Method of calculation of satellite flow parameters behind the point of Mach reflection

Anna A. Yatsenko[1], *Mikhail V.* Chernyshov[1], and *Karina E.* Savelova[1*]

[1] Baltic State Technical University «VOENMEH», 190005 Saint-Petersburg, Russian Federation

Abstract. A mathematical model for the calculation and express analysis of triple configurations of air shock waves arising from an "elevated" explosion of a condensed high-energy material is presented. The considered blast waves with a long duration of the positive phase (compression phase) can take place in industrial and agricultural applications: in the explosion of fuel-air mixtures, explosive dust in elevators and granaries, as well as high-energy materials for agricultural and industrial applications based on ammonium nitrate and explosive saltpetre mixtures with petroleum products. The developed mathematical model uses exact and semi-empirical relationships developed earlier in the theory of optimal shock wave systems.

1 Introduction

During the detonation of a charge of a condensed high explosive (HE) or a high-energy material (HEM) elevated above the solid surface, a branched structure of explosive shock waves is formed. It is so-called triple configuration, which consists of three shocks with a common (triple) point and a contact discontinuity [1-4]. Despite the equality of static pressures of co-current flows separated by a contact discontinuity behind triple configurations, their velocities, dynamic pressures (velocity heads), temperatures [3–6], and many other parameters are not equal. Due to significant differences in the velocities and dynamic pressures of these co-current flows, the wind loads on humans, animals, and structures exposed to the blast wave [3–8] above and below the triple point T of the Mach reflection (Fig. 1a) also differ. The degree of damage to an object caused by wind loads (for example, due to the transfer of a human body in a wake behind the wave) becomes dependent on its initial height and the height of the explosion epicenter.

In this study, we obtained a mathematical model for calculation and express analysis of triple configurations of shock waves that arise during an explosion, the epicenter of which is located at a certain finite height H above the solid surface. This model is suitable for calculating and comparing the velocities and dynamic pressures of co-current gas flows behind different sequences of shock waves. These significantly different parameters, in turn, characterize the so-called tertiary impact of the blast wave (due to the translational movement of the body with subsequent impact).

* Corresponding author: karinkamurz@yandex.ru

Explosions under consideration with a long duration of the positive (compression) phase of the blast wave can take place in industry and agriculture, in particular, during the explosion of fuel-air mixtures [9], explosive dust in elevators and granaries [10], as well as HEMs for agricultural and industrial purposes based on such well-known fertilizer as ammonium nitrate and explosive mixtures of saltpeter with petroleum products, for example, with diesel fuel [8].

Despite the same pressure behind the main (Mach) blast wave and behind the system of the incident wave and the reflected one, the dynamic pressures of flows separated by a contact discontinuity τ (Fig. 1,a), as it was shown in [3, 4], are significantly different. It leads to different degrees of damage to people, technical objects, and field and forest fauna. The dependence of the degree of blast damage to small and large animals on the parameters of the shock wave was studied in laboratory conditions, in particular, in [1, 11, 12].

2 Propagation of shock waves at the elevated blast

Shock wave s_1 which appears at the elevated blast of the charge O of some condensed HE or HEM situated at the height H over ground (Figure 1,a), reaches the surface at the point A of its normal reflection. After it, the wave s_1 distributes along the surface, accompanied by regular (for example, in point B) reflection of the wave s_2. At the further removal of blast wave front s_1 from blast epicenter (in point C) its regular reflection transforms to irregular (Mach) one: new shock wave (main one, Mach one) s_3 appears. At Mach reflection, shock waves s_1, s_2 and s_3 always have the common (triple) point T; so they form the triple-shock configuration at this point. Further propagation of incident blast wave front s_1 leads to elevation of the triple point (its trajectory t is shown in Figure 1,a), to increase of size of Mach wave s_3, and to gradual confluence of the incident wave (s_1) and the reflected one (s_2). Shock-wave structure of the elevated blast is discussed in details in [13-15].

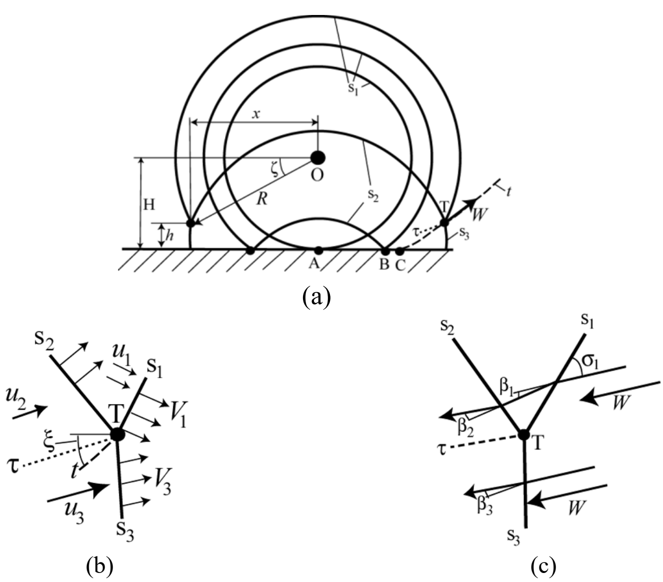


Fig. 1. Shock-wave structure that forms at the elevated blast (a) and flow reversal scheme at the transition from the unsteady triple configuration (b) to the steady one (c)

Gas streams behind the triple configuration, which are divided by contact surface τ, have the same static pressures, as well as equal normal (with respect to the slipstream τ), velocity components. But their full velocities (u_2 and u_3, Figure 1,b), dynamic pressures and many other parameters are not the same. This sometimes leads [3-5] to a large difference in wind (aerodynamic) loads on an object located at the same distance R from the explosion epicenter, but at different heights h, since the aerodynamic drag force acting on the object is proportional to the square of flow velocity. Thus, the damaging translational effect of co-current flows behind blast waves (due to the transfer and impact of the body as a whole) depends on the ratio of the heights of the explosive charge (*H*) and the damaged object (*h*).

Despite the equality of overpressures and almost the same specific pressure impulses behind reflected wave and Mach one, a significant difference in the velocities of the co-current flows behind them leads to a dependence of the damaging effect on the height of the object. As it was shown in [15], the shock-wave structure that occurs at explosion in a half-open (with an open top) explosion-proof urn is similar to a near-surface explosion of some (slightly smaller) charge without such a protective structure. Thus, the shock-wave structure of an elevated explosion, including the triple configuration of air shock waves, seems to be a topical subject of research.

3 Mathematical model of the triple configuration of steady shocks

Model of the triple configuration of the propagating shocks is based on well-known relations for the analogous steady structure in supersonic flow (Figure 1,c). That steady configuration consists of three steady shocks (s_1, s_2 and s_3) in supersonic flow with Mach number $M>1$, as well as the tangential discontinuity (the slipstream) τ emanating from their common (triple) point T [3-5]. Parameters of the steady shocks $s_{1\ldots3}$ are mutually dependent due to conditions of equalities of static pressures and co-directionality of flow velocity vectors across the slipstream τ:

$$J_1 J_2 = J_3, \qquad (1)$$

$$\beta_1 + \beta_2 = \beta_3. \qquad (2)$$

Here $J_i = p_i / p_j$ is relation of static pressures after the shock (p_i) and before it (p_j); $i=1\ldots3$, and β_i is flow deflection angle on shock surface which depends on shock strength (J_i) and flow Mach number (M_j) before it:

$$|\beta_i| = \operatorname{arctg}\left[\sqrt{\frac{(1+\varepsilon)M_j^2-\varepsilon-J_i}{J_i+\varepsilon}}\,\frac{(1-\varepsilon)(J_i-1)}{(1+\varepsilon)M_j^2-(1-\varepsilon)(J_i-1)}\right] \qquad (3)$$

Shock strength also determines Mach number M_i of gas flow behind the shock:

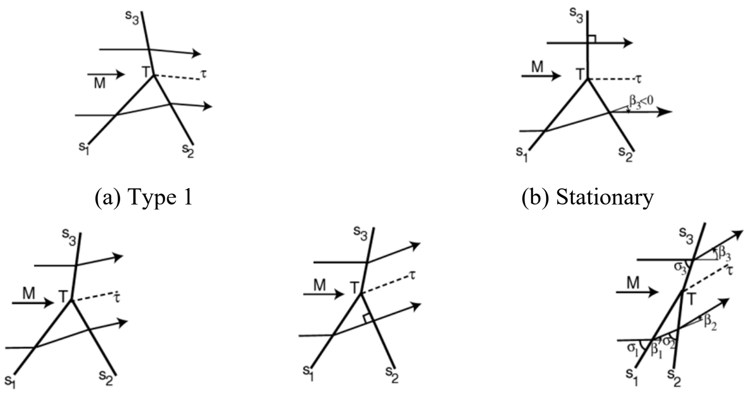
$$M_i = \sqrt{\frac{(J_i+\varepsilon)M_j^2-(1-\varepsilon)(J_i^2-1)}{J_i(1+\varepsilon J_i)}}\,. \qquad (4)$$

Here $\varepsilon = (\gamma-1)/(\gamma+1)$, and γ is the ratio of gas specific heats (in this study, we consider the air blast waves, so $\gamma = 1,4$).

Relations (1-4), Hugoniot shock adiabat and gas equation of state determine all other flow parameters after the shock in triple configuration. In particular, we can calculate the velocities u_2 and u_3 of co-current gas streams at both sides of the slipstream from the following relations:

$$u_2 = a_0 M_2 \sqrt{E_1 E_2 J_3}\ ,\quad u_3 = a_0 M_3 \sqrt{E_3 J_3}\ , \tag{5}$$

here $E_i = (1+\varepsilon J_i)/(J_i+\varepsilon)$, and a_0 is sound velocity in an undisturbed flow before the triple configuration.



(a) Type 1 (b) Stationary

(c) Type 2 (d) Transitional (e) Type 3

Fig. 2. Classification of triple-shock configurations (a) Type 1, (b) Stationary Mach one, (c) Type 2, (d) Transitional TC-2-3, (e) Type 3

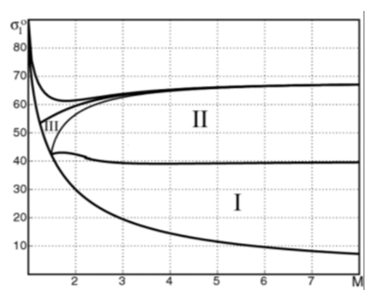


Fig. 3. Domain of existence of triple-shock configurations of various types

Depending on direction of flow deflection at separate shocks, we differ the triple configuration of the first type (see Fig. 2,a), of the second one (Fig. 2,c), and of the third one (Figure 2,e). All three shocks deflect the flow at the same direction at the configuration of the third type; flow deflection direction at the shock s_2 differs from others in second-type configurations, and flow deflection direction at the shock s_1 differs at triple configurations of the first type. We consider the configurations with normal shocks s_3 ($\beta_3 = 0$, Figure 2,b – so-called stationary Mach configuration [15, 16]) and s_2 ($\beta_2 = 0$, Figure 2,d – transitional TC-2-3) as the transitional ones.

Corresponding the relations (1-4) the strength J_1 of the incident shock and initial flow Mach number M defines both type of the triple configurations, parameters of shocks and flows behind them quite unambiguously (with the exception of some easily removable cases [18, 19]). Figure 3 demonstrates the domains I-III of existence of the triple configurations of various types at the plane “initial flow Mach number – incident shock slope angle”. Here the slope angle of the incident shock is the following one: $\sigma_1 = \arcsin\sqrt{(J_1+\varepsilon)/\left[(1+\varepsilon)M^2\right]}$.

The region II corresponds to second-type triple configuration, which, as a rule, forms at Mach shock reflection.

3.1 Method of calculation of the triple configuration of propagating shocks

To generalize the results of analysis of steady triple configurations on analogous structures of propagating shock, we apply the flow reversal [20] in relation to triple point (Figure 1,a-c).

The strengths J_1 and J_3 of the shocks s_1 and s_3, which move at the stagnant media, depend on the velocities V_1 and V_3 of the normal propagation of their wave fronts:

$$J_1 = (1+\varepsilon)M_{D1}^2 - \varepsilon\,,\ \ J_3 = (1+\varepsilon)M_{D3}^2 - \varepsilon\,,\ \ M_{D1} = V_1/a_0\,,\ \ M_{D3} = V_3/a_0\,. \quad (6)$$

The condition

$$V_1/\sin\sigma_1 = V_3/\sin\sigma_3 = W\,, \quad (7)$$

guarantees that the triple point T always belongs to both waves. Here W is the velocity of the triple point, $\sigma_1 = \pi/2 - \xi - \varsigma$ (see Figures 1,a-b), $\sin\varsigma = (H-h)/R$, $\mathrm{tg}\,\xi = y'(x)$, and $y(x)$ is equation of trajectory of the triple point T. If the dependence $V_1(R)$ is known, and equation of motion of the point T is given, the strengths of the waves s_1 and s_3 (and, consequently, the value $J_2 = J_3/J_1$) can be defined at any moment of time.

At flow reversal (Figure 1,c) triple point becomes stagnant, and gas stream flows on it with the velocity W in the direction opposite to the trajectory of its movement shown in Figure 1,b. The relations

$$J_1 = (1+\varepsilon)\left(W\sin\sigma_1/a_0\right)^2 - \varepsilon\,,\ J_3 = (1+\varepsilon)\left(W\sin\sigma_3/a_0\right)^2 - \varepsilon\,. \quad (8)$$

determine the strengths of the waves s_1 и s_3 in such equivalent steady triple configuration. As it is seen from (6, 7), those strengths do not change comparing with the considered unsteady triple configuration of propagating shocks. The system (1-5) allows us, due to relation (5), to determine both flow velocities across the slipstream τ, and their projections of co-ordinate axis in the equivalent configuration of the steady shocks:

$$u_{2x} = u_2\cos(\beta_3 - \varsigma),\ u_{2y} = u_2\sin(\beta_3 - \varsigma),$$
$$u_{3x} = u_3\cos(\beta_3 - \varsigma),\ u_{3y} = u_3\sin(\beta_3 - \varsigma)\,. \quad (9)$$

At the reverse transition to the coordinate system bound to stagnant gas before the propagating blast waves (Figure 1,b), we add the velocity of the triple point and the velocities of flows before the steady triple configuration:

$$\vec{U}_2 = \vec{u}_2 + \vec{W}\,,\ \vec{U}_3 = \vec{u}_3 + \vec{W}\,. \quad (10)$$

It allows us to determine the vectors $\vec{U}_2$ and $\vec{U}_3$ of flow velocities after the unsteady triple-shock configuration and to compare their values.

Two empirical relations close the system (6-10), which determines flow velocities behind the triple configurations of the propagating shocks. One of them is the dependence of incident overpressure $\Delta p_1 = (J_1 - 1)\cdot p_0$ (amplitude of the wave s_1) on the distance R from blast epicenter, which determines, according to (6), the velocity V_1 of its propagation [5, 6, 15, 16]:

$$\Delta p_1 = 1{,}38 R_*^{-1} + 0{,}543 R_*^{-2} - 0{,}035 R_*^{-3} \text{ at } 0{,}05 < R_* \le 0{,}3\,, \qquad (10)$$

$$\Delta p_1 = 0{,}607 R_*^{-1} + 0{,}032 R_*^{-2} + 0{,}209 R_*^{-3} \text{ at } 0{,}3 < R_* < 1{,}0\,, \qquad (11)$$

$$\Delta p_1 = 0{,}065 R_*^{-1} + 0{,}397 R_*^{-2} + 0{,}322 R_*^{-3} \text{ at } 1{,}0 \le R_* < 10\,. \qquad (12)$$

Here $R_* = R/G^{1/3}$ (m/kg$^{1/3}$) is distance from blast epicenter in Sadovsky-Hopkinson reduced variables; G (kg) is blast energy in TNT equivalent; Δp_1 (MPa) is amplitude (overpressure) of the wave s_1. Similarly to the variable R_*, we introduce in this study the reduced values of charge epicenter height $H_* = H/G^{1/3}$, height of the triple point $h_* = h/G^{1/3}$, reduced distance $x_* = x/G^{1/3} = \sqrt{R^2 - (H-h)^2}\,/G^{1/3}$ between blast epicenter projection and triple point projection on horizontal solid surface. The dimensionless distances $\bar{R} = R_*/H_* = R/H$, $\bar{h} = h_*/H_* = h/H$, $\bar{x} = x_*/H_* = x/H$ are determined analogously and applied also.

The second empirical value that closes the system of equations (6-10), is the relation $h = h(x)$ for trajectory of the triple point T. According to experimental data, it has the following form:

$$\frac{1-\bar{h}}{\bar{x}} = \mathrm{ctg}\left[\alpha^* + 1{,}2\ln\frac{\bar{R}}{1{,}3}\right], \qquad (13)$$

here $\alpha^* \approx 2\pi/9$ [23].

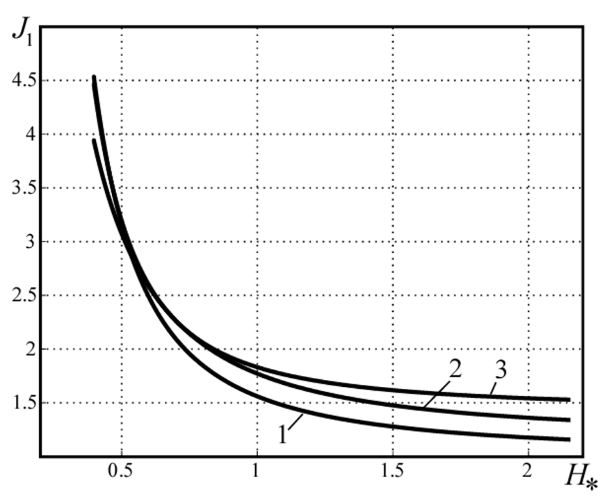


Fig. 4. Dependence of incident shock strength at point of regular / Mach reflection transition on reduced blast height (H_*, m/kg$^{1/3}$), by the empirical relation (13), by von Neumann criterion and by detachment criterion of reflection transition (curves 1-3, correspondingly)

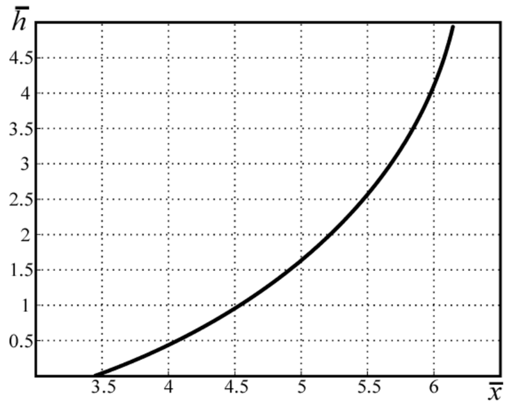


Fig. 5. Triple point trajectory, according to empirical relation (13)

4 Conclusion

We obtained the mathematical model based on exact and semi-empirical relations, which describes the shock-wave structure of a near-surface explosion and makes it possible to estimate wind loads behind various shock waves that form this structure. A detailed analysis of the results of calculations based on the proposed model can lead to the development of recommendations for the safest placement of biological and other objects in the event of an accidental explosion threat. The developed model can also help to determine the optimal parameters of blast-protection devices that give a directed effect to the blast wave and mitigate its damaging effect.

5 Acknowledgements

The study was carried out at the expense of a grant of the Russian Science Foundation No. 22-29-20269, https://rscf.ru/en/project/22-29-20269/, and a grant from the St. Petersburg Science Foundation in accordance with the agreement dated April 15, 2022 No. 57/2022.

References

1. R. Merrifield *Fire and explosion hazards to flora and fauna from explosives* Journal of Hazardous Materials **A74**, 149-161 (2000)
2. V.N. Uskov, P.S. Mostovykh *Triple configurations of traveling shock waves in inviscid gas flows* Journal of Applied Mechanics and Technical Physics **49(3)**, 347–353 (2008)
3. V.N. Uskov, M.V. Chernyshov *Special and extreme triple shock-wave configurations* Journal of Applied Mechanics and Technical Physics **47(4),** 492–504 (2006)
4. M.V. Chernyshov *Extreme Triple Configurations with Negative Slope Angle of the Reflected Shock* Russian Aeronautics **62(2)**, 259–266 (2019)
5. M.V. Chernyshov, K.E. Murzina, S.A. Matveev and V.V. Yakovlev *Shock-wave structures of prospective combined ramjet engine* IOP Conf. Series: Materials Science and Engineering **618**, 012068 (2019)
6. B.E. Gelfand and M.V. Silnikov *Barothermal Action of Blasts* St. Petersburg: Asterion 658 (2006)

7. B.E. Gelfand and M.V. Silnikov *Blast Safety* St. Petersburg: Asterion 392 (2006)
8. L.A. Shushko and Yu A. Kaganer *Calculation of the intensity of shock air waves in the near range of the explosion action* Combustion, Explosion and Shock Waves **34(6)** 671-678 (1998)
9. RD 03-409-01 2001 *Methodology for assessing the consequences of emergency explosions of fuel-air mixtures*
10. PB 14-159-97 1997 *Safety rules for hazardous production facilities for the storage and processing of grain*
11. M.V. Chernyshov *Empirical estimates of the probability of high-explosive damage to humans and animals in the explosion of condensed matter* Problems of Technosphere Risk Management **1(5)**, 40-48 (2008)
12. M.V. Chernyshov and F.M. Kompan *Probabilistic assessment of the degree of high-explosive destruction of a biological object* Military Enginery. Series 16. Counter-terrorism Technical Devices **5-6**, 50-54 (2008)
13. J.M. Powers and H. Krier *Attenuation of blast waves when detonating explosives inside barriers* Journal of Hazardous Materials **(13)** 121–133 (1983)
14. M. Omang, S.O. Christensen, S. Børve and J. Trulsen *Height of burst explosions: a comparative study of numerical and experimental results* Shock Waves **(19, 2)** 135–143 (2009)
15. B.E. Gelfand, M.V. Silnikov and M.V. Chernyshov *On the efficiency of semi-closed blast inhibitors* Shock Waves **20(4)**, 317–321 (2010)
16. A.L. Adrianov, A.L. Starykh and V.N. Uskov *Interference of Stationary Gasodynamic Discontinuities* Novosibirsk: Nauka 180 (1995)
17. G. Ben-Dor 2007 Shock Wave Reflection Phenomena (Berlin – Heidelberg – NewYork: Springer) p 342
18. M.V. Chernyshov, A.S. Kapralova and K.E. Savelova *Ambiguity of solution for triple configurations of stationary shocks with negative reflection angle* Acta Astronautica **179,** 382–390 (2021)
19. M.V. Chernyshov and A.S. Kapralova *Triple configurations of pursuit shock waves in conditions of ambiguity of the solution* Journal of “Almaz – Antey” Air and Space Defence Corporation **4**, 46–52 (2017)
20. V.N. Uskov *Running One-Dimensional Waves* St. Petersburg: BSTU “VOENMEH” **1** 232 (2013)
21. J. Henrych *The Dynamics of Explosion and Its Use* Amsterdam: Elsevier, 562 (1979)
22. A. Ullah, F.Ahmad, H.W. Jang, S.W. Kim and J.W. Hong *Review of analytical and empirical estimation for incident blast pressure* KSCE Journal of Civil Engineering **21**(6), 2211–2225 (2017)
23. I.A. Balagansky and L.A. Merzhievsky *The Effect of Weapons of Destruction and Ammunition* Novosibirsk: Novosibirsk State Technical University, 408 (2004)