Sensitivity analysis of underwater launch based on Morris method

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Abstract. The discharge velocity, pressure difference and acceleration of underwater launch performance are affected by many factors, including depth, sailing speed, burning rate, water temperature, density and other operating parameters. The existing underwater launch sensitivity analysis is a local sensitivity qualitative analysis based on single-step perturbation analysis, which can only investigate the influence of a single uncertain factor on the model results in a small range. In this paper, based on the underwater launch model under multi-factor coupling, 10 input parameters were selected to calculate the multi-factor sensitivity of the underwater launch model based on the three objectives of velocity, pressure difference and acceleration at the time of discharge, and the Morris global sensitivity analysis method was used to calculate the sensitivity of the underwater launch model, and the sensitivity of the 10 input parameters to the model results was sorted. The results show that the five input parameters of burning rate, depth, propellant temperature, density and water temperature have the highest sensitivity to velocity, pressure difference and acceleration, and the sensitivity order is slightly different. The influence of water temperature on the three targets is almost evenly distributed in the range of values, and the other parameters have different sensitive ranges for different targets.

1. Introduction

Underwater load launching technology is a launching method that accelerates the load underwater to a certain speed to get out of the launcher barrel through the combustion and expansion of the working medium. Now it has been developed to a more advanced variable depth underwater launch technology, adding an energy adjustment control mechanism on the original basis. The useful energy reaching the tail of the load is regulated by controlling the amount of cooling water injected to achieve emission at different depths. Due to the huge economic cost and labor cost of underwater launch test, it becomes an important means to study underwater launch technology to construct mathematical model of underwater launch simulation to simulate the underwater launch process and obtain the underwater launch state parameters[1-4]. The mathematical simulation model of underwater launch needs to be diagnosed and predicted by sensitivity analysis to ensure the accuracy and reliability of the model.

Sensitivity analysis is an analytical calculation method based on mathematical model, which draws relevant conclusions by calculating the influence degree of input parameters on model output results. It is used to evaluate the influence of each input parameter of the model on model results qualitatively or quantitatively, including the influence of model output and model performance[5]. Sensitivity analysis can be divided into local sensitivity analysis and global sensitivity analysis[6]. Local sensitivity analysis only examines the impact of small changes in each input parameter of the model within a local range on the model results. Only when the model input and output are linear or approximately linear dependent, can local sensitivity analysis better grasp the impact of each input parameter of the model on the overall performance of the model. Global sensitivity analysis studies the global impact of each input parameter of the model on the model results within its allowable variation range, and it does not require the model to be a linear system. Therefore, global sensitivity analysis also includes examining the impact of the interaction between model parameters on the model results[7]. Global sensitivity analysis can be divided into qualitative analysis and quantitative analysis. Qualitative analysis is to study the relative size of response of different parameters to output variables, such as Morris method, Delta Test method, etc. Quantitative analysis can obtain the contribution rate of different parameters to the output variables, such as Sobol method and EFAST method[8-11].

In the calculation of previous underwater launch sensitivity studies, researchers only used local sensitivity analysis, and only calculated the influence of small changes in the value range of input parameters on the output results, without considering the interaction between input parameters. In this paper, the input parameters of the underwater launch model were analyzed based on the Morris global sensitivity analysis method, and the significant order of the influence of the input parameters (depth Ht, speed Vt, water temperature Ts, burning rate Vr, propellant...
temperature Ty, density ρ, atmospheric pressure P, air initial temperature Tk, friction coefficient μ and structural inner diameter D) on the exit velocity, pressure difference and acceleration was determined, which provided a basis for the subsequent quantitative analysis of underwater launch sensitivity.

2. Underwater launch model

2.1. Basic Assumptions

Underwater launch is a complex process of energy exchange and energy conversion, and there are many factors that affect the change of the whole working system. The following assumptions are made when establishing the model of underwater launch:

(1) Without considering the change of gas parameters along the pipeline, the flow of gas along the pipeline is regarded as the transmission process of working medium energy from the power source to the launching cylinder.

(2) The working medium gas, water vapor and the air in the launching cylinder can be uniformly mixed every instant to exchange energy and form a mixture of uniform state parameters at each point.

(3) For the energy loss such as heat transfer, the energy coefficient is taken into account, and its value is assumed to be a constant in the whole working process.

(4) For the macroscopic kinetic energy contained in the working medium gas, kinetic energy coefficient is introduced, and its value is assumed to be a constant in the whole working process.

2.2. Underwater launch model

2.2.1. Equation of flow.

In the launching process, the flow per second \(G_g\) and working medium quality \(m_g\) are

\[G_g = \frac{1}{C^*} A \mu_f P_c\]  \hspace{1cm} (1)

\[m_g = \int_0^t G_g \, dt + m_{g0}\]  \hspace{1cm} (2)

In the equation(1), \(C^*\) is the characteristic velocity, which represents the influence of the combustion process of working medium on the mass flow rate, and its dimension is the same as that of the velocity; \(A\) is the cross section area of the conveying pipeline; \(\mu_f\) is the flow coefficient; \(P_c\) is the pressure in the combustion chamber.

2.2.2. Equation of energy.

The mixed gas formed by the working medium gas, water vapor and air builds up the pressure in the closed space, overcomes the resistance of the load movement, and ejects the load out of the launching cylinder. The energy equation is as follows.

\[U_{g1} + U_{j1} + U_{a1} + Q = U_{g2} + U_{j2} + U_{a2} + \int_0^1 PdV\]  \hspace{1cm} (3)

Cooling water is added in the working process, and the cooling water absorbs the latent heat of evaporation into water vapor. In this process, a certain saturation pressure will correspond to a certain saturation temperature. The Clausius-Claperon equation is used to calculate the change of saturation pressure with temperature.

\[\frac{dP_{sat}}{P_{sat}} = \frac{h_{fg}}{R} \frac{dT_{sat}}{T_{sat}^2}\]  \hspace{1cm} (4)

2.2.3. Equation of motion.

The actual gas state equation of 55 Martin-Hou is used to describe the state of the working medium gas in the emission cylinder.

\[P = \frac{RT}{V-h} + \frac{A_1 + b_1T + C_1e^{A_2T}}{V-b_2} + \frac{A_2}{V-b_3} + \frac{b_3T}{V-b_4}\]  \hspace{1cm} (5)

2.2.4. Equation of state.

The kinetic energy of the emitted load is calculated by subtracting the work done by the resistance during the motion. The resistance includes gravity, friction, atmospheric pressure, hydrostatic pressure, lateral load force and additional inertial force.

\[\int_1^2 PdV = \frac{1}{2} MV^2 + FL\]  \hspace{1cm} (6)

\[F = Mg + F_z + P_gS_L + \rho gS_LF_h + \psi Ma\]  \hspace{1cm} (7)

3. Underwater launch sensitivity analysis

This paper takes the input parameters of the underwater launch model as the research object, and uses Morris method to analyze the global sensitivity of the parameters. The effects of depth Ht, speed Vt, water temperature Ts, burning rate Vr, propellant temperature Ty, density ρ, atmospheric pressure P, air initial temperature Tk, friction coefficient μ and structural inner diameter D on the velocity, pressure difference and acceleration of underwater ejector cylinder are studied. The specific steps are as follows.

3.1. Morris sample design

Assuming that the number of sampling points of a single parameter is \(P\), the parameter values are discretized to obtain discrete points \(\left\{0, \frac{1}{P-1}, \frac{2}{P-1}, \ldots, 1-\Delta\right\}\), \(\Delta = \frac{1}{P-1}\). In this paper, \(P=6\) is taken, and the value
The range of input parameters is evenly divided into \((P-1)\) isometric intervals.

### 3.2. Generate a randomized matrix

The randomization matrix of the sampling matrix \(B^*_\{l,n\}\) can be expressed as:

\[
B^*_\{l,n\} = (M_{\{l,n\}} \times Z^*_\{l,n\}) + \frac{\Delta}{2} \times J^*_\{l,n\} \times P^*_\{n,n\} \tag{8}
\]

In equation (8), \(l = n + 1\), \(n\) is the number of parameters; \(B^*_\{l,n\}\) is a \(l \times n\) dimensional matrix, which is characterized by the difference between two adjacent rows and only one parameter value, the other parameter values are the same; \(M_{\{l,n\}}\) is a \(l\) dimensional column vector in which all elements are \(1\); \(Z^*_\{l,n\}\) is the \(n\) dimensional row vector, \(Z^*_\{l,n\} = [x_1, \cdots, x_1, \cdots, x_n]\). The parameter \(x_i\) is randomly taken from \(\left\{0, \frac{1}{p-1}, \frac{2}{p-1}, \cdots, 1 - \Delta\right\}\). \(P^*_\{n,n\}\) is a \(n \times n\) dimensional random permutation matrix, each row and column of the matrix has only one value of \(1\), and the rest values are all \(0\). \(J^*_\{l,n\}\) is a \(l \times n\) dimensional matrix, and the matrix \(J^*_\{l,n\}\) calculation equation is

\[
J^*_\{l,n\} = 2B^*_\{l,n\} - J^*_\{l,n\} \times D^*_\{n,n\} + J^*_\{l,n\} \tag{9}
\]

In equation (9), the matrix \(B^*_\{l,n\}\) is a strictly lower triangular matrix with all element \(1\); \(J^*_\{l,n\}\) is a matrix where all the elements are \(1\); \(D^*_\{n,n\}\) is a \(n\) dimensional diagonal matrix, and each diagonal element can only be randomly valued \(\pm 1\) with equal probability.

In this paper, the number of input parameters of underwater launch model is 10. According to equation (8) and (9), a random sample matrix \(B^*_{\{11,10\}}\) can be obtained by programming with Matlab. In this paper, 6 random sample matrices are obtained after 6 times of sampling. The random sample matrix \(B^*_{\{11,10\}}\) is listed below, and other generated random sample matrices will not be described again.

\[
B^*_{\{11,10\}} = \begin{bmatrix}
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
H_2 & V_1 & T_3 & V_4 & T_6 & \rho_5 & P_6 & T_k_4 & \mu_6 & D_6 \\
\end{bmatrix}
\]

### 3.3. Calculation of basic sensitivity factors

Assuming that the value of the \(i\)-th parameter changes in two adjacent rows of the randomized matrix \(B^*_\{l,n\}\), the basic influence factor calculation equation of the \(i\)-th parameter is as follows:

\[
SN_i = \frac{y(z_1, \cdots, z_{i+1}, \cdots, z_n) - y(z_1, \cdots, z_i, \cdots, z_n)}{\Delta} \tag{10}
\]

In equation (10), \(y(x)\) is the objective function. All \(l\) group of adjacent row elements in the randomized matrix \(B^*_\{l,n\}\) are taken as input parameters of the model, and the basic factors of all \(n\) parameters can be obtained according to equation (10), a basic influence factor can be obtained for each sample with this parameter using a randomized matrix \(B^*_\{l,n\}\). The expressions of the absolute mean \(E_i\) and standard deviation \(\sigma_i\) of the basic impact factor for the response of the \(i\)-th input parameter are:

\[
E_i = \sum_{j=1}^{r} \left| SN_y \right| \left( r \right)^{-1} \tag{11}
\]

\[
\sigma_i = \left[ \sum_{j=1}^{r} \left( SN_y - \sum_{j=1}^{r} SN_y / r \right) \left( n \right)^{-1} \right]^{0.5} \tag{12}
\]

In formula (12), The absolute value of \(SN_y\) is taken in order to avoid the positive and negative balance at \(SN_y < 0\).
4. Conclusion

For parameter sensitivity, the absolute mean $E_i$ mainly describes the importance of the input variable $x_i$. The greater the value of $E_i$, the greater the influence of parameter $x_i$ on the model output results. Standard deviation $\sigma_i$ describes the importance of a variable from the other hand. If variable $x_i$ has a higher standard deviation, it indicates that parameter $x_i$ affects the output value of the model and the interaction with other parameters is greater, indicating that the impact of this parameter on the output of the model is non-linear.

4.1. Sensitivity ranking

Morris global sensitivity analysis method and underwater launch model were used to conduct sensitivity analysis on the input parameters of 10 underwater launch models, and the sensitivity factors of input parameters to the three targets of exit cylinder velocity, pressure difference and acceleration as well as the absolute mean and standard deviation of input parameters were calculated. The results are shown in the following table.

### Table 1. Calculation table for velocity sensitivity of underwater launch.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>$SN_i$</th>
<th>Absolute mean</th>
<th>Standard deviation</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$Ht$</td>
<td>2.093</td>
<td>-2.102</td>
<td>2.148</td>
<td>2.311</td>
</tr>
<tr>
<td>$Ty$</td>
<td>-0.954</td>
<td>0.2475</td>
<td>-0.481</td>
<td>0.2585</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.191</td>
<td>0.3575</td>
<td>0.261</td>
<td>0.2385</td>
</tr>
<tr>
<td>$Ts$</td>
<td>0.2020</td>
<td>-0.204</td>
<td>-0.207</td>
<td>0.2095</td>
</tr>
<tr>
<td>$D$</td>
<td>-0.105</td>
<td>-0.112</td>
<td>0.1115</td>
<td>0.113</td>
</tr>
<tr>
<td>$Vt$</td>
<td>-0.077</td>
<td>0.0755</td>
<td>0.085</td>
<td>-0.083</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.064</td>
<td>-0.069</td>
<td>-0.062</td>
<td>0.0745</td>
</tr>
<tr>
<td>$P$</td>
<td>0.0405</td>
<td>0.037</td>
<td>-0.043</td>
<td>-0.042</td>
</tr>
<tr>
<td>$Tk$</td>
<td>0.013</td>
<td>-0.012</td>
<td>-0.013</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

### Table 2. Calculation table of pressure difference sensitivity of underwater launch.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>$SN_i$</th>
<th>Absolute mean</th>
<th>Standard deviation</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$Ht$</td>
<td>0.09182</td>
<td>-0.0909</td>
<td>0.0901</td>
<td>0.09205</td>
</tr>
<tr>
<td>$Vr$</td>
<td>-0.0059</td>
<td>-0.0010</td>
<td>-0.0179</td>
<td>-0.0308</td>
</tr>
<tr>
<td>$Ty$</td>
<td>-0.0286</td>
<td>-0.0139</td>
<td>-0.0082</td>
<td>-0.0079</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.00117</td>
<td>0.0095</td>
<td>0.0155</td>
<td>0.0135</td>
</tr>
<tr>
<td>$Ts$</td>
<td>0.00704</td>
<td>0.0069</td>
<td>-0.0076</td>
<td>0.0075</td>
</tr>
<tr>
<td>$D$</td>
<td>-0.0036</td>
<td>-0.0036</td>
<td>0.00369</td>
<td>0.00459</td>
</tr>
<tr>
<td>$P$</td>
<td>1.81E-3</td>
<td>1.84E-3</td>
<td>1.82E-3</td>
<td>1.84E-3</td>
</tr>
<tr>
<td>$Tk$</td>
<td>-4.3E-4</td>
<td>2.2E-4</td>
<td>4.6E-4</td>
<td>3.7E-4</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.1E-4</td>
<td>-7.5E-5</td>
<td>-8.5E-5</td>
<td>-2.5E-5</td>
</tr>
<tr>
<td>$Vt$</td>
<td>1E-5</td>
<td>1E-5</td>
<td>1E-5</td>
<td>1E-5</td>
</tr>
</tbody>
</table>

### Table 3. Calculation table of acceleration sensitivity of underwater launch.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>$SN_i$</th>
<th>Absolute mean</th>
<th>Standard deviation</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$Ht$</td>
<td>2.364</td>
<td>-4.031</td>
<td>1.973</td>
<td>4.6655</td>
</tr>
<tr>
<td>$Ty$</td>
<td>-1.148</td>
<td>1.363</td>
<td>2.519</td>
<td>1.076</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.252</td>
<td>4.388</td>
<td>0.133</td>
<td>0.236</td>
</tr>
<tr>
<td>$Ts$</td>
<td>0.1635</td>
<td>-0.323</td>
<td>-0.206</td>
<td>0.177</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0854</td>
<td>-0.174</td>
<td>-0.083</td>
<td>0.427</td>
</tr>
<tr>
<td>$D$</td>
<td>-0.123</td>
<td>-0.202</td>
<td>0.195</td>
<td>0.117</td>
</tr>
<tr>
<td>$Tk$</td>
<td>0.028</td>
<td>-0.509</td>
<td>0.079</td>
<td>-0.044</td>
</tr>
<tr>
<td>$P$</td>
<td>0.0345</td>
<td>-0.037</td>
<td>-0.031</td>
<td>0.026</td>
</tr>
<tr>
<td>$Vt$</td>
<td>0</td>
<td>-5E-4</td>
<td>-5E-4</td>
<td>5E-4</td>
</tr>
</tbody>
</table>

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4.2. Result analysis

Based on the results and discussions presented above, the conclusions are obtained as below:

(1) In the calculation of underwater ejector velocity sensitivity (Table 1), it can be seen that the influences of burning rate (Vr), depth (Ht), propellant temperature (Ty), density (ρ) and water temperature (Ts) on the ejector velocity are in order from high to low, and the influence of water temperature on the ejector velocity is approximately evenly distributed within the value range. The burning rate, depth, propellant temperature and density are sensitive in the value range.

(2) In the calculation of the pressure difference sensitivity of the underwater launch cylinder (Table 2), it can be seen that the influences of depth (Ht), burning rate (Vr), propellant temperature (Ty), density (ρ) and water temperature (Ts) on the pressure difference of the outlet cylinder are in order from high to low, and the influences of depth, density and water temperature on the pressure difference of the outlet cylinder are approximately evenly distributed within the value range. The burning rate and propellant temperature are sensitive to each other.

(3) In the calculation of pressure difference sensitivity of underwater launch cylinder (Table 3), it can be seen that the influences of burning rate (Vr), depth (Ht), propellant temperature (Ty), density (ρ) and water temperature (Ts) on the discharge cylinder velocity are in order from high to low, and the influences of propellant temperature and water temperature on the discharge cylinder pressure difference are approximately evenly distributed within the value range. The burning rate, depth, propellant temperature and density are sensitive in the value range.

References


2. Numerical investigation of a muzzle multiphase flow field using two underwater launch methods


