Noise reduction through a waveguide structure consisting of expansion chambers with a geometrical defect

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Abstract. Noise control helps to make working environments safer and keep operations in line with health and safety standards. Exhaust noise is the main component of noise pollution in urban environments. In this paper, we focus on noise control by improving acoustic attenuation performance using a one-dimensional waveguide structure composed of simple periodic expansion chambers with a geometric defect. This defect is located at the center of the periodic structure and results from a modification in the length of the central chamber of the system. The objective is to study the properties of acoustic transmission and transmission loss and to examine the effect of defects in a periodic acoustic structure. The system's spatial periodicity enables us to design wide band gaps where sound waves cannot propagate. This characteristic is very important for reducing noise in our environment. The effect of the cross-sectional ratio on the band gap behavior was also examined in this work. In addition, we have shown that the presence of a defect in a regular structure leads to a perturbation of the structure's spatial periodicity. This leads to the creation of defect modes or resonance modes in the band gaps. We also controlled the number and amplitude of defect peaks within the band gap by varying the length of the defect. The results of this work are of interest for various applications, such as the creation of wide acoustic bands, low-frequency noise reduction, and acoustic wave filtering. ***Keywords*:** Noise control, expansion chamber, acoustic band gap, defect modes

1 Introduction

Noise pollution influences both the environment and human health. The developed world has more infrastructure and resources to deal effectively with noise pollution [1]. In contrast, the challenge of balancing rapid development with the need to control noise pollution is more acute in developing countries, presenting difficulties for investment in sustainable development initiatives such as noise reduction. Indeed, the importance of investment in noise reduction is directly related to the importance of its negative effects on health and the environment. The greater its negative effects are, the more investment strategies appear. In this context, several methods are used to control noise pollution. The use of periodic structures to attenuate acoustic waves is one of the famous methods used in noise reduction. LF filtering can be achieved by combining resonators, which leads to high sound attenuation. Therefore, several resonators of different sizes are used to provide large bands [2-6]. A study of sound transmission in periodic waveguides is carried out theoretically using Bloch's wave functions and verified experimentally. We can apply Bloch's wave function to infinite and finite cases [7]. The band structure of acoustic waves in periodic waveguides, the axial reversal symmetry limitation and the reciprocal allowable Bloch wave solver have been studied [8]. Multiple expansion chambers can provide a higher level of sound absorption at lower frequencies. Feedback silencers are widely used as tailpipe silencers designed with specific dimensions to reduce noise [9-12]. Mufflers are distributed periodically in the duct in which perforations along the lateral surface of the duct are commonly associated with the muffler to provide more attenuation. The result of periodic tracking is a change in transmission loss, which could be used to control duct noise [13]. Based on work on acoustic structures, a one-dimensional (1D) phononic crystal constructed by a girder system with cross-sections that vary periodically was presented in [14]. In many applications, the expansion chamber is widely used in the automotive industry to reduce engine exhaust noise, thereby limiting noise pollution in the urban environment [15,16]. The acoustic performance of periodic microperforated expansion chambers was evaluated [17]. The numerical results show that the microperforation completely changes the attenuation behavior of the structure compared to that of the conventional chamber muffler. An experimental validation of a number of geometric factors on transmission loss was presented in [18]. The periodic provision of an expansion chamber system with an integrated microperforated panel (PECMPP) with a high airflow capacity, which could be used in the acoustic barrier structure [19], proposed to improve the acoustic isolation bandwidth of the system. In recent years, the study of structures containing geometric or material defects has also received particular attention due to their important applications [20-22]. The appearance of defect resonance modes explains the

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breakdown of the system's spatial periodicity [3].The impact of defects on the acoustic transmission spectrum has recently been studied by researchers using closed resonator systems and opened waveguides [3,23]. Antraoui et al. [24] examined the characteristics of resonance modes in a structure consisting of a discontinuity of waveguides of different geometric dimensions. They also discovered that the creation of these modes makes it possible to extract new bands transmitted inside the band gaps to design very fine transmission frequencies. Hongbo et al. [25] calculated the band structure in a 1D phononic crystal slab with semicircular holes. They showed that the structure used provides topological states of adjustable width and generates defect states or defect modes, which can be used to filter and guide waves. However, the effect of a defect in the expansion chamber structure on the acoustic transmission spectrum is still very rare. By way of comparison, Z.A. Zaky et al. [26] studied a proposed gas PnC sensor with a sequence of expansion chamber tubes with a defect. They showed that the presence of the defect produces resonant peaks in the interstices. They also showed that acoustic velocity is a more important factor than variations in the mass density of gas samples in the location of resonant peaks because of its strong effect on acoustic wave propagation.

In this study, we have mainly used the assumption of a low-frequency plane wave. The aim is to study the properties of defect modes in a periodic structure composed of expansion chambers attached to a principal waveguide. We first examined the effect of the periodicity of the perfect structure on the transmission and transmission loss spectra. Bloch's theorem shows that spatial periodicity has a significant effect on the appearance of very wide acoustic bandgaps. The creation of these band gaps is important for the complete reflection of the acoustic wave propagating through the system. Increasing the cross-sectional ratio of waveguides is also examined in this work. We have shown that increasing the cross-section ratio is a very significant factor in increasing the transmission loss coefficient and therefore in reducing the noise that affects the environment and human health. In the second part of this work, we analyzed the behavior of defect modes through an expansion chamber structure. In this case, we introduced a large expansion chamber that represents the defect at the center of the perfect periodic structure. Our results show that varying the defect size, particularly the defect length, allows the existence of localized states of resonance modes appearing in the acoustic band gap. More specifically, we have shown that the presence of an even number of defects leads to the creation of a single defect mode in the middle of the band gap with a very high transmission rate. On the other hand, the presence of an odd number defect in the regular structure creates two defect picks in the same acoustic band gap. The results are very useful for filtering and guiding acoustic waves.

2 System design and theoretical methods

A geometric defect sandwiched between two 1D periodic structures is designed and studied theoretically in this section. Figure 1 shows a schematic of a defect situated in the middle of two identical periodic waveguide structures. Each unit cell consists of a simple expansion chamber of length L and cross-section s_2 attached to a main guide of half-length and cross-sections $L_1/2$ and s_1, respectively. In this work, we deliberately created defects by modifying the geometric dimensions of the system. As shown in Figure 1, the defect is created by varying the length of the central expansion chamber $L_D \neq L$.

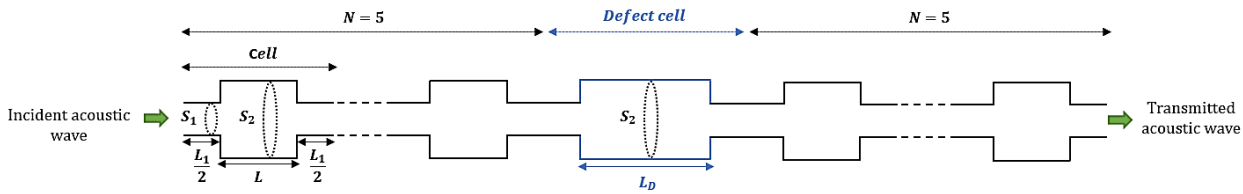

Fig. 1. Schematic illustration of a periodic waveguide structure consisting of expansion chambers with a geometrical defect..

In this section, we applied the familiar transfer matrix method of our system containing a defect [22,24,27]. We have also used Sylvester's theorem to calculate the matrix of a perfectly periodic structure [28].

$$M_C = \begin{pmatrix} M_{C11} & M_{C12} \\ M_{C21} & M_{C22} \end{pmatrix} = M_{C_1} M_{C_2} M_{C_1} \quad (1)$$

With:

$$M_{C_1} = \begin{pmatrix} cos(k\,L_1/2) & j\,z_{c_1} sin(k\,L_1/2) \\ j\,\frac{sin(kL_1/2)}{z_{c_1}} & cos(k\,L_1/2) \end{pmatrix} \quad (2)$$

and

$$M_{C_2} = \begin{pmatrix} cos(kL) & jz_{c2}\,sin(kL) \\ j\,\frac{sin(kL)}{z_{c2}} & cos(kL) \end{pmatrix} \quad (3)$$

We used Bloch's theorem to compute the acoustic wave dispersion relation, which describes spatial periodicity by the following relation [3,24].

$$cos(\emptyset) = \frac{(M_{C11}+M_{C22})}{2} \quad (4)$$

where $\emptyset = Kd$ is the Bloch phase and $d = L_1 + L$ is the length of the network period with $L_1 > L$.
The periodic structure repeated by a number of periods N is calculated by Sylvester's theorem in equation (5) below.

$$M_C^N = \begin{pmatrix} M_{C11}U_N - U_{N-1} & M_{C12}U_N \\ M_{C21}U_N & M_{C22}U_N - U_{N-1} \end{pmatrix} \quad (5)$$

With $U_N = \sin(N\emptyset)/\sin(\emptyset)$.

The characteristic acoustic impedances of the input and output waveguides are z_i and z_o, respectively. In this case, the system consists of the same inlet and outlet air guides (see Figure 1), i.e. $z_i = z_o = z_{c_1}$. Consequently, the rate of transmission of the perfect periodic structure is computed according to the elements of the TMM and the Chebyshev polynomial:

$$T = \left| \frac{2}{M_{C11}\, U_N - U_{N-1} + y_{c1} M_{C12} U_N + z_{c1} M_{C21} U_N + M_{C22} U_N - U_{N-1}} \right|^2 \quad (6)$$

The transmission loss coefficient (TL) is given below,

$$TL = 20\, log \left| \frac{1}{T} \right| \quad (7)$$

In the presence of a defect, the global matrix of a defect placed in the middle of two perfect periodic structures is given by:

$$M' = M_C^N M_{DEFECT} \quad (8)$$

With:

$$M_{DEFECT} = \begin{pmatrix} cos(kL_D) & jz_{c2}\, sin(kL_D) \\ j\frac{sin(kL_D)}{z_{c2}} & cos(kL_D) \end{pmatrix} \quad (9)$$

M_{DEFECT} is the transfer matrix of the defect chamber

3 Results and discussion

3.1 Perfect periodic structure

We consider here that the acoustic waves propagate along the structure with a normal incidence angle equal to 0. This research focuses on the role that the periodicity of our system plays in the reflection of acoustic waves in specific frequency intervals known as acoustic band gaps. The design of these bands is essential for trapping acoustic transmission and thus reducing noise in the environment. In Figure 2, we plot the band structure (Figure 2(a)), acoustic transmission (Figure 2(b)) and transmission loss coefficient (Figure 2(c)) according to frequency, for a periodic structure of finite expansion chambers with $N = 11$ periods and a cross-sectional area ratio $m = s_2/s_1 = 4$. Figure 2(a) shows the band structure of the real part responsible for the appearance of the pass bands that allow the acoustic wave to propagate within the system. Figure 2(a) also shows the imaginary part of the Bloch vector, which coincides exactly with the ranges of the band gaps. Figure 2(b) also shows the appearance of four zero-transmission bands separated by five transmitted bands at positions fully compatible with those of the band structure shown in Figure 2(a). As shown in Figure 2(b), the width of the first and fourth band gaps is very large compared to the width of the second and third band gaps. This is also consistent with the band gap distribution in Figure 2(a). The formation of band gaps is due to the spatial periodicity of the structure, which leads to multi-destructive interference with the rigid walls of the waveguides. Figure 2(c) shows the transmission loss coefficient varying with frequency. The results in this figure show significant compatibility with the results obtained previously in Figures 2(a) and 2(b). Specifically, we observe that the transmission loss coefficient TL is very high in the zones of the first and fourth forbidden frequency intervals corresponding to the transmission spectrum shown in Figure 2(b). These results clearly show that our periodic expansion chamber system is a very good choice for controlling the position and width of the band gap by creating wide band gaps. These bands have the property of perfectly reflecting the acoustic wave and therefore producing selective silences to improve the acoustic noise reduction properties.

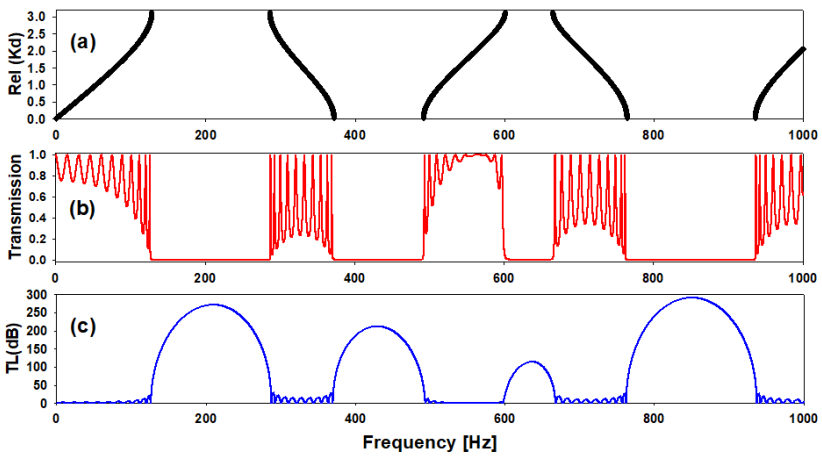

Fig. 2. (a) Band structure, (b) transmission spectrum and (c) transmission loss coefficient versus frequency for a perfect periodic structure with $N = 11$ periods.

We looked at the effect of varying the cross sectional ratio $m = s_2/s_1$ on the properties of the transmission loss coefficient. In Figure 3, we plot the spectrum of the coefficient of transmission loss versus frequency for four different cross sectional ratios: $m = 2$ (Figure 3(a)), $m = 4$ (Figure 3(b)), $m = 8$ (Figure 3(c)) and $m = 10$. The objective of this study is to show the crucial role of our waveguide system in controlling sound intensities through evaluating the transmission loss coefficient by changing the value of m. Here, we

examined the evolution of the *TL* in the frequency range between 700 and 1000 Hz corresponding to the fourth band gap shown in Figure 2 above. When the value of m increases from 2 to 10, the decrease in the *TL* coefficient increases very significantly, as shown in Figures 3((a)-(d)). For $m = 2$, the maximum *TL* amplitude is equal to 138.5 dB, occupying a frequency interval between 799 and 898 Hz. Then, for $m = 4$, the maximum *TL* amplitude is equal to 290 dB with a frequency interval between 762.1 and 937 Hz. Consequently, when the section ratio is increased to $m = 10$, the maximum *TL* decreases sharply to $TL = 850.1\ dB$ with a large extension of the frequency interval from 724.9 to 975.2 Hz. Clearly, these results indicate that the cross sectional ratio m is a crucial parameter for increasing the *TL* transmission loss coefficient. This leads to an extension and increase in the width of the band gaps. This result is in agreement with the study by M. El Malki and A. Khettabi who showed that periodicity and also the cross-section ratio allow the acoustic band gap to be widened using a different method called the Green's Function" [29]. Finally, our results show that increasing the band gap width by modifying the *TL* allows us to control the acoustic noise using our periodic expansion chamber system.

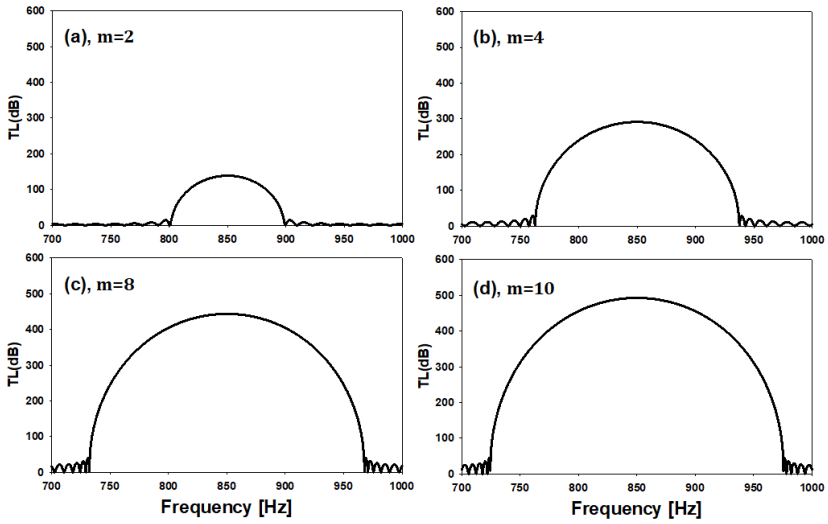

Fig. 3. *TL* variation with frequency for various values of the cross-sectional ratio $m = s_2/s_1$. (a) For $m = 2$, (b) $m = 4$, (c) $m = 8$ and (d) $m = 10$.

3.2 Perfect periodic structure with defects

In this last part, we introduce a geometric defect in the middle of the periodic structure (see Figure 1) to analyse the properties of the resonance modes in the acoustic band gap. The frequency of the incident wave is between 700 and 1000 Hz, and the acoustic wave propagates at normal incidence. The transmission spectrum of the proposed defect-free structure is shown in Figure 4(a). Figure 4(a) shows the creation of a wide band gap of 173.4 Hz with left and right edges at frequencies of 763.3 Hz and 936.7 Hz, respectively, without the appearance of a defect mode. This shows that when the length of the defect is equal to the normal length of the chamber, i.e., $L_D = L$, the structure operates as a perfect periodic structure. Figures 4((b)-(d)) illustrate the variation in the transmission spectrum across the structure when a defect of length $L_D \neq L$ is positioned within the middle of the periodic structure. As shown in the figure, we progressively increased the length of the central chamber by three values, namely, $L_D = 2L$, $L_D = 3L$ and $L_D = 4L$. Figure 4(b) shows that the existence of this type of defect leads to an extension of the band gap over a width of 1276.3 Hz with the appearance of a defect mode located in the middle at a resonance frequency of 850 Hz. This defect peak appears in the middle of the band gap with a high transmission rate equal to $T = 0.95$ and a quality factor of $QF = 85000$, which is obtained by calculating the quotient between the frequency of the resonance peak and the total width at the half-maximum of the resonance peak. The creation of a defect mode is the result of a coincidence between the resonance mode of the cell presenting the defect and the resonance mode of the periodic system. This coincidence causes a disturbance in the spatial periodicity of the structure and therefore leads to the production of licalised defect modes in the acoustic band gap through structural reflection of the propagated acoustic wave. Interestingly, the presence of a single defect peak in the band gap is due to the even number of defect guides ($L_D = 2L$) inserted in the structure. Furthermore, Figure 4(c) shows the creation of two defect peaks in the same band gap for an odd number of defect guides ($L_D = 3L$). Consequently, when the number of defects introduced into the system is odd, we obtain two defect peaks in the band gap and a single peak when the number of defects is even. This important property is also confirmed in Figure 4(d), where we found a single defect mode at the center of the band gap for $L_D = 4L$.

The parity of the number of defects is therefore a very important parameter for controlling the defect peak number and position in the band gap. Finally, our results have enabled us to design one or more multichannel acoustic transmissions.

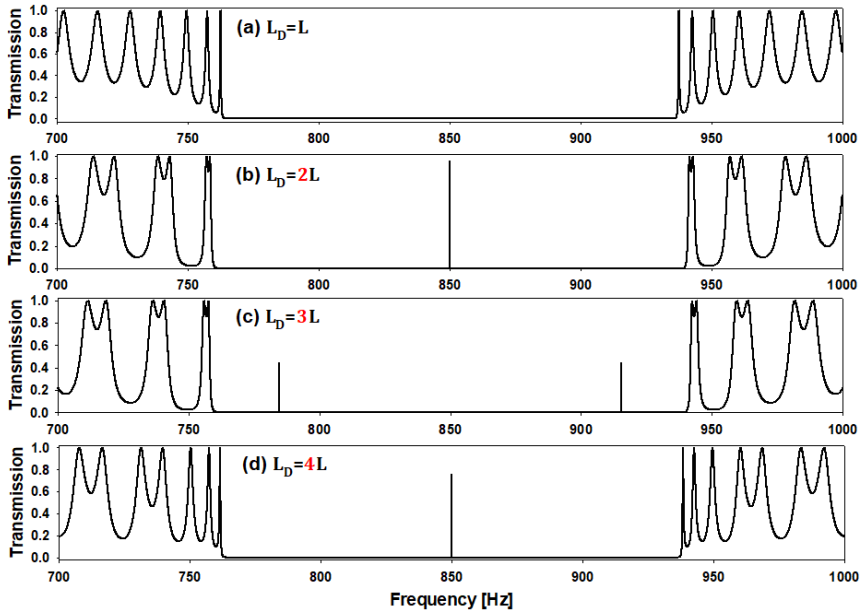

Fig. 4. Variation in the transmission with frequency for different defect lengths L_D. (a) For $L_D = L$, (b) $L_D = 2L$, (c) $L_D = 3L$ and (d) $L_D = 4L$.

4 Conclusions

This research focuses on the study of the acoustic characteristics of defect modes through a system composed of expansion chambers. We first examined the effects of the spatial periodicity of our system on the total reflection of the acoustic wave. Using Bloch's theorem, we have shown that the periodicity of the system leads to the creation of specific frequency intervals called acoustic band gaps. The design of these bands is essential for trapping the acoustic transmission and therefore the total reflection of the acoustic wave. We also used an important technique to increase the band gap width to reduce noise by modifying the waveguide cross-sectional ratio. The results demonstrated that increasing the cross-sectional area increases the sound transmission loss coefficient. The interest in these results lies in controlling the position and width of the band gap to improve the acoustic noise reduction properties. In the second work, we inserted a geometrical defect. The variation of the defect leads to the emergence of resonance modes corresponding to the modes of the defect in the band gap. The results show that inserting an even number of defects (expansion chamber defects) produces a single resonance mode at the center of the band gap, with a high transmission rate. On the other hand, the introduction of an odd number of defects creates two defect modes in the same band gap. The creation of these defect modes is very useful for the design of multichannel acoustic filters with narrow transmission inside the band gap.

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