Construction and implementation of a mathematical model for damping vibrations of high-rise buildings and structures using reactive dampers

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Abstract. The problem of developing a computational model of an active vibration damping system of a high-rise building during an earthquake using a simple and suitable calculation scheme for high-rise buildings is considered. An approximate finite element model is presented that adequately describes the dynamic characteristics of a high-rise building. A mathematical model of a steel lattice tower is considered as an example. The ratio equations and displacement difference equations describing the behavior of reactive dampers installed in the tower are presented. An approach to optimizing the parameters of relative motion is presented. The calculation of vibration damping of a tower with reactive dampers installed on one and two levels is shown. The classification of forces with different physical properties and affecting the movement of mechanical systems is presented.

1 Introduction

There are two main types of high-rise buildings: high-rise buildings and tower-like high-rise structures. When solving problems of studying the dynamics of skyscrapers and high-rise buildings by direct methods, tens of thousands of iterative solutions of huge systems of equations are required. In such cases, in order to save time, there is an urgent need to find an approximate mathematical model of a building or structure, which in its static and dynamic properties closely corresponds to traditional finite element models. Such problems arise, for example, when solving the problem of active damping of vibrations of a high-rise structure under the influence of an earthquake [1-7]. The approximate model should include a system of equations describing the stress-strain state of the structure and effectively controlling the relative movement of the structure.

2 Subject, methods and materials

Let's study the behavior of a building subject to non-stationary kinematic displacements, equipped with damping devices.

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Based on the analysis of a number of calculation schemes, we will take an approximate calculation scheme of a plate-core frame as a system with a rigid overlap disk, elastic columns rigidly fixed on it and a core of rigidity approximated by a rigid rod (Fig. 1). If the overlap plate is considered as a disk of absolute rigidity, then each such disk will be in flat motion. If s is the number of floors, then the total number of degrees of freedom of a high-rise building will be t = 3s.

Let the origin of the coordinates of all the tiers of the building pass through the center of the elevator block (see Fig. 1). Assume that this is the center of rotation of the overlap disk and connect this center of rotation with each column by a radius vector. Then, for each nodal point of the column interface with the floor slab in the layer, the horizontal displacement caused by rotation is determined by the following dependence:

$$\left.\begin{aligned} u_{i,j}^{rot} &= -b_i \varphi_j \cdot \sin\gamma_i = -y_i \varphi_j, \\ v_{i,j}^{rot} &= +b_i \varphi_j \cdot \cos\gamma_i = x_i \varphi_j. \end{aligned}\right\} \tag{1}$$

Here b_i - is the distance from the origin to the column axis; γ_i – the angle of inclination b_i to the x axis.

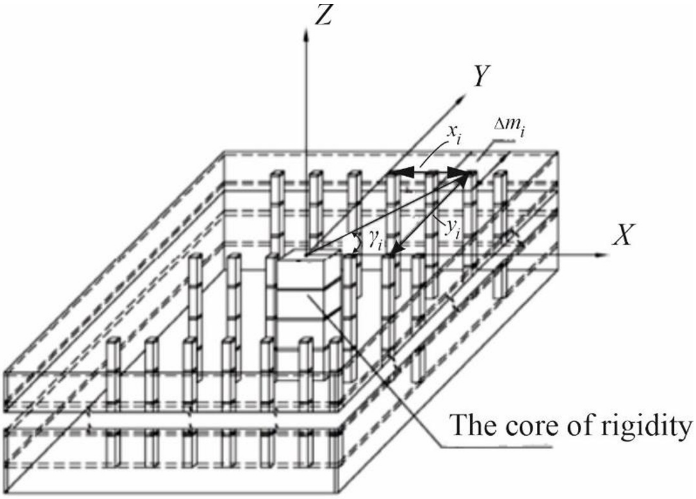

Fig.1. A diagram of a high-rise building (ceilings are conventionally shown as transparent).

The stiffness matrix of one tier of a high-rise building will look like:

$$K_{lev} = \begin{bmatrix} r & -r \\ -r & r \end{bmatrix}, \tag{2}$$

$$r=\begin{bmatrix} \sum_{i=1}^{n}\frac{12EI_{iy}}{l^3} & 0 & 0 \\ 0 & \sum_{i=1}^{n}\frac{12EI_{ix}}{l^3} & 0 \\ 0 & 0 & \sum_{i=1}^{n}\frac{GI_{iz}}{l}+\sum_{i=1}^{n}y_i^2\frac{12EI_{iy}}{l^3}+\sum_{i=1}^{n}x_i^2\frac{12EI_{ix}}{l^3} \end{bmatrix}, \tag{3}$$

where x_i, y_i – are the coordinates of the i–th column of the tier; n – is the number of columns on the floor.

With sequential numbering of floors and movements, the stiffness matrix of a high-rise building will look like:

$$K=\begin{bmatrix} k_{11}^1+k_{11}^2 & 0 & 0 & k_{14} & 0 & 0 & 0 & 0 & - & 0 \\ 0 & k_{22}^1+k_{22}^2 & 0 & 0 & k_{25} & 0 & 0 & 0 & - & 0 \\ 0 & 0 & k_{33}^1+k_{33}^2 & 0 & 0 & k_{36} & 0 & 0 & - & 0 \\ k_{41} & 0 & 0 & k_{44}^2+k_{44}^3 & 0 & 0 & k_{47} & 0 & - & 0 \\ 0 & k_{52} & 0 & 0 & k_{55}^2+k_{55}^3 & 0 & 0 & k_{58} & - & 0 \\ - & - & - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & 0 & - & k_{s-3,s-3}^{(s/3)-1}+k_{s-3,s-3}^{s/3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 & k_{s-2,s-2}^{s/3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 & 0 & k_{s-1,s-1}^{s/3} & 0 \\ 0 & 0 & 0 & 0 & 0 & - & k_{s,s-3} & 0 & 0 & k_{s,s}^{s/3} \end{bmatrix}, \tag{4}$$

where the superscript means the floor number. In this case, the displacement vector will be written as:

$$U=\left\{u_1v_1\varphi_1...u_sv_s\varphi_s\right\}^T. \tag{5}$$

The matrix of masses concentrated at the overlap levels can be written as:

$$M=diag\left\{m_1m_1J_{z1}^m...m_sm_sJ_{zs}^m\right\}, \tag{6}$$

where

$$J_{zj}^m=\sum_{i=1}^{n}(\Delta m_i)(x_i^2+y_i^2). \tag{7}$$

Here j is the floor number.

The system of reactive dampers will be installed on a rigid protective plate. The damper can be attached to the center of gravity (origin) of the coated disk to dampen linear motion or to the central edge of the protective plate to create a pair of forces to dampen rotational motion. The liquid damper must have radial outlets and openings to create a pair of forces.

The design solution of a high-rise tower is conveniently approximated by a system of hinge-rod elements [2]. For a hinge-rod system, the stiffness matrix of the final element and the matrix of internal damping have the following form:

$$k_i=\frac{EA_i}{l_i}\begin{bmatrix} e_i & -e_i \\ -e_i & e_i \end{bmatrix}, \tag{8}$$

$$b_{i,mp} = \frac{EA_i\chi_y}{2l_i}\begin{bmatrix} e_i & -e_i \\ -e_i & e_i \end{bmatrix}, \tag{9}$$

where χ_y is the coefficient characterizing the viscosity of the material; E is the modulus of elasticity of the material; A_i and l_i are the cross-sectional area and length of the i – th element, respectively;

$$e_i = \begin{bmatrix} (c_1c_1)_i & (c_1c_2)_i & (c_1c_3)_i \\ (c_2c_1)_i & (c_2c_2)_i & (c_2c_3)_i \\ (c_3c_1)_i & (c_3c_2)_i & (c_3c_3)_i \end{bmatrix}. \tag{10}$$

Here c_j are the guiding cosines of the inclination of the i – th element to the j – th axis of the general coordinate system.

With a nodal mass distribution of the hinge-rod system, the mass matrix will have a diagonal structure:

$$M = diag\{m_1m_1m_1...m_nm_nm_n\}. \tag{11}$$

where n is the number of node movements.

The use of vibration damping systems and devices is necessary to increase the dynamic strength, stability, durability and reliability of mechanical systems such as high-rise towers and buildings. The operation of these systems and devices is based on the emergence of reaction forces or forces that change the rhythm of movement. These forces have different physical properties.

We classify the forces that affect the movement of mechanical systems and have various physical properties (Table 1).

Table 1. A classification of some force influences affecting the movement of mechanical systems and having different physical nature.

№	Name of the force	Mathematical formula for calculating force
1	Gravitational force (gravitational force)	$m\overline{g}$
2	The strength of elasticity	$c\cdot\Delta(t)$
3	Reactive power	$F_i = V_g \cdot \frac{dm}{dt}$
4	Electromagnetic force	$\frac{\partial T}{\partial q}$
5	Inertia force (conditional force)	$-m\cdot\overline{a}(t)$
6	Friction force	$-\alpha\cdot v(t)$

The following designations are used in the table: m is the inert mass of the object; c is the stiffness of the elastic system; Δ – movement of the elastic system; V_g – the velocity of the air flow at the nozzle of the injector; T – the energy of the magnetic field; q – the gap of the electromagnetic drive; a – acceleration of the material point; α – the coefficient of dynamic resistance; ; v – the velocity of the point of the system.

Pendulum shock absorbers are affected by a combination of two forces: gravity and inertia. Spring–loaded dynamic single– and multi-mass shock absorbers also work on the basis of a combination of two forces (elastic and inertial). Pendulum and spring shock dampers instantly convert the impact energy into an elastic force pulse. Damping devices use friction to dissipate vibration energy.

The above-mentioned vibration dampers are usually used to passively control the dynamics of structures and components of machines.

Reactive and/or electromagnetic forces can be used to create active dampers.

There are two main types of active vibration dampers:

1) systems with controlled passive damping characteristics;

2) systems generating controlled active damping forces.

Vibration dampers were proposed in [1, 7]. Such dampers act due to the reaction force of water jets. The magnitude of the reaction force can be adjusted by changing the cross-sectional area of the water jet and the overpressure in the tank. This type of dampers is very environmentally friendly and effective in short and medium duration impacts. Reactive vibration dampers are especially effective in motion disturbances such as unsteady loads and seismic impacts [1,7].

The disadvantages of liquid active dampers are the relatively large initial mass and the time required to open and close the drain valve. Reactive dampers, on the contrary, are devoid of these disadvantages, since they are installed using a reactive charge system rigidly connected to the floor slab.

The motion of a mechanical system with reactive dampers is subject to kinematic effects, which can be described by matrix finite element equations:

$$M\ddot{U}+B\dot{U}+KU=-M\ddot{\Delta}+F\,, \tag{12}$$

where B – is the damping matrix; U - is the vector of relative (deformation-induced) displacements; Δ - portable (seismic) movements; F – vector of reactive effects,

$$F=\left\{000\ldots F_{iu}F_{iv}F_{i\phi}00\ldots F_{ku}F_{kv}F_{k\phi}0\ldots 0\right\}^{T}. \tag{13}$$

Here F_i is a reactive force or a pair of forces acting in the direction u, v, φ at the level of the i – th floor of the building,

$$F_i=V_g\cdot\frac{dm}{dt}. \tag{14}$$

The equation of motion in the i – th direction will have the form:

$$m_i\ddot{u}_i+\sum_{k=1}^{n}b_{ik}\dot{u}_k+\sum_{k=1}^{n}k_{ik}u_k=-m_i\ddot{\Delta}+F_i\,. \tag{15}$$

Using a finite-difference approximation of derivatives in time coordinate[6], we rewrite equation (15) as:

$$\begin{gathered}m_i\frac{2\left(\alpha u_{i,t\text{-}\Delta t}-\left(1+\alpha\right)u_{i,t}+u_{i,t+\Delta t}\right)}{\alpha\left(1+\alpha\right)\left(\Delta t\right)^2}+\sum_{k=1}^{n}b_{ik}\frac{\left(u_{k,t+\Delta t}-u_{k,t-\Delta t}\right)}{\left(1+\alpha\right)\Delta t}+\\+\sum_{k=1}^{n}k_{ik}\cdot u_{k,t}=-m_i\ddot{\Delta}_t+F_{i,t}.\end{gathered}, \tag{16}$$

where α - is the coefficient of time step change.

With a known value $F_{i,t}$, the entire vector $u_{i,t+\Delta t}$ is easily calculated $U_{t+\Delta t}$. The task of damping vibrations is to reduce the relative movements of the structure caused by portable seismic impacts.

Let's set the task of optimizing the parameters of relative motion:

$$f\left(u_{j,t+\Delta t},\dot{u}_{j,t+\Delta t}\right)\rightarrow \min \tag{17}$$

By

$$F_{\min}\leq F_i\leq F_{\max}, \tag{18}$$

where $f\left(u_{j,t+\Delta t},\dot{u}_{j,t+\Delta t}\right)$ is the objective function consisting of optimized motion control parameters; j is the displacement number; $\left(u_{j,t+\Delta t},\dot{u}_{j,t+\Delta t}\right)$ - movements and velocities of the central point of overlap of the tier (pole), including the angular velocity of rotation.

3 Results and discussion

We will determine the motion parameters and adjust the dampers to achieve the best results. We use one or two dampers to suppress vibrations.

This task is solved at each step of the movement along the time coordinate to achieve optimal control of the dynamics of the building.

To conduct numerical experiments on solving dynamic FEM problems and graphical interpretation of the results in the MATLAB environment, the DA (Dinamika Active) software package has been developed. During numerical experiments, the active liquid vibration dampener proposed in [3] was investigated (Fig. 2). In particular, the work of one and two dampers installed on a mechanical system exposed to seismic effects was considered. To maximize the effect of active dampers, the parameters involved in the control of vibration dampers were studied.

Active liquid dampers work by creating a variable reaction force that prevents relative (flexural) (oscillatory) movement of the structure. The reaction force is created by hydraulic fluid ejected from the tank through valves when valves located in opposite positions are alternately opened (closed) under the action of overpressure p_0.

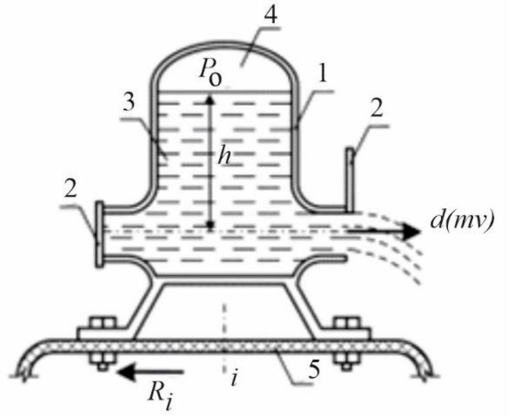

Fig. 2. Operation diagram of the active liquid damper: 1 – damper capacity; 2 – valve; 3 – working fluid; 4 – gas under pressure; 5 – protected structure.

The mechanical system is a 10-storey beam structure: one damper is installed on the 10th floor, two dampers on the 10th and 7th floors, respectively. The kinematics of vibration excitation corresponds to the Brawley seismic acceleration map (California, USA). The parameters under study include the displacement of structural elements, the required reaction forces and the initial mass of the dampers. The results are presented in table 2.

Table 2. The results of studies of the parameters of the displacement of structural elements, the required reactive force and the initial mass of the damper.

Parameters		Number of vibration dampers	
		One damper	Two dampers
The effect of the damper 's operation	Reducing the movements of the 10th floor by	94.95 %	99.97 %
	Reducing the movements of the 9th floor by	84.03 %	99.77 %
	Reducing the movements of the 7th floor by	43.72 %	99.98 %
	Reducing the movements of the 5th floor by	1.56 %	88.84 %
	Reducing the movements of the 1st floor by	61.95 %	49.41 %
Required reactive power	One damper	22 кН	6 кН
	Two dampers	-	16 кН
The initial mass of the extinguishers as a percentage of the mass of the structure	One damper	3.27 %	1.09 %
	Two dampers	-	2.62 %
	The amount	3.27 %	3,.1 %

The results of studies of the damper settings for different target functions are presented in Table 3.

Table 3. The results of studies of the damper settings for different target functions.

Control parameter	Reducing movement			
	10th floor		7th floor	
	One damper	Two dampers	One damper	Two dampers
$U_{10\,floor} \rightarrow \min$	94.93 %	94.97 %	43.71 %	98.10 %
$U_{10\,floor} - U_{floor} \rightarrow \min$	86.67 %	94.95 %	36.54 %	93.62 %
$U^2_{10\,floor} - U^2_{7\,floor} \rightarrow \min$	94.92 %	87.10 %	43.70 %	88.25 %
$U^2_{10\,floor} + U^2_{7\,floor} \rightarrow \min$	94.95 %	99.97 %	43.72%	99.98%
$R_{res} = F = m_i \cdot \ddot{\Delta}$	78.58 %	79.42 %	29.55 %	80.57 %

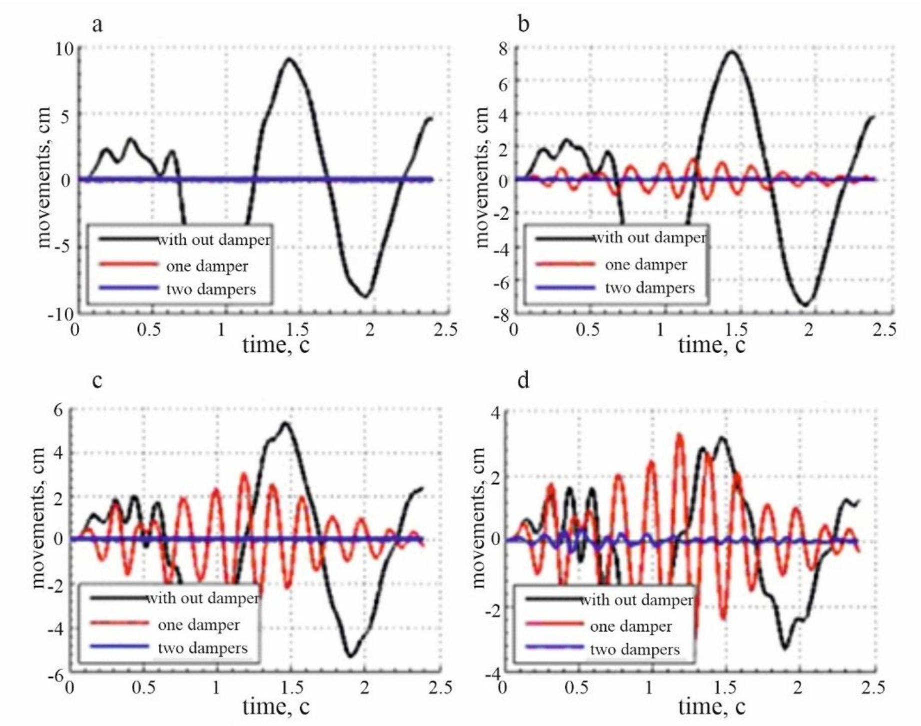

Fig. 3. Moving floors of a tower with one and two dampers: a – 10th floor; b – 9th floor; c – 7th floor; d – 5th floor.

4 Conclusions

An approximate finite element model of a high-rise building is presented, which adequately describes its dynamic characteristics.

The results of numerical studies of high-rise towers with dampers are considered. It is shown that one active liquid vibration dampener installed on the upper floor of the structure effectively reduces the oscillation range of the upper zone of the structure. When installing a second vibration dampener, the efficiency of damping the oscillation span of the entire system increases, despite the increased load on the structure due to the addition of the mass of the second extinguisher. In the case of installing two vibration dampers, the most effective is to adjust them to a minimum of the sum of the squares of the average floor movements. The conducted research confirms that an active vibration dampener is an effective means of controlling the process of damping seismic vibrations.

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