Survivability of reinforced concrete frame systems with complex stressed elements

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Abstract. On the energy basis, using the diagrammatic method, the solution of the problem of determining the survivability parameter of the reinforced concrete frame structure of a multistory building with complex-stressed elements under static-dynamic deformation caused by a special impact is obtained. Determination of the parametric load value, at which in the most stressed spatial section at the considered loading mode one of the criteria of the special limit state comes from the system of canonical equations of the extraordinary version of the mixed method. Criterion check of survivability of the considered physical and structurally nonlinear frame system is performed using diagrams "parametric load - generalized reference reaction" for the spatial section of the complex stressed reinforced concrete element. Comparison of the experimental and design survivability parameters gives an assessment of the efficiency and reliability of the proposed design dependencies. It has been shown that with the adopted initial hypotheses, the proposed method for calculating the survivability of frames with complex stressed elements satisfactorily describes the process of their deformation and destruction under the considered impacts.

Keywords: reinforced concrete frame, complex stress state, experimental and theoretical studies, special impact, calculation model, survivability parameter.

1 Introduction

Due to the constantly increasing types and intensity of special impacts on buildings and structures, the number of scientific publications in the field of protecting real estate objects from progressive collapse is growing in Russia and many foreign countries. In a number of countries, the solution to this problem has reached the level of development of new generation regulatory documents. At the same time, a number of areas in research and solutions to the problems of this problem remain insufficiently studied. This concerns both the fundamental problems of the theory of survivability of physically and structurally nonlinear systems, and the solution of a number of problems of an applied nature. Thus, in

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well-known publications, for example, in Russian [1-3] and foreign [4-6], the results of mainly numerical or numerical-analytical studies of structural systems during their structural restructuring are given. A small number of publications of recent years are devoted to the creation of analytical models of deformation of such systems [7-10]. At the same time, only some publications consider the physical side of the problem of force resistance to deformation and destruction of structures made of various materials, especially of such a complex material as reinforced concrete [11-13]. Moreover, these publications consider the mode deformation of reinforced concrete elements with destruction along the normal section under the simplest stress states. At the same time, as shown in work [14], the force resistance of structural systems under the considered impacts, among other things, is largely determined by the nature of the stress state in their elements. The number of experimental studies on the survivability of reinforced concrete structures under special impacts is extremely limited. In our country, such studies were carried out mainly on models of structural systems [15-18]. For natural structures, experimental studies were carried out in some foreign countries, for example [19-23]. But they are made according to different methods, for the simplest stressed-deformed states. They set different goals and, accordingly, obtained different results, sometimes qualitatively different. For example, this relates to the assessment of dynamic effects when removing bearing elements from the structural system [14, 24], taking into account the violation of the continuity of concrete and the modulus of deformations and limit deformations in reinforced concrete elements in beyond-limit states of their deformation, etc. In this regard, this article provides a methodology for the analytical determination of the survivability parameter of reinforced concrete frame structures of multi-storey buildings in a complex stressed state in their elements, as well as the results of comparing the calculation using this methodology with experimental data.

2 Method

The statically indeterminate frame-rod system of the multi-storey building frame (Figure 1) is taken as the research object. In the current Russian and foreign regulatory documents, the special limit state of reinforced concrete elements of such structures is determined by criteria limiting the limit deformations and limit movements of elements located in the so-called zone of possible local destruction after the sudden removal of one of the bearing elements of the structural system. Exceeding the limits of these parameters is considered as exhaustion of the bearing capacity of the element. At the same time, it is not difficult to see that in a statically indeterminate structural system, when a limit state occurs in the section of one structural element, there is no loss of survivability of the entire frame system or a significant part thereof. Here, as in [25, 26], we will define survivability as the ability of the system to redistribute the load between its bearing elements when one of the elements of the structural system is suddenly removed from operation.

Evaluation of survivability of a statically indeterminate frame reinforced concrete structural system with complex stressed elements is performed using the survivability parameter of the $\lambda$. This parameter is determined by the relative value of the parametric load, at the action of which the value of one of the criteria exceeds the permissible value in any of the considered complex stress sections of the system element, at its static-dynamic loading mode.

The definition of the survivability parameter is $\lambda$ considered in relation to the fragment of the complex stress element of the crossbar located in the area of removal of the extreme column of the frame system. When building the design model, the following initial hypotheses are adopted:
- the area of possible local destruction of the flat or spatial frame system is limited by the span of the crossbar or crossbars adjacent to the removed column at the spatial structural system;
- a special limit state in a complex hinge occurs when the element is destroyed along the spatial section due to the combined action of bending and torque moments;
- when calculating the complex stress element, the ratio between bending and torque in this element remains constant before and after cracking, i.e. the diagrams "bending moment-curvature", "torque-angle of rotation" have affine similarity at all stages of loading the structure.

Fig. 1. Diagram of frame structural system (a) and fragment of complex stressed element in the area of possible local destruction (b)

The adopted scheme of the frame structures, the scheme of its loading and reinforcement corresponded to the experimental structures tested for the operational load and special impact [27, 28].

A diagram of the states of the spatial section of the reinforced concrete crossbar of the considered structurally nonlinear frame is presented in the coordinates "parametric load λ - generalized reference reaction $R_{sup}$" (Figure 2). Consider the two most characteristic stages of the mode static-dynamic loading of the structural system:

1 – static loading of the frame with the applied design load in the form of $\lambda mP_i$;
2 – shows the dynamic loading of the frame with a special effect in the form of a sudden removal from the structural system of the first floor post.

At the first stage of calculation, a design load in the form of $\lambda mP_i$ forces is applied to a statically indeterminate frame-rod structural system. Let these forces be applied at arbitrary points located at distances $a$, $b$ from the supports along the span of each crossbar, in general with eccentricity $e$, in relation to the longitudinal axis of the crossbars (see Fig. 1a).

The design load is parametric, i.e. its change is proportional to the accepted $\lambda$ parameter. Figure 3 shows the points $c_k (k=1,2,3,n)$ in which it is assumed to form spatial plastic joints in the most stressed cross-bar cross-sections when these cross-sections in reinforcement or compressed concrete reach the limit deformations from the combined action of the torque and bending moments at the parametric growth of the studied parameter of survivability of the $\lambda$. These points are not difficult to pre-assign, since they are located in the intersection nodes of the elements, in the places of application of concentrated forces, in places with a change in the rigidity of the elements of the considered rod system.
So, in the process of loading the frame-rod system to a certain load $\lambda_{crP_1}$ (see Fig.2), in the most stressed spatial section of a complexly stressed element with initial bending stiffness $B_0$ and torsion $G_0$, the first spatial crack is formed and the stiffness of the element will decrease to the values $B_{crc}$ and $G_{crc}$, respectively.

We determine the value of the parametric load $\lambda_{mP_1}$, at which one of the criteria of a special limiting state occurs in the most stressed spatial cross-section of the frame elements under the considered static-dynamic loading regime. The introduced index $m$ determines the level of relative load $\lambda_{mP_1}$, at which, as a result of the instantaneous removal of one of the elements of the statically indeterminate frame system under study, the most stressed spatial section perceives the limiting bending and torque when they act together. When a static load of a given level of $\lambda_1P_1$ is applied to a loaded frame structure and a special impact in the form of a sudden removal of a first-floor column from the structural system and subsequent structural restructuring of the frame system, a spatial plastic hinge is formed in the most stressed section loaded with this effect and, accordingly, the degree of static indeterminability of the frame system will change by an amount of $n-1$ (in the diagram Fig.2 point $\lambda_{d,n-1}$), but the destruction of the rigidly pinched frame element considered in the zone of possible local destruction will not occur. A further increase in the static load at $m=2$ and, accordingly, $\lambda_2P_1 > \lambda_1P_1$ will lead to the fact that not one, but at least two spatial plastic hinges are formed in the studied structural element in the most stressed sections and, accordingly, the degree of static indeterminability of the frame system will change by the value $n-2$ (in the diagram Fig.2, the point $\lambda_{d,n-2}$), but the kinematic variability of this rigidly pinched frame element will not occur. When three plastic hinges are formed sequentially on the frame crossbar under consideration, the kinematic variability of this rigidly pinched frame element simulating the zone of possible local destruction will occur. At the same time, if in the crossbar located in the zone of possible local destruction after the formation of the third plastic hinge, the deformations in the reinforcement do not reach the limit values of a special limit state and there is no violation of its anchoring on the crossbar supports, then the entire continuous structure of the crossbar in question will turn from rigid into a changeable cable-stayed system with elements working on tension.

The calculation of unknown forces in the considered substructure ($M_j, Z_i$) was performed using an extraordinary mixed method [29]. According to this method, the solution of the problem is constructed as follows. The initial initial substructure system (Figure 3a) is described by a hinge-rod model in which the places of possible disconnection of connections are replaced by complex hinges and, accordingly, unknown angular $M_j (j = 1, 2, \ldots, k)$, $T_j (j = 1, 2, \ldots, k)$ and linear $Z_i (i = k+1, k+2, \ldots, n)$ by connections (Figure 3b).
In the case of a spatial system of a multi-storey building frame, the zone of possible local destruction is represented by a spatial substructure, and accordingly, the main system of the mixed method is selected (Figure 3c, d). At the same time, in accordance with the accepted hypothesis on affine similarity of deformation diagrams of complex stress spatial section, the number of unknowns in the system under consideration is reduced and the values of torques $T_j$ ($j = 1, 2, ..., k$) are calculated through the similarity factor $\eta = M/T$. This option is convenient enough to solve this problem when using Mathcad. The solution does not require a change in the system of governing equations, but the number of unknowns is always reduced by one regardless of the location of the cross-section in which the connection is switched off. The estimated survivability parameter $\lambda$ the described static-dynamic deformation diagram "parametric load $\lambda$ – generalized reference reaction $R_{sup}$" (see Fig. 2) for an arbitrary section of the frame-rod system are calculated using a system of determining canonical equations of the considered version of the extraordinary version of the mixed method, the main system of which is shown in Fig. 3b and d. Taking into account the parametric load recording form adopted in [26, 29], the load coefficients in the considered version of the mixed method are presented in the form of two terms (1):

$$
\begin{align*}
\sum_{j=1}^{k} \delta_{ij} M_j + \sum_{m=k+1}^{N} \delta_{im} Z_m + \Delta_{iq} + \delta_{ip} \lambda = 0, & \quad (i = 1, 2, \ldots, k) \\
\sum_{j=1}^{k} r_{ij} M_j + \sum_{m=k+1}^{N} r_{im} Z_m + R_{iq} + r_{ip} \lambda = 0, & \quad (i = k + 1, \ldots, n)
\end{align*}
$$

(1)

where $\delta_{ij}$, $\delta_{im}$, $r_{ij}$, $r_{im}$ - corresponding unit displacements and reactions in the canonical equations for the given system; $r_{im} = 0$; $\Delta_{iq}$, $R_{iq}$ - load coefficients (respectively displacements and reactions) from the constant load $q$; $\delta_{ip}$ - displacements in the direction of remote links from the external parametric load $P_i$ at $\lambda = 1$. In the considered system $r_{ip}$ - reaction in the i-th superimposed connection of the main system from the external parametric load $P_i$ at $\lambda = 1$. 

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**Fig. 3.** The given (a) and main (b) system for calculating the substructure of the zone of possible destruction of a flat frame system; c, d – the same for calculating the substructure of the zone of possible destruction of a spatial frame system.
In matrix form, the system of canonical equations has the form (2):

\[
\begin{align*}
A \cdot M + B \cdot Z + \Delta_q + \delta_p \cdot \lambda &= 0 \\
C \cdot M + 0 + R + r_p \cdot \lambda &= 0
\end{align*}
\]  

(2)

Using the matrix form of the representation of equations (2), as well as considering the properties of the canonical equations of the mixed method \( C = -B^T \) (the index "T" means the transposition operation), the system of equations (2) can be written as follows (3):

\[
\begin{bmatrix} A & B \\ -B^T & 0 \end{bmatrix}\begin{bmatrix} \bar{M} \\ \bar{Z} \end{bmatrix} + \begin{bmatrix} \bar{\lambda}_q \\ \bar{\lambda}_p \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{\rho}_p \end{bmatrix}
\]

(3)

Its solution has the form (4):

\[
\begin{bmatrix} \bar{M} \\ \bar{Z} \end{bmatrix} = \begin{bmatrix} M_q \\ Z_q \end{bmatrix} + \begin{bmatrix} \bar{m}_q \\ \bar{z}_p \end{bmatrix} \lambda
\]

(4)

where

\[
\begin{bmatrix} M_q \\ Z_q \end{bmatrix} = \begin{bmatrix} A & B^{-1} \\ -B^T & 0 \end{bmatrix}\begin{bmatrix} \bar{\lambda}_q \\ \bar{\lambda}_p \end{bmatrix}, \quad \begin{bmatrix} \bar{m}_q \\ \bar{z}_p \end{bmatrix} = \begin{bmatrix} A & B^{-1} \\ -B^T & 0 \end{bmatrix}\begin{bmatrix} 0 \\ \bar{\rho}_p \end{bmatrix}
\]

(5)

By solving the system of equations (3) and determining unknown ones, it is possible to calculate the moments in the disconnecting links (spatial hinges) of the considered frame subassembly from the total action of the given and considered parametric loads (6):

\[
M_j = M_{jq} + m_{jp} \lambda, \quad (j = 1, 2, ..., k)
\]

(6)

where \( M_{jq} \) and \( m_{jp} \) are elements of the column matrix \( \bar{M}_q \) and \( \bar{m}_p \) (the " \( \rightarrow \) " sign here and in formulas (3) – (5) denotes a vector quantity).

The criterion check of survivability of the considered physical and structurally nonlinear frame system when one of the supporting vertical elements is disconnected from operation is performed by the diagram method using the built diagrams of static-dynamic deformation of the spatial section of the complex stressed reinforced concrete element "parametric load \( \lambda \) - generalized reference reaction \( R_{sup} \)" (see Fig. 2).

In accordance with the calculation method proposed in work [30], from the equations of equilibrium and deformation dependencies in the spatial design section (Figure 4), a connection was established between the generalized reference reaction \( R_{sup} \), external load and limiting internal forces in this section.

\[\text{Diagram}\]

\[\text{Equations}\]
Recording the equilibrium equations of moments of internal and external forces in the vertical section passing perpendicular to the longitudinal axis of the reinforced concrete element along the edge of the compressed zone closing the spatial spiral crack, relative to the $y$ passing through the point of application of the resultant forces in the stretched reinforcement, an unknown value of the limit bending dynamic moment $M_{j,ult}^d$ in the considered section is determined.

When the limit value of dynamic bending $M_{j,ult}^d$ or in compressed concrete or stretched reinforcement reaches the limit values determined by [30] in the spatial section of the element, a plastic hinge is formed in the most stressed section $j$ after sudden removal of one of the supports from the structural system. At the same time, for all forces in the disconnecting complex plastic hinges (moment ties) of the calculated frame sub-structure, a system of inequalities will be satisfied:

$$M_j^d = \left|M_{jq}^d + m_{jp}^d \lambda \right| \leq M_{j,ult}^d, \quad (j = 1, 2, ..., k), \tag{7}$$

where $M_{j,ult}^d$ - are limit values of bending dynamic moment in the considered spatial plastic hinge.

When calculating the left side of the inequality system (7), the value $M_j^d$ is taken from the absolute value: the negative value of the moment $M_j^d$ indicates that the direction of action of this force will be opposite to its limit value adopted in the main system.

The minimum value of the parametric load - the survivability parameter $\lambda_m$, $(m = 1, 2, 3)$ at which, after disconnecting from the operation of the element, a special limit state is reached in the most loaded cross-section of the crossbar $C_j$ can be found from expression (8)

$$\lambda_m = \min \left( \frac{M_{jq}^d}{m_{jp}^d} \pm \frac{|M_j^d|}{m_{jp}^d} \right), \quad (j = 1, 2, ..., k) \tag{8}$$

In the numerator of formula (8), the sign "-" is placed when the direction of the moments $M_{jq}^d$ and $m_{jp}^d$, matches, the sign "+" - if they are opposite.

When the $j$-th link is disconnected from operation, the degree of static uncertainty of the frame is reduced by one. Constraints (1) and $j$-th unknown are excluded from the system of equations (7). The original matrix of mixed method equations (3) are transformed as follows:

1. matrix A excludes the $j$-th row and the $j$-th column;
2. the $j$-th line is excluded in matrix B;
3. in column matrices, the values of the cargo coefficients $\tilde{\Lambda}_q$ and $\tilde{R}_q$ are specified using formulas (9):

$$\Delta_{iq}^{(l)} = \Delta_{iq}^q + \delta_{ip} \lambda_m + \delta_{iq} \left( \pm M_{j,ult}^d \right)$$
$$R_{iq}^{(l)} = R_{iq}^q + r_{iq} \lambda_m + r_{iq'} \left( \pm M_{j,ult}^d \right) \tag{9}$$

In expressions (9), the sign before the limit dynamic moment $M_{j,ult}^d$ is taken in accordance with the sign of the moment $m_{jp}^d$.

For the convenience of automation of calculations, reformatting the original system of equations is more rational without excluding the $j$-th unknown. To do this, replace the main element $\delta_{ll}$ of matrix A in formula (3) with one, and replace the elements $\Delta_{iq}$ and $\delta_{ip}$ the corresponding load column matrices with zeros. In the system of equations thus obtained, the following disconnecting connection is determined when the limit dynamic force is reached. Next, the system of equations is transformed again in the same way, but the original system
will already be the system obtained after the previous step of solving the problem. In this case, at the second and subsequent steps, we obtain an increment of the parametric load, the value of which at the \( i \)-th step will be determined by the formula (9):

\[
\lambda_{mi} = \lambda_{m,i-1} + \Delta_{m,i-1}
\]  

Thus, the criterion of survivability of the investigated structural frame system in the zone of possible local destruction is a sequential change in the \( \lambda_m \) parameter \((m=1,2,3)\) during the transition from a statically indeterminate frame system with complex stressed elements to a system that varies as spatial plastic joints are formed sequentially. In fact, checking the survivability criterion reduces to calculating the determinant of the matrix of coefficients at unknown at each step, after performing transformations of the system of equations. As a result, when the determinant of the matrix is zero, the system turns into geometrically variable and its survivability is considered exhausted.

### 3 Results

To assess the efficiency and reliability of the proposed design dependencies, the survivability parameter of the experimentally tested model of the reinforced concrete frame was calculated [27, 28]. with the following input data (Figure 5a): frame span \( l = 1050 \) mm, frame floor height \( h = 450 \) mm, cross-bar cross-section height \( h = 100 \) mm, cross-bar cross-section width \( b = 100 \) mm. Longitudinal reinforcement of the crossbar and above the first floor is made of rods with a diameter of 6 mm. Transverse reinforcement is taken in the form of closed clamps made of wire with strength equal to A240 with diameter of 2 mm. The pitch of the transverse reinforcement on the support quarters of the span is taken as 40 mm in the middle part of the span - 60 mm. The concrete of the frame structure is B15. The frame is loaded with two concentrated forces applied to the girders in each span. Frame girders above the first floor are loaded with the same forces, but applied with \( e_1 \) eccentricity (see Fig. 1a).

The design analysis of the frame was carried out according to the diagrams of two levels. On the first level, as a rod system, using a frame model with rigid insert nodes. At the second stage, according to the secondary design scheme, a frame fragment was calculated that simulates the zone of possible local destruction in the form of a crossbar adjacent to the removed column (see Fig. 1a) using the design scheme (see Fig. 3a). As a result of the calculation for the system under consideration, the design parameter of \( \lambda \) survivability (curve 1) was calculated at two loading stages: the first at a given static load, the second at instantaneous loading of the frame caused by the removal of the extreme column (Figure 5b). Here is also a diagram of the change in the survivability parameter, based on the results of tests of the considered frame design under the same loading mode (curve 2). The design pattern of destruction of the reinforced concrete frame model obtained during \( \lambda=1.8 \) corresponded to the experimental one (Figure 5c).

It can be seen that with the accepted design hypotheses, the proposed method for calculating the survivability parameter of frames with complex stressed girders satisfactorily describes the process of deformation and destruction of the structural system in the beyond-limit state under the considered special effect. The difference in the experimental and design value of the parametric load and, accordingly, the parameter of survivability of the \( \lambda \) causing its local destruction in the zone adjacent to the removed column in the considered calculation example was 17.7%.
Fig. 5. Formwork and reinforcement scheme of reinforced concrete frame (a); experimental and design dependence of the generalized support reaction of the crossbar over the remote corner column $R_{sup}$ on the survivability parameter $\lambda$ (b); general view of frame failure (c)

4 Conclusions

1. Proposed method of determining survivability parameter of physically and structurally nonlinear reinforced concrete frame structural systems of frames of multi-storey buildings with elements experiencing complex resistance - bending with torsion at static-dynamic mode of their loading with special effect.

2. With the use of an extraordinary mixed method of structural mechanics of rod systems, design dependencies are obtained to determine the parametric load, at which, in the most stressed spatial section of frame elements, under the considered loading mode, the criteria of a special limit state are achieved and, as spatial plastic hinges are formed sequentially, the transition from a statically indeterminate frame system to a geometrically variable one occurs.

3. Comparison of the results of calculation of survivability parameters obtained according to the proposed method with the results of experimental determined survivability parameters showed satisfactory (17.7%) their agreement and confirmed the possibility of using the proposed method for practical calculations of survivability of frames of multi-storey buildings with complex stressed elements.

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