Calculation formulas for contact curves of technological machine

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Abstract. The problem of describing the contact curves of technological machines has been solved. At the same time, contact interaction was studied taking into account changes in the technological parameter and deformation of the materials of the contacting bodies. The obtained dependencies make it possible to find rational parameters of the technological process with sufficient accuracy. The obtained models indicate that the shape of the contact is influenced by such parameters as strip thickness, roll radii, external forces, as well as parameters characterizing the deformations of the contacting bodies.

1 Introduction

Roller modules are used in various technological machines, which consist of two work rolls and a strip of processed material. Depending on the processing requirement, one or both rollers of the roll module have an elastic coating.

Most roll modules perform two functions simultaneously: the main and auxiliary ones. The latter most often is to transport a process product.

Under the main function of the roll module, certain contact stresses appear at each point of the contact zone, which are the determinants for finding the main indicators of the roll module and the technological operation. The main contact task in the roll module is modeling contact stresses, which are determined primarily by the shape and magnitude of these curves, especially in the case when the rolls have an elastic coating. In this regard, to solve the main contact problem in the roll module, a solution to another contact problem is required - an analytical description of the contact shape.

The problem of analytical description of the form of contact is a type of poorly formalized problem, since there are no strict formulas describing the deformation of contacting bodies, hydrodynamic phenomena in fibrous media during spinning, friction and wear processes at contact points, etc.

In [1,2], the analytical description of the shape of the roll contact curves is associated with the ratio of the deformation rates of contacting bodies; to determine these deformation rates an additional experimental study is necessary.

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In the studies conducted in, the problem of simulating the laws of contact stress distribution is solved based on the preliminary selection of the formula for the roll contact curves. Generally, the arc of a circle, ellipse, and parabola are used [1].

The contact interaction of a deformable material and the roller with elastic coating can be considered by analogy with the elastic wheel rolling on the deformable ground [2]. The issues of interaction between an elastic wheel and deformable ground were studied quite extensively [3-18], however, some issues remained unclear. In some publications on this issue, the rear part of the contact curve is plotted as a straight line, and in other publications, it is presented as a curve; however, analysis of the changes in the curve due to the forces and moments acting on the wheels is incomplete. In some studies, it is believed that the contact line of an elastic wheel with deformable ground is part of a circle, and in others, it is represented by a complex line that is not part of the circle; this line has different shapes; for example, in [18], the shape of the contact line is given as a truncated circle, ellipse, and third-degree polynomial.

The analysis showed that at present there are many different models of wheel-ground interaction. Often, calculations using these models require data that can only be obtained through complex experimental studies.

Thus, at present there are no analytical dependences of contact curves.

This work is devoted to determining the analytical dependence of the contact curves of a symmetrical roller module (Figure 1).

![Diagram of the upper part of the symmetrical roller module.](image)

**2 Materials and methods**

The shape of the roll contact curve depends on the external forces acting on the roll, the strain properties of the interacting body materials, and the friction properties of surfaces [19-28].

The properties of each body (processed material or roll coating) in the module are characterized by a compression line during interaction and a line that determines the restoration of its deformation (shape). These lines define the relationship between the deformation of the material and the stress during compression and deformation recovery. When the material is compressed, the dependence of deformation on stress will be nonlinear.
The shape of the line when strain is restored depends on the nature of the material: it can be absolutely elastic, elastic-viscous, and plastic. In the first case, when leaving the contact zone, the material completely restores its shape, in the second case, it restores its shape partially, and in the third case - there is no restoration of strain. Accordingly, the strain recovery of a material, the stress-strain relation depends on the nature of the material.

In a roll module under static conditions, the point of maximum compression of the material of interacting bodies is located on the line of centers (on the line connecting the centers of the rolls). When the rolls rotate, due to the action of reactive forces, this point is displaced from the line of centers to the beginning of the contact zone of the rolls. It was revealed [24], that this displacement in the roll modules of technological machines is insignificant; for example, when pressing leather with rollers having an elastic coating, this displacement in the roll contact curve is only $0.09 - 1.75\%$. Therefore, we can assume that the point of maximum compression of interacting bodies lies on the line of centers.

We divide the contact curve of each roll relative to the line of centers into compression and recovery zones. In this case, the compression zone consists of lagging sliding sections 1 and sticking 2, and the recovery zone consists of sticking sections 3 and leading sliding 4. Let the contact curve of each roll be given by the following equation:

$$\rho_i = \rho_i(\phi_i), i = 1,4, -\alpha_1 \leq \phi_1 \leq -\alpha_3, \quad -\alpha_3 \leq \phi_2 \leq 0, \quad 0 \leq \phi_3 \leq \alpha_4, \quad \alpha_4 \leq \phi_4 \leq \alpha_2.$$  

Roller machines process materials such as fabrics, paper, leather, wool, cotton, and others. A large amount of experimental data was accumulated concerning the strain properties of such materials [23-26], in which the nature of strain is mainly specified by power functions of the following form:

$$\sigma_p = A \varepsilon_p^m,$$  

where $\sigma_p, \varepsilon_p, A, m$ – are the stresses, relative strains, and deformation coefficients of the processed material.

During the interaction process, the thickness of the processed material changes. Depending on the main function performed, another parameter – a technological one – changes as well. For example, under roller squeezing of wet materials, this parameter is the moisture content of the material being pressed.

The change in the process parameter affects the stress-strain relationship of the material. Based on this, we use the dependence that accounts for this change [2]

$$\sigma_p = A \varepsilon_p^m f(\phi),$$  

where $f(\phi)$ – is the function that determines the change in the process parameter.

Rubber and polyurethanes of various brands, and technical fabric made from felts of various origins are used for the roller coatings. In the technical literature and publications of researchers involved in studying the stress-strain relation of such materials under compression and recovery, a power-law relationship between stress and strain is mainly discussed [29-32]:

$$\sigma_r = B \varepsilon_r^k,$$  

where $\sigma_r, \varepsilon_r, B, k$ – are the stresses, relative strains, and deformation coefficients of the roll coating.

### 3 Results and discussion

Since a symmetrical roll module is being considered, it is sufficient to construct an equation for the contact curve of one of the rollers, for example, the top roller.

As a result of the slip of the processed material along the surface of the roller coating, the contact curve of section 1 has the shape of a straight line with a downward tilt relative to the horizontal line by the friction angle $\nu_1$ [25].
Research conducted in [2, 21] has shown that in the lag and advance slip zones, Amonton’s law is observed. Then the friction angle in section 1 is determined by formula \( \nu_1 = \arctan f_1 \), where \( f_1 \) — is the coefficient of friction of the processed material against the surface of the roller coating in the lag slip zone.

From Figure 1, it is clear that
\[
\nu = \phi - \beta_1, \tag{4}
\]
where
\[
\tan \beta_1 = \frac{\rho_1}{\rho_1}. \tag{5}
\]

From equalities (4) and (5), we obtain:
\[
\frac{\rho_1}{\rho_1} = \tan(\phi_1 - \nu_1)
\]
or after integration and transform, considering the initial condition \( \rho_1 = R \), for \( \phi_1 = -\alpha_1 \)
\[
\rho_1 = \frac{\cos(\alpha_1 + \nu_1)}{\cos(\phi_1 - \nu_1)} R, \quad -\alpha_1 \leq \phi_1 \leq -\alpha_3. \tag{6}
\]

Equation (5) determines the shape of the first section of the roll contact curve.

From Figure 1, it follows that at the points of section 2 of the contact curve, the relative strains of the interacting bodies are defined as:
\[
\varepsilon_p = \frac{2}{\delta_1} \left( \rho_2 - R \frac{\cos \alpha_1}{\cos \phi_2} \right), \quad \varepsilon_r = \frac{1}{H} (R - \rho_2).
\]

Using dependencies (2) and (3), we obtain:
\[
\sigma_p = A_1 \left( \frac{2(\rho_2 \cos \phi_2 - R \cos \alpha_1)}{\delta_1} \right)^{\frac{m_1}{1}} f_1(\phi_2), \tag{7}
\]
\[
\sigma_r = B_1 \left( \frac{R - \rho_2}{H} \right)^{k_1}, \tag{8}
\]
where \( A_1, m_1 \) — are the deformation coefficients of the processed material under compression, \( B_1, k_1 \) — are the deformation coefficients of the roller coating under compression, \( f_1(\phi) \) — is the function that determines the change in a process parameter under compression.

At the points of the roll contact curve, deformations of the interacting bodies occur along the normal line.

According to Newton's law, at each point of section 2 the following condition is satisfied:
\[
\frac{\sigma_p}{\cos(\phi_2 - \beta_2) k_1} = \frac{\sigma_r}{\cos \beta_2}
\]
or considering (7) and (8)
\[
A_1 \left( \frac{2(\rho_2 \cos \phi_2 - R \cos \alpha_1)}{\delta_1} \right)^{\frac{m_1}{1}} f_1(\phi_2) = B_1 \left( \frac{R - \rho_2}{H} \right)^{k_1} \cos \phi_2. \tag{9}
\]

In section 2, the values of angle \( \beta_2 \) are close to zero. Considering this, we assume that \( \sin \phi_2 \tan \beta_2 \approx 0 \).

Then from equality (9), we obtain:
\[
A_1 \left( \frac{2(\rho_2 \cos \phi_2 - R \cos \alpha_1)}{\delta_1} \right)^{\frac{m_1}{1}} f_1(\phi_2) = B_1 \left( \frac{R - \rho_2}{H} \right)^{k_1} \cos \phi_2. \tag{10}
\]

In equality (10), we make the following transforms:
\[
A_1 \left( \frac{H}{R} \right)^{k_1} \left( 1 - \delta_1 - 2(\rho_2 \cos \phi_2 - R \cos \alpha_1) \right)^{m_1} f_1(\phi_2) = \left( 1 - \frac{\rho_2}{R} \right)^{k_1} \cos \phi_2
\]
or to a first approximation
\[
A_1 \left( \frac{H}{R} \right)^{k_1} \left( 1 - m_1 \delta_1 - 2(\rho_2 \cos \phi_2 - R \cos \alpha_1) \right)^{m_1} f_1(\phi_2) = \left( 1 - \frac{\rho_2}{R} \right)^{k_1} \cos \phi_2. \tag{11}
\]

From equality (11), we find the equations of the contact curve of section 2.
\[
\rho_2 = \frac{R \cos \phi_2 + m_1 (2\rho_2 R \cos \alpha_1 + (m_1 - 1) \delta_1) f_1(\phi_2)}{(k_1 + 2a_1 m_1 f_1(\phi_2)) \cos \phi_2}, \tag{12}
\]
where \( a_1 = \frac{R}{\delta_1 B_1} \left( \frac{H}{R} \right)^{k_1} \).

Formulas (6) and (12) describe the shapes of the roll contact curve in the compression zone.

Similarly to (6) and (12), we find the equations for the roll contact curve in the strain recovery zone:

\[
\rho_3 = \frac{R \cos \psi_3 + a_2 (2m_2 R \cos \alpha_2 + (m_2 - 1) \delta_2 f_2 (\phi_3))}{(k_2 + 2a_2 m_2 f_2 (\phi_3)) \cos \psi_3}, \quad 0 \leq \phi_3 \leq \alpha_4, \quad (13)
\]

\[
\rho_4 = \frac{R \cos (\alpha_2 - \nu_2)}{\cos (\phi_4 - \nu_2)} R, \quad \alpha_4 \leq \phi_4 \leq \alpha_2, \quad (14)
\]

where \( a_2 = \frac{R}{\delta_2 B_2} \left( \frac{H}{R} \right)^{k_2} \), \( \nu_2 = \arctan f_2 \), here \( f_2 \) is the coefficient of friction of the processed material against the surface of the roller coating in the advance slip zone, \( A_2, m_2, B_2, k_2 \) — are the deformation coefficients of the processed material and the roller coating under strain recovery, \( \delta_2 \) — is the thickness of the processed material at the exit from the contact zone.

4 Conclusion

Analytical dependencies of the contact curves of the roller module of technological machines have been determined.

When developing these models, a model of deformation of the processed material was used, taking into account the change in the technological parameter of the process.

It was revealed that the contact curves depend on the geometric and deformation parameters of the strip and roll coating materials, as well as on external forces.

The obtained dependencies can be used to establish rational indicators of roller modules of technological machines.

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