulas for pinion gears can be applied in various mechanisms, including clockwork mechanisms, gun turret aiming systems, lifting and transport mechanisms, and planetary gearboxes with tracked propellers. These formulas enable effective analysis of transmission operation and optimal value selection, leading to more rational design and increased efficiency of the entire mechanism. Internal gearing technology allows for significant reductions in drive dimensions and metal consumption compared to traditional involute gears. Improving the technical characteristics of mechanical drives can be achieved by optimizing pinion gear design. This study focuses on the analysis of pinion gears, which reduce profile slippage and transmission wear, resulting in increased efficiency and convenience of operation. Additionally, pinion gears are technologically advanced and reliable. This work develops a mathematical model of a pinion transmission as a flat gearing with a constant gear ratio, providing a foundation for further research and practical applications.

1 Introduction

This work is devoted to the study of pinion gears. The use of pinion gears makes it possible to reduce the slippage of profiles and thereby reduce the wear of the transmission as a whole [1-5]. This leads to increased efficiency and makes transmission more efficient and convenient to operate. In addition, the pinion gears are technologically advanced and reliable. [6-10].

Let us consider a mathematical model of a pinion transmission as a flat gearing, the gear ratio of which is a constant [1, 2]:

$$v_j = -a_j\omega_j \sin\alpha + r\left(\dot{\alpha} - \omega_j\right) \tag{1}$$

$$\dot{r} + \alpha_j\omega_j \cos\alpha = 0 \tag{2}$$

$$k_j = \frac{(\dot{\alpha} - \omega_j)}{v_j} \tag{3}$$

where j = 1, 2 are the gear numbers; vj is the contact point's speed; aj is the centroid's radius; ωj is the angular velocity; α is the engagement angle; r is the radius from the point of contact

* Corresponding author: barbashov@bmstu.ru

to the engagement pole; kj is the tooth profile form; the point above the parameter shows its derivative in time t. The above model was successfully used in other works.

Let's find the solution of the problem using criterion of tooth profile curvature k_j = const. Then we should write (3) taking into account (1) in the formulae

$$R_j = \frac{v_j}{\dot{\alpha} - \omega_j} = r - \frac{\alpha_j \omega_j \sin\alpha}{\dot{\alpha} - \omega_j}$$

from here

$$r = R_j + \frac{\alpha_j \omega_j \sin\alpha}{\dot{\alpha} - \omega_j} \tag{4}$$

where Rj = 1/k_j — the curvature radius, and as it is easy to see directly the engagement line for the criterion v_j = 0. It is described by the formula [5]

$$\tilde{r}|_{v_j=0} = -\alpha_j \sin\alpha \pm \alpha_j \sqrt{\sin^2\alpha + c_0}$$

where the parameter c_0 has the expression:

$$c_0 = \frac{2\tilde{r}_0}{\alpha_j}\left(\frac{\tilde{r}_0}{2\alpha_j} + \sin\alpha_0\right)$$

Thus, we get

$$r = R_j - \alpha_j \sin\alpha \pm \alpha_j \sqrt{\sin^2\alpha + c_0} \tag{5}$$

2 Materials and methods

To select the sign of the function, we will set the initial conditions of the equation. To do this, we perform differentiation (5) in time. After comparing the results with (2), we get that the function α(t) fulfills the specified requirements

$$\dot{\alpha} = \frac{\omega_j}{1 \mp \frac{\sin\alpha}{\sqrt{\sin^2\alpha + c_0}}} \tag{6}$$

It's solution has the form

$$\phi_1 = \alpha - \alpha_0 \pm \left[\arcsin\frac{\cos\alpha}{\sqrt{c_0+1}} - \arcsin\frac{\cos\alpha}{\sqrt{c_0+1}}\right] \tag{7}$$

Using expressions (5) and (7), it is possible to write mathematical dependence for the profiles of the teeth of the gears

$$x_j = \alpha_j \cos\phi_j + r\sin(\alpha - \phi_j)$$
$$y_j = -\alpha_j \sin\phi_j - r\cos(\alpha - \phi_j)$$

The second link in this task is a gear rack that performs translational motion relative to the gear wheel 1

$$x_2' = r\sin\alpha$$
$$y_2' = -\alpha_j \phi_j - r\cos\alpha$$

The solution of the equation completely describes the external pinion engagement [6]. To simplify the calculations, we will introduce an additional geometric connection into the mathematical model of engagement

$$(a_1 + e)\sin\phi_1 = -(r + \rho)\cos\alpha \tag{8}$$

Here ρ is the radius of the cap; e is the displacement of the cap's center. Let's deduce the mathematical relationship between the parameters c_0 and e.

$$e = a_1\left(\sqrt{c_0 + 1} - 1\right) \tag{9}$$

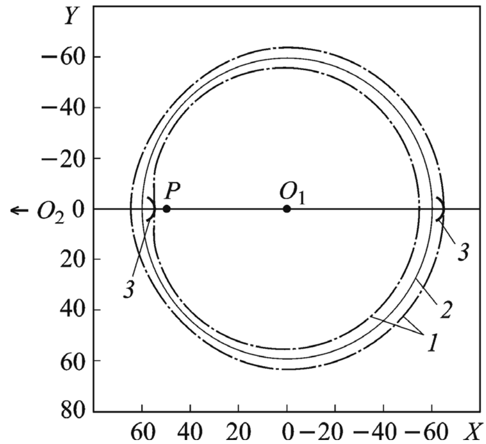

Fig. 1. Engagement lines (out—of—pole pin engagement) for the case k_1 = const ($c_0 > 0$): 1 — engagement line; 2 - line of the centers of the pin; 3 – first gear; O_1 — axis of gear's rotation; O_2 — axis of the second gear; P — pole engagement.

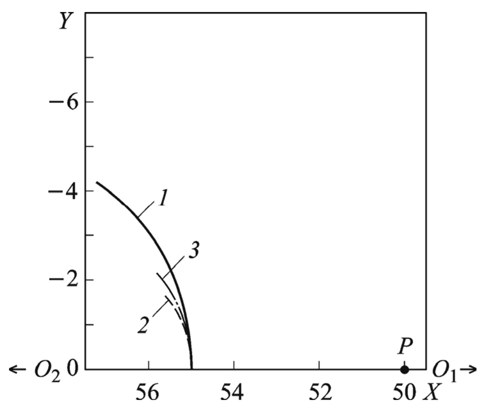

Fig. 2. Tooth profile with k_1 = const ($c_0 > 0$): 1 — profile of the first link (pinion); 2 — second gear; 3 — toothed rail; O_1 — axis of gear's rotation; O_2 — axis of the second gear; P — pole engagement.

Figure 1 shows the gearing lines for the pinion gear. The engagement in question has an axial distance $a_1 = 50$ mm and a constant gear ratio $i_{12} = 2$. The handguards are simultaneously in contact with the profiles of the teeth of the other wheel. Thus, the centers of all the cusps are located on the corresponding branches of elongated epicycloids or hypocycloids. Although all the pins can be in simultaneous engagement, the load can be transmitted as much as possible by only half of their total number. If k_j = const and v_j = const we can get next formulae, using (5) and (6):

$$v_j = -\alpha_j \omega_j \sin\alpha + \left(R_j - \alpha_j \sin\alpha \pm \alpha_j \sqrt{\sin^2\alpha + c_0}\right)\left(\frac{\omega_j}{1 \mp \frac{\sin\alpha}{\sqrt{\sin^2\alpha + c_0}}} - \omega_j\right) =$$

$$= -\alpha_j \omega_j \sin\alpha + \left(R_j - \alpha_j \sin\alpha \pm \alpha_j \sqrt{\sin^2\alpha + c_0}\right) \times \frac{\alpha_j \omega_j \sin\alpha}{-\alpha_j \sin\alpha \pm \alpha_j \sqrt{\sin^2\alpha + c_0}} =$$

$$= \frac{R_j}{r - R_j} \alpha_j \omega_j \sin\alpha \qquad (10)$$

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Hence, to v_j = const, we get $r = R_j = C \sin\alpha$. Then, using expression (10), we obtain the formulae

$$\left((C+\alpha_j)^2 - \alpha_j^2\right)\sin^2\alpha - \alpha_j^2 c_0 = 0$$

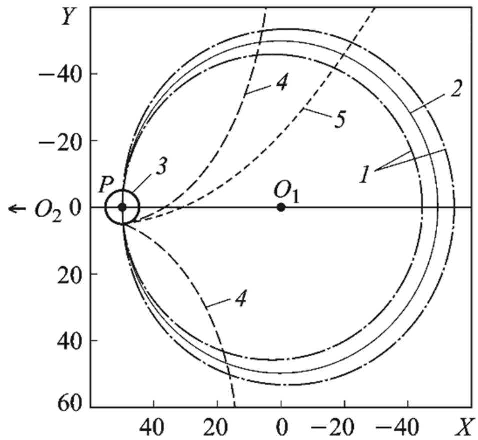

Fig. 3. Gearing lines and profiles (k_1 = const, c_0 = 0): 1 — gearing line; 2 — line of pin centers; 3 — profile of the first link (pin); 4 — second gear; 5 — toothed rail; O_1 — axis of gear's rotation; O_2 — axis of the second gear; P — pole engagement.

It shows the main advantages of the pinion engagement: smooth running and low noise compared to the involute engagement and high resource and reliability due to the fact that the load on the cap is distributed over its entire surface.

It is performed for any α while C = 2a_j, c_0 = 0. Also e should be equal zero. Then we can obtain next formulae for radius

$$r = R_j - 2\alpha_j \sin\alpha \tag{11}$$

The gearing lines and link profiles for pinion engagement with a_1 = 50 mm and i_{12} = 2 are demonstrated in Figure 3. The pinion gear has smaller dimensions with the same transmitted torque in comparison with the involute gearing; the gear ratio is from 6 to 190 and has high kinematic accuracy. The gearing line is Pascal's snail, and the profile of the first link — circle.

3 Results and discussion

Consider the case k_1 = const, k_2 = const. Then, using equations (1) – (3), we obtain a system of equations, where there is a redundant expression. His solution imposes an additional connection on the engagement parameters, which complicates the calculation. Therefore, we express the relation in the following form.

$$\dot{\alpha} - \omega_j = \frac{a_j \omega_j \sin\alpha}{r - R_j} \tag{12}$$

Hence when R_1 = const, R_2 = const

$$\omega_1 + \frac{a_1\omega_1 \sin\alpha}{r - R_1} = \omega_2 + \frac{a_2\omega_2 \sin\alpha}{r - R_2} \tag{13}$$

The equation for the gear's profile with k_1 = 0, k_2 = const is obtained from expression (13) in the form

$$r = R_2 + \frac{a_j\omega_j \sin\alpha}{\omega_1 - \omega_2} \tag{14}$$

In mathematical analysis, such a curve is called Pascal's snail. Pascal's snail is a plane algebraic curve of the 4th order. The arc length in this case can be expressed by an elliptic integral of the 2nd kind.

$$\dot{\alpha} = \omega_1 \tag{15}$$

Differentiating (14) by time t and using (2), taking into account (15) we obtain

$$\frac{\omega_1 \alpha_j \omega_j \cos\alpha}{\omega_1 - \omega_2} = -\alpha_j \omega_j \cos\alpha$$

From here $\frac{\omega_1}{\omega_1 - \omega_2} = -1$. This means that such a transmission can be made in the form of an end gear [7] with internal gearing and a gear ratio i_{12} = 1, 2. In this case, the pins are located on the second gear. Assume that the tooth profiles are straight. This will allow you to express the speed of the wheel

$$v_1 = -a_1 \omega_1 \sin\alpha$$
$$v_2 = R_2(\omega_1 - \omega_2) = const$$

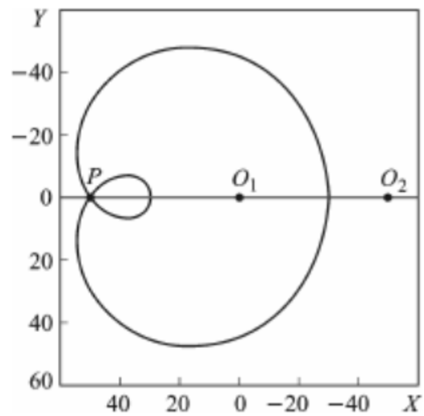

Fig. 4. The engagement line with k_1 = 0, k_2 = const: O_1 — axis of gear's rotation; O_2 — axis of the second gear; P — pole engagement.

If $k_1 = k_2$, then from (4) we have ω1 = ω2 and, consequently, $a_1 = a_2$,$k_1 = k_2 = const$, $k_1 = k_2 = \infty$; ($R_1 = R_2 = 0$). A similar expression describes the gear coupling mechanism, where there is no axial distance. Gear elements can have an arbitrary profile.

4 Conclusion

In this work formulas for the pinion gears were derived. Such transmissions can be used in clockwork mechanisms, in gun turret aiming mechanisms, in lifting and transport mechanisms and in some types of planetary gearboxes, tracked propellers. The above dependencies can be used for effective analysis of their operation and selection of optimal values. This will make the design more rational and increase the efficiency of the handguard mechanism as a whole. An internal gearing gear makes it possible to significantly reduce the dimensions and metal consumption of the drives compared to the most common involute gears.

One of the ways to improve the technical characteristics of mechanical drives is to improve the design of pinion gears.

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