Quantifying Fracture Dynamics in Clean Energy: A Novel Fractal Perspective

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Abstract. Geothermal energy, as an emergent source of power, has consistently been a focal point of scholarly investigation both domestically and internationally, with a particular emphasis on the prediction and assessment of its extraction efficiency. In common geothermal extraction projects, the seepage behavior of water within the fractures for the geothermal reservoirs significantly impacts the efficiency of resource extraction. This study introduces an interdisciplinary fractal model for geothermal extraction. An enhanced fractal theory, tailored for geothermal reservoirs, is proposed, employing four innovative fractal parameters—fracture fractal dimension, fracture tortuosity fractal dimension, fracture roughness parameter, and maximum fracture length—to quantitatively characterize the fracture structure. This refined fractal theory is applied to geothermal reservoir extraction under the complex thermal-hydro-mechanical coupling. The findings indicate that the proposed structural parameters effectively characterize the micro-macro interactions during the geothermal extraction process. Significant evolution of these fractal parameters is observed throughout the extraction process. Furthermore, there is an inverse relationship between geothermal extraction efficiency and two key fractal parameters—fracture tortuosity fractal dimension and fracture roughness parameter—while the reservoir stress is directly proportional to these parameters.

1 Introduction

As a renewable energy source, geothermal energy, characterized by its stability, longevity, and renewability, is widely acknowledged as a crucial direction for future energy development. In recent years, the exploration and application of geothermal resources have garnered considerable attention and research both domestically and internationally [1-3]. Factors such as the geological-hydrological conditions of the geothermal reservoir, the depth of geothermal extraction, and parameters of the injection and production wells significantly
influence the efficiency of heat extraction. Additionally, the multiphysical field coupling effects in geothermal reservoirs, such as reservoir stress and deformation, porosity, stratum depth, temperature, and the fracturing behavior of geothermal strata [4-6], also significantly impact the efficiency of geothermal extraction.

An increasing number of scholars are delving into the multifactorial coupling effects during the geothermal extraction process [6-8]. These effects encompass the multiphysical phenomena of thermal-hydro-mechanical coupling, including dynamic studies of physical-mechanical parameters like porosity, permeability, reservoir temperature, and water extraction temperature. This lays the groundwork for deriving various types of multiphysical field analysis models [9-11]. However, despite these advances in geothermal extraction research, quantitative nonlinear analyses of the reservoir's fracture network evolution under the disturbances of geothermal extraction and the coupled thermal-fluid-solid interactions remain challenging. Traditional methods relying on CT, digital rock reconstruction, and discrete fracture network studies for geothermal reservoirs involve significant computational efforts, typically focused on micro-nano scales. Furthermore, traditional fractal theory-based analytical methods struggle to integrate the effects of extraction disturbances and complex multiphysical field couplings [12-14].

As the primary conduit for fluid migration, the nonlinear evolution of the fracture structure in geothermal reservoirs directly impacts seepage behavior. Under the influences of extraction disturbances and the thermal-fluid-solid coupling in the reservoir, temperature changes induced by extraction alter the intensity of the reservoir's thermal expansion effect. This, in turn, leads to variations in reservoir stress and results in deformations of varying degrees. Deformations in geothermal reservoirs, such as contraction or expansion, cause the fracture network structure to undergo nonlinear evolution. The multifaceted coupled effects described above directly influence the efficiency of heat extraction. Therefore, quantitatively exploring the nonlinear evolution of the fracture network in geothermal reservoirs during extraction and elucidating the impact of this evolution on heat extraction efficiency is of paramount importance.

To address the industry bottleneck of achieving precise micro-macro quantitative analysis during geothermal extraction under multifactorial coupled influences, this study proposes an interdisciplinary geothermal extraction model. This study introduces an enhanced fractal theory suitable for geothermal reservoirs, utilizing four innovative fractal parameters (fracture fractal dimension, fracture tortuosity fractal dimension, fracture roughness parameter, and maximum fracture length) to quantitatively characterize the fracture structure. Applying this enhanced fractal theory to typical geothermal reservoirs, this model explores geothermal reservoir extraction under complex thermal-fluid-solid coupling. Subsequently, the quantitative evolution characteristics of fracture number, tortuosity, and size during extraction is investigated and the impact of fracture roughness and tortuosity on reservoir stress and extraction efficiency is analyzed. The findings of this study are anticipated to provide a scientific basis for the rational utilization and development of geothermal energy, contributing significantly to sustainable development, ensuring stability in energy supply, and enhancing energy efficiency.
2 Fractal Modeling of Geothermal Extraction under Multi-Field Effect

2.1 Fractal Characterization of Rough Fracture Networks in Geothermal Reservoir Extraction

The number of fractures whose lengths in the interval \([l, l + dl]\) can be obtained by Eqs. (1) and (2):

\[-dN(l) = Nf(l) dl = N\delta f_{\text{min}}^{l^{\delta}} l^{-(\delta + 1)} dl\]  

In most cases, the aperture of natural fractures satisfies the following relationship:

\[a = \eta l^\omega\]  

where \(a\) is the fracture aperture, \(\eta\) is the proportional coefficient for shale fracture length, and \(\omega\) is the power-law index of fracture aperture.

Based on the classical cubic law, the fluid flow through a fracture is:

\[q = \frac{a^3 l \Delta p_{f}}{12 \mu L_{f}}\]  

where \(\mu\) is the dynamic viscosity of the fluid and \(L_{f}\) is a fracture length.

And the gas flow model considering the distribution of fracture spatial location is given by:

\[q = \frac{a^3 l \left(1 - \cos^2 \theta_1 \sin^2 \theta_2 \right) \Delta p}{12 \mu L_{f}}\]  

where \(\theta_1\) is the natural fracture azimuth angle and \(\theta_2\) is the dip angle.

where \(r = l_{\text{min}} / l_{\text{max}}\) is defined as the fracture length ratio.

Combining Eqs. (1) - (4), the effective permeability for the shale is:

\[k_{f} = \frac{N \cdot \varepsilon_{D} \eta}{12A \left(3D_{f} + 1 - D_{f} \right)} \left[r^{\delta - (\delta + 1)} - 1\right]\]  

2.2 Evolution of Reservoir Stress During Geothermal Extraction

Under the combined influence of stress deformation caused by extraction disturbances, the nonlinear evolution of hydraulic pressure due to extraction water seepage, and the temperature of the geothermal reservoir, the reservoir stress control equation can be expressed as [15]:

\[\sigma_y = 2G\varepsilon_y + \frac{2G\nu}{1-2\nu} \varepsilon_{y} \delta_{y} - K\alpha_{r} T_{y}\]  

where \(\sigma_y\) is the stress tensor influenced by both water seepage and extraction disturbances, \(G\) is the shear modulus of the geothermal reservoir, \(\nu\) is the rock's Poisson's ratio, \(\alpha_{r}\) is the rock's thermal expansion coefficient, and \(T_{y}\) is the temperature tensor of the geothermal reservoir.
For the extraction of geothermal resources during water seepage, the strain in the rock is the sum of strains caused by temperature, stress, and seepage pressure, namely:

\[
\varepsilon_{ij} = \frac{1}{2} \sigma_{ij} - \frac{1}{6G} \left( \frac{1}{9K} \right) \sigma_{ii} \delta_{ij} + \frac{\alpha \Delta T}{3} \delta_{ij} + \frac{\alpha}{3K} \Delta \rho \delta_{ij}
\]  

(7)

\[
Gu_{ij} + \frac{G}{1-2\nu} u_{ij,j} - ap_{ij} - K \alpha \tau_{ij} + F_{ij} = 0
\]  

(8)

2.3 Water Seepage through Rough Fractures During Extraction

During geothermal extraction, the evolution equation for porosity, influenced by extraction disturbances and stress changes, is as follows [16]:

\[
\phi = 1 - (1 - \phi) \exp \left[ \frac{1}{K \left( p - p_o \right) + \alpha \left( T - T_o \right) + \left( \varepsilon - \varepsilon_o \right)} \right]
\]  

(9)

\[
\frac{\partial \phi}{\partial t} = -\left(1 - \phi_0\right) \left( \frac{1}{\kappa} \frac{\partial p}{\partial t} + \alpha \left( T - T_o \right) \frac{\partial \varepsilon}{\partial t} \right) \exp \left( \frac{p - p_o}{\kappa} + \alpha \left( T - T_o \right) - \left( \varepsilon - \varepsilon_o \right) \right)
\]  

(10)

Therefore, by correlating the above equations with Equation (10), the governing equation for groundwater seepage can be established as:

\[
H \frac{\partial p}{\partial t} - \nabla \left( \frac{\kappa}{\rho} \nabla p \right) = H \alpha \frac{\partial T}{\partial t} - H \kappa \frac{\partial \varepsilon}{\partial t} + Q
\]  

(11)

2.4 Water Seepage through Rough Fractures During Extraction

Neglecting the thermal filtration effect, the total heat flux during geothermal extraction comprises two components: thermal conduction and fluid thermal convection, expressed as:

\[
\dot{q}_f = -\lambda_m \nabla T + \rho_w C_s \dot{q}_s \left( T + T_w \right)
\]  

(12)

where \( C_s \) is the thermal conduction constant of the rock layer and \( \lambda_m \) is the heat convection coefficient. The energy conservation equation for heat flow and the relationship with specific heat capacity is as follows:

\[
\frac{\partial \left( (\rho C)_m (T + T_w) \right)}{\partial t} + (T + T_w) K \varepsilon \nabla \dot{q}_s + (T + T_w) K \alpha \frac{\partial \varepsilon}{\partial t} = -\nabla \dot{q}_s
\]  

(13)

\[
(\rho C)_m = \phi (\rho_w C_s) + (1 - \phi) (\rho_s C_s)
\]  

(14)

where \( (\rho C)_m \) is the specific heat capacity of the geothermal reservoir, and \( C_s \) and \( C_r \) represent the thermal constants of water and rock during the seepage process, respectively.

3 Validation of the Proposed Fractal Model

To ascertain the accuracy of the fractal multi-field coupling model, this study utilized field geothermal extraction data from studies by Wang et al. and Koh et al. [17,18]. The simulation area, depicted in Figure 1, encompasses the main simulation parameters and extraction data of the geothermal reservoir as shown in Table 1.
As demonstrated in Figure 2(a), the calculated pore pressure, radial stress, and tangential stress show good agreement with experimental results, indicating model consistency with calculations based on steady-state heat transfer. Furthermore, Figure 2(b) compares the comprehensive simulation results of geothermal extraction under the influence of multifactorial coupling with actual field project data. This comparison substantiates the accuracy of this model.

**Table 1. Primary Data of Field Geothermal Extraction.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus of rock, E (GPa)</td>
<td>24</td>
</tr>
<tr>
<td>Poisson’s ratio of rock, ν</td>
<td>0.189</td>
</tr>
<tr>
<td>Heat conductivity, 𝜆 (W/(m⋅K))</td>
<td>6.0×10⁻⁶</td>
</tr>
<tr>
<td>Constant pressure heat capacity, C_p (J/(kg⋅K))</td>
<td>900</td>
</tr>
<tr>
<td>Density of the reservoir, ρ_r (kg/m³)</td>
<td>2700</td>
</tr>
<tr>
<td>Solid bulk modulus, V(GPa)</td>
<td>48.2</td>
</tr>
<tr>
<td>Biot’s coefficient, α</td>
<td>0.49</td>
</tr>
<tr>
<td>Hydrostatic fluid pressure, N(MPa)</td>
<td>10.0</td>
</tr>
<tr>
<td>Tension strength, R_m (MPa)</td>
<td>12.1</td>
</tr>
<tr>
<td>Density of fluids, ρ_f (kg/m³)</td>
<td>1000</td>
</tr>
<tr>
<td>Rock porosity, φ</td>
<td>0.143</td>
</tr>
<tr>
<td>Dynamic viscosity, 𝜇 (Pa·s)</td>
<td>3.0×10⁻⁴</td>
</tr>
<tr>
<td>Coefficient of compressibility, a (1/Pa)</td>
<td>5.31×10⁻⁹</td>
</tr>
</tbody>
</table>
4 Results and discussion

Building on the above results, this section explores the contributions of different degrees of fracture curvature and fracture roughness to geothermal reservoir stress and production well water temperature. Figure 3 shows the contribution of different fracture roughness parameters to geothermal reservoir stress. As Figure 3 indicates, reservoir stress consistently increases under varying levels of fracture roughness. Additionally, at the same extraction time, reservoir stress is inversely proportional to the fracture roughness parameter. A larger $\varepsilon$, representing rougher fracture surfaces, impedes the flow of injected cold water. Under multi-field coupling, more difficult fluid flow leads to reduced water pressure-induced effective stress, resulting in lower reservoir stress. When $\varepsilon$ increases from 0.06 to 0.10, the maximum reduction in reservoir stress is 5.39%.

Fig. 2. Model Accuracy Validation
Figure 4 illustrates the effect of different fracture roughness on production well water temperature. As seen in Figure 4, the production well water temperature consistently decreases over different extraction periods. At the same extraction time, a greater fracture roughness parameter \( \varepsilon \) results in higher water temperatures. As previously discussed, a larger \( \varepsilon \) denotes rougher geothermal reservoir fractures, making water seepage more challenging. Consequently, with slower water seepage, more heat is extracted, raising the production well water temperature as \( \varepsilon \) increases. When the fracture roughness parameter \( \varepsilon \) increases from 0.06 to 0.10, the production well water temperature at 80 days of extraction increases by 1.78° C.

Furthermore, the influence of fracture curvature on geothermal reservoir stress and production well water temperature is also investigated. As shown in Figures 5 and 6, in the fractal model proposed in this study, the degree of fracture curvature in the geothermal reservoir is quantitatively represented by the tortuosity fractal dimension \( D_T \). As stated in Section 2, a smaller \( D_T \) indicates a more complex fracture network. Figure 5 shows the impact of different fracture tortuosity fractal dimensions on reservoir stress. It is evident that at the
same extraction time, a larger fracture tortuosity fractal dimension leads to greater geothermal reservoir stress. A larger $D_T$ implies a simpler fracture structure, thereby facilitating water seepage. Therefore, more water is injected into the geothermal reservoir from the injection well over the same period, correlating $D_T$ positively with reservoir stress. When $D_T$ increases from 1.2 to 1.6, the maximum increase in geothermal reservoir stress is 10.12%.

The influence of fracture tortuosity fractal dimension on production well water temperature is depicted in Figure 6. As shown, at the same extraction time, the production well water temperature is inversely proportional to the fracture tortuosity fractal dimension $D_T$. As mentioned, a smaller $D_T$ represents a more complexly curved fracture structure. This complexity hinders water seepage and extraction, thus prolonging seepage time and increasing the amount of heat extracted. Consequently, the production well water temperature increases. When $D_T$ increases from 1.2 to 1.6, the production well water temperature at 80 days of extraction increases by 2.39°C.

![Fig. 5. The Impact of Fracture Tortuosity Fractal Dimension on Geothermal Reservoir Stress.](image)

![Fig.6. Impact of Fracture Tortuosity Fractal Dimension on Produced Well Water Temperature.](image)
5 Conclusion

This study aims to address the industry challenge of quantitatively and comprehensively evaluating the impact of nonlinear evolution in reservoir fracture behavior on the safety and extraction efficiency of geothermal projects. Employing fractal theory adapted for porous media, this study developed an interdisciplinary geothermal extraction fractal model. This model incorporates four innovative fractal parameters (fracture fractal dimension $D_S$, fracture tortuosity fractal dimension $D_T$, fracture roughness parameter $\varepsilon$, maximum fracture length $l_{max}$) to quantitatively characterize the fracture structure, applying the refined fractal theory to geothermal reservoir extraction under complex thermal-fluid-solid coupling. The key findings are as follows:

1. The fractal geothermal extraction model proposed in this study effectively characterizes the evolution of fracture network distribution, curvature, roughness, and length in geothermal projects. The model's computational results align closely with data from actual geothermal extraction engineering.

2. Under the influence of coupled multiphysical fields, more complex fracture structures can be quantitatively represented by smaller fracture tortuosity fractal dimension $D_T$; rougher fracture surfaces can be quantitatively characterized by larger fracture roughness parameters $\varepsilon$.

3. The fracture tortuosity fractal dimension and fracture roughness parameter significantly influence the reservoir stress and produced well water temperature during geothermal extraction. For instance, when $\varepsilon$ increases from 0.06 to 0.10, the maximum reservoir stress decreases by 5.39%, and the produced well water temperature after 80 days of extraction increases by 1.78° C; when $D_T$ increases from 1.2 to 1.6, the maximum increase in geothermal reservoir stress is 10.12%, and the produced well water temperature after 80 days increases by 2.39° C.

References


