Forced oscillation of planetary mechanism synchronizer for spindle drive

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Abstract. The article is devoted to the results of a theoretical and experimental study of forced oscillation of the epicyclic-hypocyclic planetary mechanism synchronizer used to drive the spindles of the cotton machine apparatus. Spectrum of natural oscillation frequency of synchronizer is determined and mass moment of their inertia is set for stability in operating mode when satellite comes in and out of gear engagement of planetary mechanism. Periodic nature of system oscillations under action of external forces of satellite is considered. The graph of oscillations of actual excitation of the synchronizer in the form of a pulse semi-sinusoidal curve is obtained. The beam diagram is built that allows you to make delaminations from the resonant modes of operation of the synchronizer.

1 Introduction

Planetary mechanisms use tooth gear [1, 2] lantern gear [3], friction wheels [4] and belt gears [5, 6], which are used in the drive of the spindles of the cotton machine apparatus. Also, planetary mechanisms having a compact design have been widely used in the national economy: in the solution mixing mechanisms [7, 8] of the food industry, especially in the design of various mixers [9-12], for treatment facilities and reservoirs [13, 14] in the ventilation of waste water treatment [15]. In operation [16, 17] kinematic scheme of epicyclic planetary mechanism is given for their use in internal combustion engine with creation of equation of kinematic motion of satellites. At work [18], a kinematic scheme of an epicyclic-hypocyclic mechanism for driving spindles is given, in which gears are designed in the form of two toothed segments, one is designed for epicyclic motion of satellites, the other segment is made with hypocyclic toothed engagement, therefore, at one rotation of the carrier, the satellites make two different rotational movements – epicyclic and hypocyclic. Brake devices and synchronizers for smooth engagement of teeth are installed in transition zones.

2 Methods

The article proposes theoretical developments of a planetary mechanism for driving spindles, which were carried out according to the methodology of the basic positions of the theory of oscillation and stability, as well as theoretical mechanics and technological...
processes of the cotton harvester. Lagrange equation is used in oscillatory motion of synchronizer and results of machine analysis of graphs according to specially set experimental graphs. The processing of the results was carried out using mathematical statistics and computer graphics.

In the operating mode of direct rotation, under the action of periodically changing forces from the side of the rotating carrier, when the satellite enters and leaves the engagement, the synchronizer performs a forced angular oscillation about the rotation axis $D$ (Fig. 1).

The use of well-known theoretical methods of calculation with the experimental studies performed on a bench installation made it possible to establish the most successful design forms of the synchronizer of the planetary mechanism. This synchronizer ensures the normal operation of the mechanism in the absence of resonant modes.

Fig. 1. Kinematic diagram of synchronizer operation.

The aim of the study is to theoretically determine the frequencies of natural oscillations of the synchronizer, the analysis of forced oscillations and their stability in the operating mode. The synchronizer of the planetary mechanism (Fig. 1), when disturbed by the toothgear, moves through the angle $\varphi$.

For the generalized coordinate we take the angle $\varphi$.

The considered mechanical system has one degree of freedom. The kinetic energy $E$ of the synchronizer rotating around its support axis $D$ is equal to

$$E = \frac{1}{2} J_D \cdot \dot{\varphi}^2$$

(1)

where $J_D$ – mass moment of inertia of synchronizer relative to rotation axis $D$, kgm$^2$; $J_S$ – mass moment of inertia of the synchronizer relative to the center of gravity kg$\cdot$m$^2$; $m$ is the mass of the synchronizer, kg; $G$ – gravity link at the center of the synchronizer, m; $H$ –.

$$m = \frac{G}{g}$$
\( g \) – acceleration of gravity, \( 9.81 \, \text{m/s}^2 \);
\( l_1 \) – distance from support point to synchronizer center of gravity, m.

When the synchronizer is rotated by an angle \( \varphi \), the spring is shortened by \( S \), then

\[
\Delta S = l_2 \cdot d\varphi
\]

In this case, the potential energy of the system will be

\[
\Pi = C \cdot \Delta S^2 = C \cdot (l_2 \cdot d\varphi)^2
\]

where

\( l_2 \) – distance from the center of the spring axis to the hinge \( D \) of the synchronizer support axis, m;
\( C \) – coefficient of the spring constant.

Elementary operation of perturbing force

\[
\delta A \varphi = M_{\nu r} \cdot \delta \varphi
\]

Where

\( M_{\nu r} \) – torque of the carrier, \( H \cdot m \);
\( M_{\nu r} = F_{\nu r} \cdot l \)

\( F_{\nu r} \) – radial force acting on the first tooth of the synchronizer segment, \( H \);
\( l \) – distance from the thrust axis \( D \) to the top of the first synchronizer tooth, m. The force acting on

\( F_{\nu r} \) synchronizer tooth is found according to experimental data. Considering that the periodic nature of the oscillations of the system under the influence of the external forces of the satellite gear, during the rotational movement of the carrier of the planetary mechanism, occurs through the synchronizer, then it is the impulse character of the semi-sinusoidal curve and is a perturbing force \( F_{\nu r} \). And this perturbing force can be decomposed into a Fourier series, then

\[
F = F_A \left[ \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi} \left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} \right) \right]
\]

Where

\( F_A \) – axial force acting in gear engagement of cylindrical gears, \( H \);
\( \omega \) – satellite angular velocity, \( 130.08 \, \text{radian/seconds} \);
\( F = 0.048 \, \text{s} \)

Fig. 2. Sattelite Tooth Hit Curve Graph by synchronizer tooth.
Knowing the time of one circulation of the carrier of the planetary mechanism \( t = \frac{60}{n}, s \) we determine one cycle of the period \( F \) of the satellite for the carrier having the number of satellites \( z = 12 \), and the carrier rotation speed \( n = 102.5 \) cir/min, then we find the cycle time \( F = \frac{60}{z \cdot n} \).

Axial reinforcement \( F_A \) engageable cylindrical gears \( F_A = F_t \cdot \tan \alpha_W \) where \( \alpha_W \) – the gearbox engagement angle \( \alpha_W = 20^\circ \), \( F_t = 2M/d_\delta \).

\[
F_A = 2M/d_\delta \cdot \tan \alpha_W = 2Mt\alpha_W/d_\delta
\]

\[
J_D \cdot \ddot{\varphi} + cl^2 \cdot \varphi = M_v t
\]

\[
J_D \cdot \ddot{\varphi} + cl^2 \cdot \varphi = F_t \cdot l
\]

\[
J_D \cdot \ddot{\varphi} + cl^2 = F_A \cdot \left[ \frac{1}{4} + \frac{1}{2} \sin \omega t - \frac{2}{\pi} \left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{4} + \frac{\cos 6\omega t}{35} \right) \right]
\]

\[
\varphi + \frac{cl^2}{J_D} \varphi = F_A l \left[ \frac{1}{4} + \frac{1}{2} \sin \omega t - \frac{2}{\pi} \left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} \right) \right]
\]

\[
\frac{1}{J_D} \frac{d^2 \varphi}{dt^2} + \frac{cl^2}{J_D} \varphi = 0
\]

\[
\frac{d^2 \varphi}{dt^2} + \frac{cl^2}{J_D} \varphi = 0
\]

\[
r^2 + \frac{cl^2}{J_D} = 0
\]

\[
\varphi = \pm P_i
\]

\[
P_i = \frac{c}{\sqrt{J_D}} \quad \varphi_1 = \cos pt \quad \varphi_2 = \sin pt
\]

\[
\varphi = C_1(t) \cos pt + C_2(t) \sin pt
\]
\[ (t)\cos pt + C_2(t)\sin pt = 0 \]

\[ -pC_1'(t)\cdot \sin pt + pC_2'(t)\cdot \cos pt = \]

\[ = \frac{F_A\cdot l}{J_D}\left[ \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi}\left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} \right) \right] \]

\[ C_1(t) = -C_2(t)\cdot \tan pt \]

\[ C_2'(t)\cdot \sin pt \cdot \tan pt + C_2(t)\cos pt = \]

\[ = \frac{pA\cdot l}{J_D\cdot p}\left[ \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi}\left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} \right) \right] \]

\[ C_2(t) = \frac{\cos pt}{\sin^2 pt + \cos^2 pt} \cdot \]

\[ C_1'(t) = \frac{F_A\cdot l\cdot \sin pt}{J_D\cdot p}\left[ \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi}\left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} \right) \right] \]

\[ C_1(t) = \frac{F_A\cdot l}{J_D\cdot p}\int \sin pt\left[ \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi}\left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} \right) \right] dt = \]

\[ = \frac{F_A\cdot l}{J_D\cdot p}\left\{ \frac{1}{\pi\cdot p}\cos pt - \frac{1}{4}\left[ \frac{1}{p - \omega}\cdot \sin(p - \omega) t - \frac{1}{p + \omega}\sin(p + \omega) t \right] \right. \]

\[ - \frac{1}{3\pi}\left[ \frac{1}{p + 2\omega}\cos(p + 2\omega) t + \frac{\cos}{p + \cos 2\omega}(p - 2\omega) t \right] \]

\[ - \frac{1}{15\pi}\left[ \frac{1}{p + 4\omega}\cos(p - 4\omega) t \right] \]

\[ - \frac{1}{35}\left[ \frac{1}{p - 6\omega}\cos(p - 6\omega) t + \frac{1}{p - 6\omega}\cos(p - 6\omega) t \right] \left\} + k_1 \]

\[ C_2(t) = \frac{F_A\cdot l}{J_D\cdot p}\int \cos pt\left[ \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi}\left( \frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \frac{1}{35}\cos 6\omega t \right) \right] dt = \]
\[
\frac{1}{35\pi} \cos 6\omega t \, dt
= F_A \cdot l \left[ \frac{1}{\pi p} \sin pt + \frac{1}{4} \left[ - \frac{1}{p + \omega} \cos(p + \omega) t + \frac{1}{p - \omega} \cos(p - \omega) t \right] \right. \\
- \frac{1}{3\pi} \left[ \frac{1}{p - 2\omega} \sin(p - 2\omega) t + \frac{1}{p + 2\omega} \sin(p + 2\omega) t \right] \\
+ \frac{1}{15} \left[ \frac{1}{p - 4\omega} \sin(p - 4\omega) t + \frac{1}{p + 4\omega} \sin(p + 4\omega) t \right] \\
+ \frac{1}{35\pi} \left[ \frac{1}{p - 6\omega} \sin(p - 6\omega) t + \frac{1}{p - 6\omega} \sin(p - 6\omega) t \right] \right] + k_2
\]
\[ \begin{align*}
&F_A l \left( \frac{1}{\pi} - \frac{2p}{\pi} \left[ \frac{1}{3(p^2 - 4\omega^2)} + \frac{1}{15(p^2 - 16\omega^2)} + \frac{1}{35(p^2 - 36\omega^2)} \right] + k_1 \right) = 0 \\
&F_A l \left( \frac{1}{2(p^2 - \omega^2)} \right) + \omega_2 p = 0
\end{align*} \]

\[ k_1 = \frac{F_A l}{J_D p} \left( \frac{1}{\pi} - \frac{2p}{\pi} \left[ \frac{1}{3(p^2 - 4\omega^2)} + \frac{1}{15(p^2 - 16\omega^2)} + \frac{1}{35(p^2 - 36\omega^2)} \right] \right) \]

\[ k_2 = \frac{F_A l \omega}{2J_D p(p^2 - \omega^2)} \]

\[ \varphi = \frac{F_A l}{J_D p} \left( \frac{1 - \cos \omega t}{\pi p} + \frac{\sin \omega t}{2(p^2 - \omega^2)} \right) - \frac{2p}{\pi} \left[ \frac{\cos 2\omega t - \cos \omega t}{3(p^2 - 4\omega^2)} + \frac{\cos 4\omega t - \cos \omega t}{15(p^2 - 16\omega^2)} + \frac{\cos 6\omega t - \cos \omega t}{35(p^2 - 36\omega^2)} \right] - \frac{\omega_2 \sin \omega t}{2(p^2 - \omega^2)} \]

\[ T = \frac{2\pi}{l} \sqrt{\frac{J_D}{c}} \]

The period of forced oscillations of the synchronizer system is

\[ T = \frac{2\pi}{l} \sqrt{\frac{J_D}{c}} \]

get $\omega_{dis} = 20.5$ Hz, and at $z = 15$, $\omega_{dis} = 25.6$ Hz. Considering that the synchronizer is affected by one factor from the side of the pinion gear and when the teeth engage with the synchronizer, the multiplicity of changes in frequencies, forced vibrations will be equal to $K = 1$, then $\omega_{dis.1} = 20.5$ Hz $\omega_{dis.2} = 25.6$ Hz.

To define the adjustment coefficient $Z_H = \omega_{dis}/P$ find numerical values of natural frequency $P$ of synchronizer. To do this, experimentally setting the long arc of the synchronizer $l = 0.054$ m and its weight $G = 0.19$ kg, determine the moment of inertia of the synchronizer $I_D = 0.0000311$ N\(\cdot\)m\(\cdot\)s\(^2\) and spring constant $C = 0.078$ N/m. By substituting these values, we get $P_1 = 2.7$ Hz — for a 12 satellite carrier and $P_2 = 3.25$ Hz — for a 15 satellite carrier. By matching these values, we see that $\omega_{dis} \neq P$, at the same time $\omega_{dis} > P$, then $Z_{H_1} = \omega_{dis}/P_1 = 7.58$ and $Z_{H_2} = 7.88$. Thus, the mechanism relates to a system of forced high frequency oscillations. Find the dynamic coefficient of the mechanism by the formula:

\[ \mu = \left| \frac{1}{1 - \frac{\omega_{dis}}{P}} \right| \]

That is, from the graph we see that the amplitude of the resonant oscillations is very small and is located on the right side of the graph within $\omega_{dis} > P_1 = 7.58$. If the tuning coefficient value $z = 7.58$, then $\mu = 0.017$. 
That is, from the graph we see that the amplitude of the resonant oscillations is very small and is located on the right side of the After planetary determination of the spectrum of natural frequencies of the system according to the method of E.L. Ayrapeto-M.D. Genkin for gears [19-33], the so-called "beam diagram" is built, which allows us to conclude that it is possible to detach from resonant modes. Along the abscissa axis of the beam diagram, the rotation speed of the carrier \( n \) is deposited, and along the ordinate the frequency of the disturbing factor \( \omega_{(\text{dis.1})} \) and \( \omega_{(\text{dis.2})} \). Further we apply the operating speed of the satellite carrier \( n = 100 \ldots 150 \) cir/min, the forbidden zones of the natural frequencies of the synchronizer \( P_1, P_2, P_3, P_4 \) on the beam chart in the form of straight lines parallel to abscissa axis. The intersection points of the lines, for example, \( P_1 \) and \( P_2 \) determine resonant speed modes of rotation of the 12 satellite carrier. Since the operating speed of the satellite carrier \( n = 100 \ldots 150 \) cir/min, the forbidden zones of the natural frequencies of the synchronizer \( P_1 \) and \( P_2 \), which fluctuate in the interval \( P = 20 \ldots 30 \) Hz. Therefore, the natural frequency of the synchronizer must be lower than \( P_2 \). For the 15 satellite carrier, the forbidden zones of natural frequencies are synchronizers \( P_3 \) and \( P_4 \), which fluctuate in the interval \( P = 25,6 \ldots 38 \) Hz. From this beam diagram you can select different values of moments of inertia \( I_D \) synchronizer and spring constant \( C \). By setting the condition \( \omega_{\text{dis}} \geq P \), we get \( 20,5 \leq l \sqrt{C/I_D} \) or \( l \leq 20,5 \sqrt{I_D/C} \). Selecting different values of moments of inertia \( I_D \) and spring constant \( C \) determined optimal length of arc \( l \) of synchronizer. At the selected values of \( I_D \) and \( l \), spring constant \( C \) was calculated: \( C \leq I_D \).

### 3 Conclusions

Forced oscillations of synchronizer are investigated taking into account external forces of satellite of epi-hypocyclic planetary mechanism when entering gear engagement of internal segment wheel. The graph of oscillations of actual excitation of the synchronizer when entering the crown wheel gear is obtained. Resonant high-speed modes of the carrier of the planetary mechanism were determined and a forbidden zone of natural frequencies of the synchronizer was established, \( P_1 = 20,5 \) Hz, \( P_2 = 25,6 \) Hz, so for a 12-satellite carrier, the natural frequency \( P_1 \) should be lower than \( 20,5 \) Hz, and for a 15-satellite carrier lower than \( 25,6 \) Hz.

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