Calculation of the producing surfaces of milling cutters for processing gears of eccentric-cycloidal engagement

Vadim Kuts\textsuperscript{1} and Julia Malneva\textsuperscript{1}

\textsuperscript{1}Southwest State University 50 let Oktyabrya Avenue, 94 305040 Kursk, Russia

Abstract. The paper considers a new method for processing curved gears of eccentric-cycloidal engagement with a curved profile using a special tool. The surfaces of wheels of eccentric-cycloidal engagement with a curved profile for small and large wheels are simulated. The scheme of forming a small wheel of eccentric-cycloidal engagement using a shaped milling cutter with a constructive radial feed is shown. The producing surface of the milling cutter is obtained, which allows to fulfill all the conditions of shaping when implementing the proposed scheme for shaping a small wheel of eccentric-cycloidal engagement with a curved profile. For a large wheel, a shaping scheme using the rolling method using a worm wheel is proposed. Modeling of the producing surface of a worm mill for shaping the surface of a large wheel of eccentric-cycloidal engagement using the rolling method is performed. This approach can be used in the design of milling cutters designed for processing curved gears of eccentric-cycloidal engagement of various sizes. The use of such tools will make it possible to make the transition to serial and mass production of gears with this engagement.

1 Introduction

Currently, the creation and application of new types of gears, as well as the design of new gears based on them, are widely discussed. A special example of such gears is an eccentric-cycloidal gearing with a curved profile [1] (Fig. 1).

In [1] and on the Internet [1], the main advantages of this type of engagement are shown in comparison with traditional ones (involute engagement, Novikov engagement, etc.) and despite this, eccentric-cycloidal gearing has not been widely used. One of the reasons for this is the lack of technologies (machines and tools) that ensure the mass production of gears with a curved profile. The application of schemes for shaping the surfaces of these gears based on multi-coordinate spatial movements, using CNC machines and a universal tool [1] working by the touch method or a specialized tool [1] working by the method of copying the tooth profile, does not allow achieving high performance indicators and can be effective. It is used only in conditions of single and small-scale production. For the

\* Corresponding author: kuc-vadim@yandex.ru

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).
transition to serial and mass production, it is necessary to develop new high-performance shaping schemes and on their basis create specialized mills and tools.

![Image of eccentric-cycloidal engagement with a curved profile](image)

**Fig. 1** of eccentric-cycloidal engagement with a curved profile [1]

This work is devoted to the modeling of the producing surfaces of specialized tools (cutters) using the example of several high-performance schemes for forming wheels of eccentric-cycloidal gear with a curved profile (Fig. 2), in particular for a small wheel of eccentric-cycloidal transmission (see Fig. 1) consider the shaping scheme with the use of shaped milling cutters with a constructive radial feed, and for a large wheel, a processing scheme according to the method of rolling with the use of a worm saw.

### 2 Modelling of surfaces of wheels of eccentric-cycloidal engagement with a curved profile

In [1], formulas were given for modeling the surfaces of gears of eccentric-cycloidal engagement with a curved profile, Fig. 2 shows the results of modeling these surfaces.

![Image of modeled surfaces](image)

**Fig. 2.** Modeling of surfaces of wheels of eccentric-cycloidal engagement with a curved profile according to [1]: a) small wheel; b) large wheel; c) in engagement
Figure 3 shows a diagram of the shaping of a small eccentric-cycloidal engagement wheel using shaped milling cutters with a constructive feed.

![Diagram of shaping process](image)

**Fig. 3.** The scheme of forming a small wheel of eccentric-cycloidal engagement with the use of a shaped milling cutter with a constructive radial feed

### 3 Modelling of the milling cutter's producing surface

In accordance with this scheme, the axes of the tool and the cut wheel are parallel, the rotation frequency of the tool is equal to the rotation frequency of the processed wheel and the shaping of the wheel surface will be performed in one turn of the cutter. We will determine the producing surface of the instrument based on the construction of a family of envelope surfaces, for which we will compile the basic equation of shaping [1-6] a small wheel of eccentric-cycloidal engagement with a curved profile in accordance with the scheme (see Fig. 3)

\[
\bar{r}_e(v, \alpha) = A_e \cdot \bar{r}_c(v, \alpha), \tag{1}
\]

where \( \bar{r}_e(v, \alpha) \) is the vector equation on top of the processed small wheel of eccentric-cycloidal engagement [1];
- \( v \) and \( \alpha \) are the parameters of the wheel surface;
- \( A_e \) - matrix of the forming system;
- \( \bar{r}_c(v, \alpha) \) - the equation of the producing surface of the milling cutter.

The matrix of the shaping system in accordance with the shaping scheme (see Fig. 3) it can be calculated as

\[
A_e = A^{(6)}(\theta_1) \cdot A^{(6)}(H) \cdot A^{(6)}(-\theta), \tag{2}
\]

where \( A^{(6)}(\theta_1) \) - the matrix of rotation of the workpiece around the 0Z axis by an angle of \( \theta_1 \);
\begin{align}
A^{(6)}(\theta) &= \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & 0 & 0 \\
\sin(\theta) & \cos(\theta) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (3)
\end{align}

\begin{align}
A^{(3)}(H) - & \text{ matrix for setting the axial distance between the axes of rotation of the workpiece and the cutter by the value } H \\
A^{(3)}(H) &= \begin{bmatrix}
1 & 0 & 0 & H \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} ; \quad (4)
\end{align}

\begin{align}
A^{(6)}(-\theta) - & \text{ the matrix of rotation of the milling cutter around the axis by an angle } \theta \\
A^{(6)}(-\theta) &= \begin{bmatrix}
\cos(\theta) & \sin(\theta) & 0 & 0 \\
-\sin(\theta) & \cos(\theta) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} . \quad (5)
\end{align}

The equation of the milling cutter producing the surface is obtained from equation (1)
\begin{equation}
\bar{r}_f(v,\alpha) = A^{(6)*} \cdot \bar{r}_f(v,\alpha) . \quad (6)
\end{equation}

The equation (6) thus obtained will contain four parameters \(v,\alpha,\theta,\theta\) which is redundant (only two parameters are required), and requires the imposition of connections on any two of the four parameters. Therefore, we will impose a kinematic connection on the workpiece rotation parameter in accordance with the proposed shaping scheme 
\begin{equation}
\theta = \theta . \quad (7)
\end{equation}

The number of parameters will be reduced to three \(v,\alpha,\theta\) and equation (6) at this stage can be considered as a vector function of three arguments \(\bar{r}_f(v,\alpha,\theta)\). On the parameter \(\theta\) let's impose an envelope connection by solving with respect to the parameter \(\theta\) an equation of the form
\begin{equation}
\frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial v} \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \alpha} \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \theta} = 0 \ . \quad (8)
\end{equation}

or in matrix form
\begin{equation}
\begin{bmatrix}
\bar{r}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial v} & \bar{r}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \alpha} & \bar{r}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \theta} \\
\bar{j}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial v} & \bar{j}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \alpha} & \bar{j}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \theta} \\
\bar{k}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial v} & \bar{k}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \alpha} & \bar{k}_f \cdot \frac{\partial \bar{r}_f(v,\alpha,\theta)}{\partial \theta} \\
\end{bmatrix} = 0 . \quad (9)
\end{equation}

Solving equation (8) or (9), it was found that
\begin{equation}
\theta = \alpha \quad (10)
\end{equation}
and after replacing $\theta$, the equation of the producing surface of a milling cutter with a structural radial dacha is obtained

$$\vec{r}_f(v, \alpha) = \begin{bmatrix}
\varepsilon \cdot \cos v - \left( H - \frac{d}{2} \right) \cos \alpha \\
\varepsilon \cdot \sin v - \left( H - \frac{d}{2} \right) \sin \alpha \\
l \cdot v \\\frac{2\pi}{1}
\end{bmatrix}, \quad (11)$$

where $\varepsilon$ is the eccentricity of the small wheel being processed; $d$ is the diameter of the circumference of the tooth section; $l$ is the width (length) of the wheel.

Figure 4 shows the result of modeling the milling cutter's producing surface according to (11) for a small eccentric-cycloidal engagement wheel with a curved profile at the specified parameters [1] $d=20$ mm, $\varepsilon=5$ mm, $l=50$ mm and $H=200$ mm.

Fig. 4. Modeling of the milling cutter's producing surface: a) the initial view of the producing surface; b) in contact with a small wheel, at $\theta=0$; c) in contact with a small wheel, at $\theta=\pi$.

Thus, the resulting producing surface of the milling cutter (11) allows you to fulfill all the shaping conditions when implementing the proposed shaping scheme. For a large wheel, consider the shaping scheme using the rolling method with the use of a worm wheel (Fig. 5). 

---

**References:**

[1] E3S Web of Conferences 531, 01004 (2024)
In accordance with this scheme, the producing surface of the milling cutter is set at a distance $H$ from the axis of the large wheel, and the axis of the milling cutter is rotated by an angle $\beta$ advising the angle of elevation of the curved tooth surface line. The rotation speed of the big wheel will be in $Z+1$ times less than the rotation speed of the milling cutter, where $Z$ is the number of teeth of the large wheel and the milling cutter performs translational movement along the axis of the wheel.

Then, the shaping matrix included in the basic shaping equation (1) for this scheme is written as

$$A_x = A_x^{(6)}(\theta_1) \cdot A_x^{(3)}(z) \cdot A_x^{(3)}(H) \cdot A_x^{(4)}(\beta) \cdot A_x^{(6)}(\theta) ,$$

(12)

where $A_x^{(3)}(z)$ is the matrix of translational movement of the milling cutter along the axis of the large wheel by the value $z$;

$A_x^{(4)}(\beta)$ - the rotation matrix of the milling cutter axis is returned to the angle $\beta$ corresponding to the angle of elevation of the curved surface of the teeth.

The equation of the milling cutter producing the surface is represented similarly (6) in the form

$$\vec{r}_x(v,\alpha) = A_x^{-1} \cdot \vec{r}_x(v,\alpha) ,$$

(13)

Where $\vec{r}_x(v,\alpha)$ is the vector equation on top of the processed large wheel of eccentric-cycloidal engagement [10, 11].

The angle $\beta$, which advises the angle of elevation of the curved surface of the teeth, is defined as

$$\beta = \arccos \left[ \begin{bmatrix} \frac{\partial \vec{r}_x(v,\alpha)}{\partial v} & \frac{\partial \vec{r}_x(v,\alpha)}{\partial v} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial \vec{r}_x(v,\alpha)}{\partial v} & \frac{\partial \vec{r}_x(v,\alpha)}{\partial v} \end{bmatrix}^{-1} \right]_{v=0,\alpha=0} ,$$

(14)
where $\vec{r}^{3Y}(v,\alpha)$ is the projection $\vec{r}_u(v,\alpha)$ of the vector function onto the coordinate plane $0X_uY_u$.

$$
\vec{r}^{3Y}(v,\alpha) = \begin{bmatrix}
\vec{r}_u \cdot \vec{r}(v,\alpha) \\
\vec{r}_v \cdot \vec{r}(v,\alpha) \\
0 \\
1
\end{bmatrix}.
$$

(15)

To obtain the equation of the generating surface, we define the following relations:

$$
z = 0; \quad \theta_i = -\frac{\theta}{Z+1}. \quad (16)
$$

The envelope relation, in relation to this scheme, is superimposed on parameter $v$, for which we make up an equation similar to equation (8) or (9) and solve it with respect to parameter $v$. Taking into account the established envelope relationship, the equation of the milling cutter's producing surface can be generally represented as

$$
\vec{r}_j(\alpha,\theta) = \vec{r}_j(v = f(\alpha,\theta),\alpha,\theta). \quad (18)
$$

Figure 6 shows an example of changing the connection $v = f(\alpha,\theta)$ for a large wheel with parameters $d=20$ mm, $e=5$ mm, $l=50$ mm, $a=200$ mm, $Z=15$ and $H=200$ mm.

![Fig. 6. An example of the implementation of the envelope connection $v = f(\alpha,\theta)$: 1) by $\theta=0$; 2) by $\theta=\pi/2$; 3) by $\theta=\pi$](image)

And Figure 7 shows a model of a section of the producing surface at specified parameter intervals $\alpha=[-2\pi; 2\pi]$ and $\theta=[0; 2\pi]$. 

7
Fig. 7. Modeling of the producing surface of a worm mill for shaping the surface of a large wheel of eccentric-cycloidal engagement using the rolling method: a) the initial surface; b) in contact with the surface of the large wheel.

This producing surface of the milling cutter also allows you to fulfill all the shaping conditions when implementing the proposed shaping scheme [1-14].

4 Conclusion

Thus, in accordance with the proposed schemes for shaping small and large curved gears of eccentric cyclic transmission, methods for calculating the points of producing cutters were developed: shaped milling cutters with constructive radial feed and worm milling cutters working by the rolling method. This approach can be used in the design of milling cutters designed for processing curved gears of eccentric-cycloidal engagement of various sizes. The use of such tools will make it possible to make the transition to serial and mass production of gears with this gear.

Acknowledgments

The work was carried out within the framework of the implementation of the development program of the Southwestern State University of the Priority 2030 project.

References