Defect mode properties of an acoustic structure made up of periodic expansion chambers containing defects

Mohamed El Malki*, *Ilyas* Antraoui, and *Ali* Khettabi

Laboratory of Materials, Waves, Energy and Environment, Department of Physics, Faculty of Sciences, Mohammed First University, Oujda 60000, Morocco

Abstract.
Noise pollution is an environmental menace in agricultural, industrial, commercial, and residential facilities of all countries around the globe particularly developing countries. This paper examines noise reduction of an acoustic band gap structure made from expansion chambers. The interface response predictions of the transmission loss are presented and compared with the experiment, and the boundary element method (BEM) data obtained from the literature showing good agreement at low frequencies. A wider band gap with and stronger muffling effects at a lower frequencies is shown, which depends on the geometry of the expansion chamber and the periodicity. Furthermore, the periodicity is broken by the introduction of defects to analyse the narrow frequency transmission bands within the band gaps. In this work, the defect is artificially produced and controlled by the length and/or the cross-section of the central waveguide segment. The influence of dimension parameters on the wave suppression band gaps is analyzed. We show that the defect mode transmission moves within band gaps. It location can be controlled by the dimension of the defective expansion chamber. The closest to the middle of the band gap, the narrowest becomes. The results presented in this work can help to guide the achievement of broader acoustic band gaps in waveguide systems.

1 Introduction

Since the 1970s, the World Health Organization (WHO) has considered environmental noise an important problem [1]. A definition of environmental noise is that emitted from all noise sources, except noise at the industrial workplace [2]. Environmental noise exposure has several impacts on human health and the environment [3]. One of the important efforts for noise reduction of heavy machines, cars and HVAC systems (Heating, ventilation, and air conditioning) is ducted silencing. This later is based on the use of different impedances to attenuate noise by reflections induced by geometrical discontinuities.

The wave propagation in periodic structures is an important subject in wave guiding and filtering. Band gap structures have received attention from many researchers. Much of this research goal is to examine and create or design acoustic or photonic band-gap structures. This last has the objective of manipulating the properties of the waves. Many practical cases have been developed for electromagnetic waves [4–6]. The ability to manipulate acoustic waves has led to many suggested applications [7–8]. Yet, research in this area is far less extensive.

The expansion chambers behave like a simple model of a low pass filter, and has applications in automotive engineering. An expansion chamber is the simplest model of an exhaust system. Expansion chambers mufflers have some distinguished performance characteristics. Namely, broad anechoic band, corrosion resistance, high temperature resistance, long service life, and important pressure resistance. Hence, working with these silencers is very common.

The propagation of acoustic waves through expansion chambers has been studied [9-12]. The methodology outlined in these works has the ability to optimize the design of periodic resonators to reduce noise in engineering applications. The evaluation of dispersion helps predict band gaps. The band structure constructed by varying cross section of a hard-walled cylindrical duct is analyzed [13]. The method of null-field [14] is used to study the band gap of a periodic waveguide. The resonance phenomena of parallel plate waveguides with periodic corrugations is describe using an interesting comparison between three methods [15].

The effects of noise elimination by unit cell of an expansion muffler and the periodicity along the main waveguide has it influence on band gap widths and muffling effects at lower frequencies [16]. In this case, many parameters are investigated to control bandwidth, such as the resonance frequency, the period including dimensional parameters of the expansion chamber are investigated. In addition, a gas–liquid duct silencer can enrich the acoustic performance of low-frequency noise attenuation [17], the results show that ultra-broad bands and ultra-low frequencies can be reduced by silencers with a small size. A broadband and strong sound

* Corresponding author: m.elmalki@ump.ac.ma

attenuation can be achieved by periodic the perforated and micro-perforated periodic expansion chambers muffler [18-22] which offer a promising application in duct noise control. Other efficient designs used Helmholtz resonators [23-27], and extended-tube resonators [28-30] to reduce noise at low frequencies. Yet, the band gaps caused by periodic expansion chambers seems to be much narrower than those of Helmholtz resonators.

The achievement of broadband, and low-frequency sound attenuation using periodic structures has been a challenging problem in many application cases. Sound attenuation can be also controlled by existence of artificial defects inside the periodic structure. However, only few studies exist [31]. In addition, the complexity of the appearance of defect mode in practical cases is still an important problem. Munday [32] studied the defect states caused by a defective expansion chamber called diameter-modulated waveguide. In the previous work, Munday discussed the limitations of experiment that restrain the design of the narrowest and higher transmission defect states predicted by the theoretical calculations. He also explained important steps, which could be considered to avoid the previous experimental limitations.

Recently, a defective phononic crystal made of alternating waveguides is used to detect hazardous greenhouse gases. The simulation results show the importance of longitudinal acoustic speed on the position of the resonant peaks [33].

In this paper, we numerically study the properties of acoustic forbidden band and defect states in periodic expansion chambers. Localized modes are important characteristics in many acoustic/photonic band-gap system applications. Defect modes are created by broking the periodicity of expansion chambers. While the periodic acoustic waveguides properties have been widely studied, few focuses has been assigned to that of defect states in acoustics. In this work, the defect is constructed by modifying the length and/or the cross-section of the waveguide located in the middle of the system. The corresponding narrow band of transmission moves inside band gaps as function of the defect dimension changes. The closest to the middle of the forbidden bands, the narrowest becomes.

2 Theoretical model

One-dimensional (1D) investigation is performed to determine the acoustic properties of the periodic expansion chambers. While the changing effect of the waveguides impedances on the transmission loss is important at low frequencies, it is useful to consider a one-dimensional plan wave assumption due to its simplicity.

The formalism of the interface response function (IRF) is used to determine transmission coefficient and the transmission loss, which can evaluate the acoustic performance of the structure in terms of noise reduction. The acoustic band gap structure based on waveguide array is created by joining multiple mufflers (Figure 1).

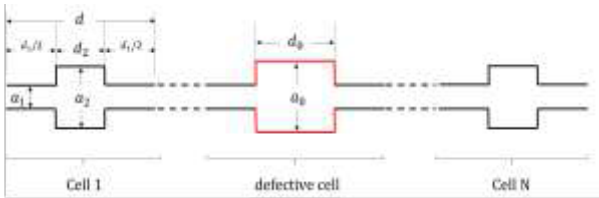

Fig. 1. One-dimensional (1D) model of the N unequally sized waveguides. The periodic structure contains one defective cell composed of a different expansion chamber is introduced in the middle of the structure.

Each cell of the periodic structure contains two waveguides with different dimensions. In practical case, joining adjacent cells can cause additional reflections due to the impedance that represent the small gap between cells. However, masking tape used to connect the sections can effectively reduce this effect compared to the interlocking PVC pipe connectors [32].

We call a_i and d_i $(\mathrm{i} = 1 - 2)$ the diameter and length of each waveguide, respectively. Namely, the main waveguide and the expansion chamber. In the general case, d_1 and d_2 are different $(d_1 \neq d_2)$, we note that $d = d_1 + d_2$ is the length of the elementary cell. A defect is created at a cell, in the middle generally, by changing the dimension of the central expansion chamber to provoke more reflections. Let a_0 and d_0 $(\mathrm{i} = 1 - 2)$ be the diameter and length defective expansion chamber.

Each cell is the association of two finite waveguides. By using IRF theory [10], the transmission coefficient corresponds to the common unit cells can be written as:

$$\mathrm{T} = \left| \frac{1}{\cos(kd_2) + \mathrm{j}\frac{1}{2}\left(\frac{z_2}{z_1} + \frac{z_1}{z_2}\right)\sin(kd_2)} \right|^2 \tag{1}$$

The transmission loss can be determined as

$$\begin{aligned} \mathrm{TL} &= 10\log_{10}\left|\frac{1}{\mathrm{T}}\right| \\ &= 10\log_{10}\left[1 + \frac{1}{4}\left(\frac{s_2}{s_1} - \frac{s_1}{s_2}\right)^2 \sin^2(kd_2)\right] \end{aligned} \tag{2}$$

The creation of a dimensional defect inside the periodic expansion chambers can make reduce noise. Here, the studied defect is within the expansion chambers located in the middle of N cells. The effect of the change in its length and cross-section will be examined. The interface response theory allows us to make the necessary operations to form the defective structure and then obtain its transmission coefficient. The calculation of the interface response function in the real interface space of the structure containing a defect requires defining the disturbance operator which authorizes all operations at the level of the interfaces affected by the disturbances. These operations are explained in [11] in detail.

The perturbation due to the insertion of the defective expansion chamber in the structure is given by:

$$\gamma = \frac{S_1}{F_1}\left(\frac{F_2 C_2}{S_2} - \frac{F_{02} C_{02}}{S_{02}}\right) \tag{3}$$

where

$F_i = -\mathrm{j}\omega/z_i$, $C_i = \cosh(kd_i)$, $S_i = \sinh(kd_i)$, $(\mathrm{j} = \sqrt{-1})$, $k = \omega/c_0$. z_i is the characteristic impedance of the waveguide i, ω the angular frequency of the incident acoustic wave, and c_0 refer to the speed of sound.

The defective structure transmission coefficient of N unit cells can be obtained by following the expression detailed in [34]:

$$T = \left| \frac{2\sin(kd_1)(t^2-1)t^N}{(\alpha_1^2-\alpha_1^2 t^{2N})(t^2-1)+\beta\gamma} \right|^2 \quad (4)$$

where

$$\beta = t(\alpha_1^2 - \alpha_1^2 t^{2N}) - \alpha_1\alpha_2\left(t^{2(N-n+1)} + t^{2n}\right),, (1 < n < N) \quad (4a)$$

$$\gamma = \frac{S_1}{F_1}\left(\frac{F_2 C_2}{S_2} - \frac{F_{02} C_{02}}{S_{02}}\right) \quad (4b)$$

$$\alpha_1 = 1 - te^{jkd_1} \quad (4c)$$

$$\alpha_2 = t - e^{jkd_1} \quad (4d)$$

$$t = e^{jKd_1} \quad (4e)$$

n is an integer representing the location of the defective cells.

3 Numerical results

3.1 Numerical validation of the transmission loss

The acoustic wave attenuation of the expansion muffler is investigated numerically and experimentally. In Fig. 2, a comparison of the transmission loss is made between the interface response prediction (black dotted symbols), the experiment, and the boundary element method (BEM) [35], which were extracted from its publication for a given geometrical constriction (Table 1).

Table 1. Geometric parameters of the structure.

Parameter	Value
Expansion chamber diameter a_2 [mm]	164.4
Expansion chamber length d_2 [mm]	257.2
Duct diameter a_1 [mm]	49
Velocity [ms^{-1}]	340.0

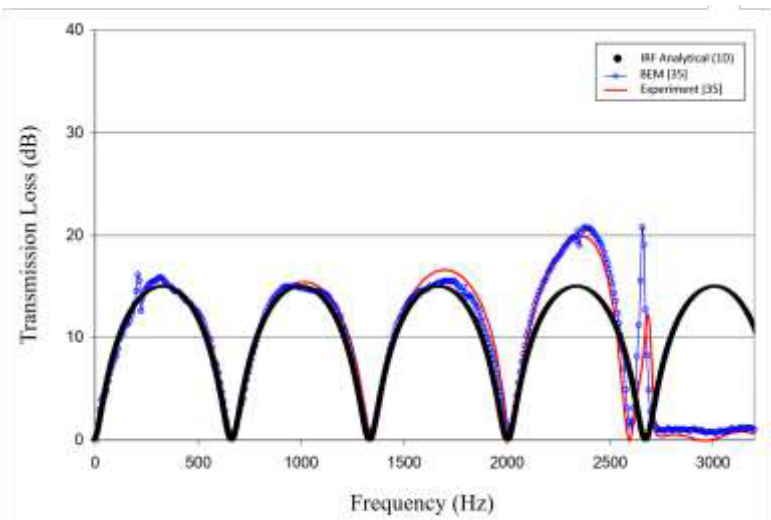

Fig. 2. The transmission loss of the reactive expansion chamber. Comparison of the 1D analytical IRF predictions (black circle), and the BEM vs. the experiment (red line) and blue dotted symbols represent the BEM and the experiment of [35].

Clearly, the results are in good agreement especially in the frequency range {0 – 2000 Hz}.

The TL curve has troughs at frequencies where $f_n = nc_0/2d_2 (n = 1, 2, 3, \dots)$ [10]. In this example, the fundamental frequency is at 660Hz, and the harmonics are at 1322Hz, 1983Hz.

As high frequency range, the 1D analysis fails due to multi-dimensional effects. The first higher order mode occurs around the frequency 2634Hz.

3.2 Effect of the variation in length and section of the dimension defect

Here, we numerically discuss the presence of a defective expansion chamber effects inside the periodic waveguides filled with air. Figure 3 shows the acoustic wave transmission for a defective cell located in the middle of the 10 identical expansion chambers. This defect is created changing the length of one expansion chamber located in the center of the structure. The case where $d_0 = d_2$ ($d_2 = 2d_1$ = 400mm) corresponds to the perfect case where no defect is introduced. The band structure in this case has two pass-bands of widths equal to 160Hz and 230Hz, respectively. These two bands are surround a central band gap of 60Hz in its width. The control of acoustic wave propagation mainly depends on the resonance of the expansion muffler and the periodicity given by all the regularly spaced mufflers [10, 16].

A localized mode appears inside the forbidden band with important transmission when a defective expansion chamber is connected. For $d_0 < d_2$ (Figure 3(a)), increasing the defective length makes the defect mode moves towards the lower frequencies with an increase in its transmission. On the other hand, the transmission decreases for $d_0 > d_2$ (Figure 3(b)). The movement direction of the localized mode is towards low frequencies as the expansion chambers are low-pass filters. In addition, the band gap is wider by when defective length d_0 increases.

The quality factor (Q-factor) measures the width of the peak, and is defined as $Q = f_0/\Delta f$, where f_0 is the peak

frequency, and Δf is the full width at half maximum of the resonance peak. Table 2 resumes the results of the Q-factor of the defective length d_0.

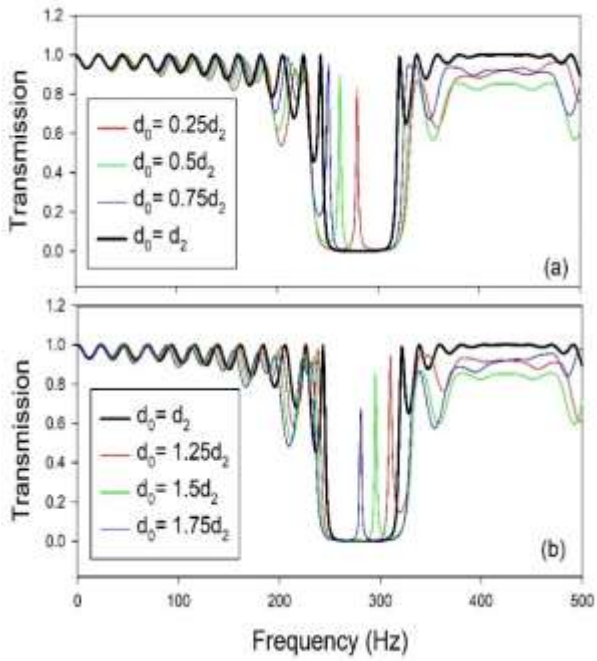

Fig. 3. Variation of transmission coefficient for different values of defect length (d_0). The defect has the form of an expansion chamber located between five identical unit cells (n = 5) for $d_2 = 2d_1$, and s_2/s_1 = 1.5.

Table 2. The Q-factor of the defect peak resulting from various lengths of the defective expansion chamber.

Length of the defect	Q-factor
$d_0 = 0.25d_2$	137.03
$d_0 = 0.5d_2$	99.16
$d_0 = 0.75d_2$	47.51
$d_0 = 1.25d_2$	77.5
$d_0 = 1.5d_2$	93.65
$d_0 = 1.75d_2$	95.40

The Q-factor is important for $d_0 = 0.25d_2$ and $d_0 = 1.75d_2$. Hence, the defect peak is very narrow when it is localized in the center of the band gap. In general, the Q-factor of the defect peak generated by a defective side branches [24-30] is very narrow compared to that of the defective alternating waveguide.

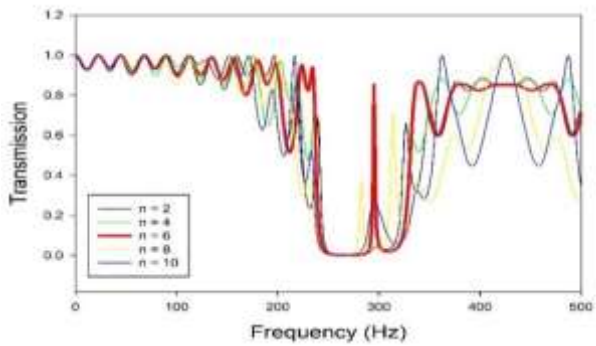

Fig. 4. Defect position variation for $d_2 = 2d_1$, and s_2/s_1 = 1.5, $d_0 = 1.5d_2$, and $N = 11$.

In addition, Figure 4 shows the variation effect of the defective expansion chamber position within the structure. The closer the defect is to the middle of the structure, the greater localization of its transmission. These because the defect is more spatially confined when it is close to the structure center.

Figure 5 shows the variation of the defective section s_0 of the expansion chamber. The localized mode confined around the frequency 295Hz. Increasing the defective section in this case reduces the defect transmission. In addition, the transmission bands are affected by the defective section compared to that of the defective length.

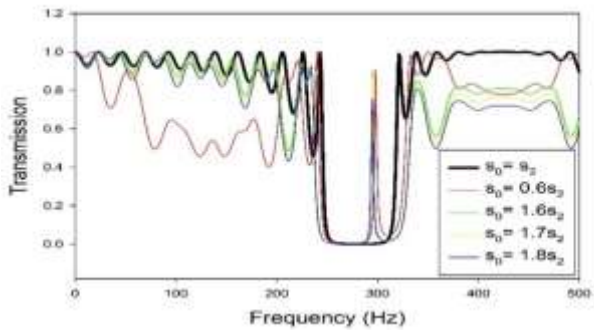

Fig. 5. Variation of defect mode transmission for different defective cross-sections s_0. The defect is confined in 295Hz within the first band gap for $d_2 = 2d_1$, and s_2/s_1 = 1.5, and $N = 11$.

4 Conclusion

This work concerns noise reduction using a periodic structure composed of expansion chambers. Stronger muffling effects is shown at a low frequencies given by wider band gap, which depends on the geometry of the expansion chamber and the periodicity.

The transmission loss of an expansion chamber predicted by 1D analytical analysis is compared with the experiment, and the boundary element method (BEM) data obtained from the literature and they show good agreement at low frequencies.

The effect of defects on the acoustic performance of a periodic reactive silencer based on expansion chambers is investigated. The defective expansion chamber gives rise to a localized mode inside the band gap when the periodicity is broken, the effect of its length and cross-section is examined. We show the defect mode transmission movement within band gaps, it location can be controlled by the dimension of the defective expansion chamber. The closest to the middle of the band gap, the narrowest becomes.

More muffling effects are added at lower frequencies as the pass-bands are affected by the variation of the defect dimension.

These results can help and guide the achievement broad acoustic band gaps filters at low frequency in a waveguide system. Yet, more efforts need to be involved to investigating the acoustic performance of the muffler under the consideration of temperature and flow effect conditions in the future. Considering these ingredient

effects, the optimal design of the periodic structure is another major subject for further studies.

References

1. WHO. Environmental noise guidelines for the European region. World Health Organization. Regional Office for Europe, (2018)
2. B. Berglund, T. Lindvall, D.H. Schwela, WHO, Guidelines for community noise, (1999)
3. A. Lead, Noise as a Public Health Hazard, (2021)
4. J.C. Chen, H.A. Haus, S. Fan, P.R. Villeneuve, and J.D. Joannopoulos, J. Lightwave Technol **14**(11) 2575–2580 (1996)
5. S.Y. Lin, E. Chow, V. Hietala, P.R. Villeneuve, and J.D. Joannopoulos, Science **282**(5387), 274–276 (1998)
6. R.F. Cregan, B.J. Mangan, J.C. Knight, T.A. Birks, P.S. Russell, P.J. Roberts, and D.C. Allan, Science **285**(5433), 1537–1539 (1999)
7. Z. Ye and E. Hoskinson, Appl. Phys. Lett. **77**(26), 4428–4430 (2000)
8. A. Diez, G. Kakarantzas, T.A. Birks, and P.St.J. Russell, Appl. Phys. Lett. **76**, 3481–3483 (2000)
9. V.D. de Lima, J. M.C. dos Santos, J.R.F. Arruda, *Passive control of noise propagation in tube systems using bragg scattering*, in Proceedings of the International Symposium on Dynamic Problems of Mechanics, DINAME, Springer, 523–530 (2017)
10. M. El Malki, A. Khettabi, AIP Conf Proc. **2074**, 020025, AIP Publishing LLC (2019)
11. A. Khettabi, I. Antraoui, AIP Conf Proc. **2074**, 020003, AIP Publishing LLC (2019)
12. M. Darche, F. Lopez-Caballero, B. Tie, JSV **572**, 118158 (2024).
13. A. Bostrom, Wave Motion, **5**(5), 59–67 (1983)
14. S. Sandstrom, J. Acoust. Soc. Am. **79**,1293–1306 (1986)
15. S. Sandstrom, J. Acoust. Soc. Am. **82**(5) 1797–1807 (1987)
16. B. Liu, J. Liu, W. Wei, H. Shen, Z. Wei, AIP Advances, **8**(11), (2018)
17. J. Liu D. Yu, J. Wen, Z. Zhang, J. Theor. Comput. Acoust. **27**(02), 1850026 (2019)
18. X. Shi, C.M. Mak, Appl. Acoust. **115**, 15-22 (2017)
19. C. Lu, C. Wan, Z. Liu, S. Du, and Y. Zhu, Appl. Acoust. **115**, 15–22 (2017)
20. A.M. Goto, J.M.C. Dos Santos, *Sound attenuation of periodic micro-perforated chamber mufflers using the spectral transfer matrix method*, in CILAMCE Proceedings of the XLIbero-Latin American Congress on Computational Methods in Engineering, ABMEC, November 11-14, Natal/RN, Brazil (2019)
21. X.L. Gai, T. Xing, Z.X. Kang, X.H. Li, B. Zhang, Z.N. Cai, F. Wang, X.W. Guan, Appl. Acoust. **161**, 107187 (2020)
22. P. Liu, S. Zuo, X. Wu, S. Chen, Y. Kong, Int. J. Mech. Sci. **234**, 107677 (2022)
23. M.L Munjal, Acoustics of ducts and mufflers with application to exhaust and ventilation system design. (John Wiley & Sons 1987)
24. A. Khettabi and M. Elmalki, IEEE 1-3 (2017, November)
25. I. Antraoui, A. Khettabi, *Defect modes in one-dimensional periodic closed resonators*, in Advances in Integrated Design and Production: Proceedings of the 11th International Conference on Integrated Design and Production, CPI, 438-445. Springer International Publishing, October 14-16, 2019, Fez, Morocco (2021)
26. M. El Malki and A. Khettabi, Mater. Today: Proc. **72**, 3786–3794 (2023)
27. M. El Malki, N. Pereira, and S. Lanceros-Mendez, E3S Web Conf. **364**, 03003 (2023)
28. A. Selamet, I. Lee, J. Acoust. Soc. Am. **113**(4), 1975–1985 (2003)
29. C. Cai, C.M. Mak, X. Shi, Appl. Acoust. **115**, 74–80 (2017)
30. M. E. Malki, N. Pereira, and S. Lanceros-Mendez, *Fano resonance in a defective network formed by helmholtz resonators with extended necks*, in Proceedings of the Digital Technologies and Applications, Springer Nature Switzerland, **669**, 308–319. January 27 to 28, Fez, Morocco (2023)
31. P. King, Acoustic properties of periodically and quasi-periodically modulated waveguides (Doctoral dissertation, MS thesis. University of Salford, 2006)
32. J.N. Munday, C.B. Bennett, W.M. Robertson, J. Acoust. Soc. Am. **112**(4), 1353-1358. (2002)
33. Z.A. Zaky, S. Alamri, E.I. Zohny, A.H. Aly, Sci. Rep. **12**(1), 21553 (2022)
34. J.O. Vasseur, A. Akjouj, L. Dobrzynski, B. Djafari-Rouhani, E.H. El Boudouti, Surf. Sci. Rep. **54**(1-4), 1-156 (2004)
35. I. Lee, Acoustic characteristics of perforated dissipative and hybrid silencers (PhD thesis, The Ohio State University, pp.129, 2005)