

# Probabilistic model of corrosion damage in bent reinforced concrete elements

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**Abstract.** When designing reinforced concrete structures operated in aggressive environments, there is a need to take into account corrosion damage and assess the overall reliability and durability of the structure. The calculation of reinforced concrete elements taking into account the corrosion of concrete and reinforcement is complicated by the stochastic nature of the parameters of corrosion processes. The conducted research is aimed at solving the urgent problem of improving the methodology of the probabilistic approach to the calculation of reinforced concrete elements under environmental and force influences. The article provides a probabilistic assessment of the reliability and durability of flexural reinforced concrete elements damaged by corrosion using a nonlinear deformation model of reinforced concrete. To take into account corrosive wear, well-known concrete deformation diagrams were used, taking into account changes in strength characteristics with increasing concentration of an aggressive environment. Reliability assessment was carried out in a specialized software package using the statistical modeling method. Based on the research results, graphs of changes in the reliability of a bendable reinforced concrete element damaged by corrosion over time were constructed.

**Keywords:** corrosive wear, statistical modeling, reliability, durability, nonlinear deformation model.

## Introduction

When assessing the reliability and durability of bent reinforced concrete elements under the influence of an aggressive environment, it is important to choose a calculation model that takes into account the development of corrosion processes. There are two methods used to model the corrosion of concrete and reinforcement: mathematical and physical-chemical. In the mathematical model, one of the possible functional dependencies of the corrosion rate, the time of action of an aggressive medium and the depth of the damaged layer are specified

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with the coefficients of operating conditions that determine the development of corrosion processes. The physicochemical model takes into account the individual experimental dependences of the mechanical properties of the material on the parameters of the aggressive environment obtained under specific operating conditions. Working with a physicochemical model involves the collection and processing of a large amount of experimental data, and strength calculations are not used in practice. The main works on the study of corrosion processes use various mathematical models.

At various times, this issue was dealt with by V.M. Bondarenko, E.A. Guzeev, A.S., Zalesov, V.I. Kolchunov, V.M. Moskvina, I.G. Ovchinnikov, A.I. Popesko, V.I. Rimshin, V.P. Selyaev, V.I. Solomatov, V.I. Travush and others [1-14]

Accounting for corrosion damage in the calculations of reinforced concrete structures is complicated by the fact that most of the design parameters are random. For most of the calculated parameters, there are no experimental data on the statistical variability of their characteristics. The accumulation of initial information makes it possible to use deterministic methods of calculation, taking into account the reliability factors determined by the variability of the initial values, or to apply the method of direct calculation of reliability. The main approaches to assessing the reliability of building structures were developed in the works of V.V., Bolotin, A.S. Lycheva, V.D. Reiser, B.S. Rastorgueva, A.R. Rzhantsyna, N.N. Skladneva, B.I. Snarkis, Yu.D. Sukhova, A.P. Kudzisa, A.G. Tamrazyan, V.S. Utkina, V.P. Chirkova and others [15-19]

During the impact of an aggressive environment on a reinforced concrete element, a number of independent processes occur, which can be taken into account separately:

1. The initial impact of an aggressive environment on the outer surface of the element without changing the stress-strain state from the acting load.
2. Diffusion of an aggressive environment into the structure of concrete with a change in its properties and stress-strain state.
3. Increasing the concentration of an aggressive environment near the surface of the working reinforcing bars to a critical level, which contributes to the onset of corrosion processes in the reinforcement.
4. Development of corrosion processes in reinforcement, leading to a decrease in its effective cross-sectional area. Change in the stress-strain state of the element due to the development of degradation processes in concrete and reinforcement corrosion.
5. The onset of the limit state due to the development of excessive deformations in concrete or reinforcement.

All these processes are random in nature and depend on the type, concentration and duration of exposure to an aggressive environment.

## Metod

The condition for ensuring reliability in a general form can be expressed through the inequality:

$$P_f(t) = P(\tilde{R}(t) - \tilde{Q}(t)) \geq [P_f]$$

where  $P_f(t)$  – probability of failure-free operation,  $\tilde{R}(t)$  – material strength characteristic,  $\tilde{Q}(t)$  – load characteristic,  $[P_f]$  – regulatory reliability level,  $t$  – building operating time.

The impact of aggressive environmental influences on the properties of concrete can be represented as a nonlinear diagram of concrete deformation, taking into account the concentration of the aggressive environment. The deformation diagram can be represented as a function (1), linking the initial modulus of elasticity of concrete  $E_b$ , the concentration of the aggressive medium  $C_x$  and deformation  $\varepsilon_b$  at each point of the element section:

$$\sigma_b = f(E_b, \nu_b, \varepsilon_b, C_x) \tag{1}$$

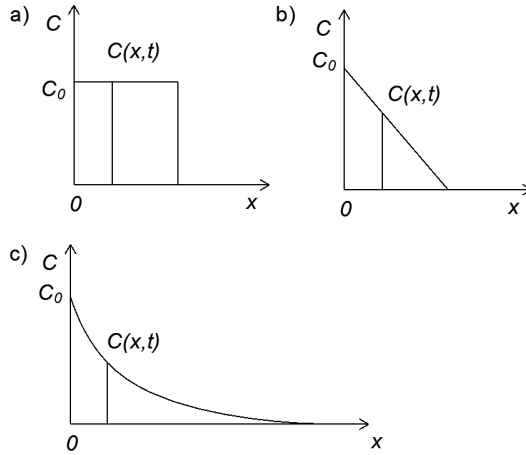
Curves of the deformation diagram can be obtained by experimental methods, by testing concrete samples at different concentrations of an aggressive medium in the material. In reality, all coefficients included in dependence (1) are random in nature, which involves processing a large amount of experimental data and determining the probability characteristics of all random variables.

$$\tilde{\sigma}_b = f(\tilde{E}_b, \tilde{\nu}_b, \tilde{\varepsilon}_b, \tilde{C}_x)$$

Under the influence of an aggressive environment on the reinforcement, corrosion processes develop, reducing its effective cross section and changing the properties of steel, and its embrittlement occurs. The change in the properties of steel during corrosion processes has little effect on the overall reliability of the structure, and it can be neglected when constructing a probabilistic model of the corrosion process. The steel deformation diagram, taking into account the stochastic nature of the physical and mechanical characteristics, can be written:

$$\tilde{\sigma}_s = \begin{cases} \tilde{E}_s \cdot \tilde{\varepsilon} & \text{at } \sigma < \tilde{\sigma}_T \\ \tilde{\sigma}_T & \text{at } \sigma \geq \tilde{\sigma}_T \end{cases}$$

During the diffusion of an aggressive substance into the porous structure of concrete, its concentration inside the material increases, chemical interaction with the components of the cement stone, changes in physical and mechanical characteristics, deformation and destruction occur. Under the influence of an aggressive environment in concrete, a regularity of distribution of the concentration of the medium over the material is formed, which also determines the law of distribution of the physical and mechanical characteristics of concrete. Currently, several models are used to describe the process of diffusion of an aggressive medium in concrete [5]: distribution of an aggressive medium by a clear front, penetration by a diffuse front, and penetration by the diffusion mechanism. To assess the reliability, the mathematically most convenient model is the model of penetration of an aggressive medium by a diffuse front.



**Fig. 1.** Scheme of penetration of an aggressive environment into concrete

a) a clear front; b) blurred front; c) diffusion penetration.

According to experimental studies [5,6], it was found that after the start of the interaction of the structure with an aggressive environment, the corrosion of reinforcement in concrete does not begin immediately, but after reaching a concentration of an aggressive environment near its surface of a certain critical value  $C_{kr}$ . The spread of an aggressive medium through an element can be described by Fick's second law in terms of the diffusion coefficient  $D$ , exposure time, and surface concentration  $C_s$ . The time after which the concentration of the aggressive medium reaches the value of  $S_{kr}$  is called the incubation period. In the case of using the model of propagation of an aggressive environment by a diffuse front, the incubation period at a known value of the critical value  $C_{cr}$  can be found by formula (2):

$$t_{inc} = \frac{\tilde{a}_s}{\left( 8 \cdot \tilde{D} \cdot \left( 1 - \frac{\tilde{C}_{kr}}{\tilde{C}_s} \right)^2 \right)} \quad (2)$$

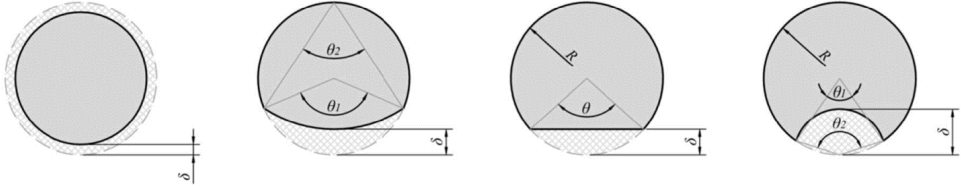
where  $a_s$  – is the thickness of the concrete protective layer.

The development of reinforcement corrosion processes in concrete can be described by several mathematical models that take into account the time of exposure to an aggressive environment, the thickness of the protective layer, the depth of corrosion, and the empirical damage parameter [7]. We accept the corrosion model, taking into account the stochastic nature of all quantities, in the form:

$$\begin{cases} \tilde{\delta} = 0 & \text{at } t < \tilde{t}_{inc} \\ \tilde{\delta} = \tilde{k} \cdot (t - \tilde{t}_{inc})^{\tilde{n}} & \text{at } t \geq \tilde{t}_{inc} \end{cases}$$

where  $\tilde{k}$  and  $\tilde{n}$  – random coefficients obtained empirically,  $\tilde{\delta}$  – the depth of corrosion damage.

The development of corrosion processes in steel reinforcement [12] can proceed in the form of continuous uniform corrosion along the entire perimeter of the rod, in the form of pitting corrosion and in the form of uneven corrosion with a flat or sickle-shaped front (Figure 2).



**Fig. 2.** Types of corrosive wear of the reinforcing bar

With this in mind, the development of corrosion processes can be described as a random function:

$$\tilde{F}_s(t) = f(\tilde{d}_0, \tilde{k}, \tilde{n}, \tilde{t}_{inc}, t)$$

where  $\tilde{d}_0$  - is the initial reinforcement diameter.

With uniform corrosion wear and the stochastic nature of the initial parameters, the corrosion development function can be described as:

$$\tilde{F}_s(t) = \begin{cases} \frac{\pi \cdot \tilde{d}_0^2}{4} & \text{at } t \leq \tilde{t}_{inc} \\ \frac{\pi \cdot (\tilde{d}_0 - 2\tilde{\delta}(t))^2}{4} & \text{at } t > \tilde{t}_{inc} \end{cases}$$

The onset of the limiting state for each point of the section of the element is determined by the achievement of limiting deformations in concrete or reinforcement:

$$\begin{aligned} |\varepsilon_{b,max}| &\leq \varepsilon_{b,ult} \\ \varepsilon_{s,max} &\leq \varepsilon_{s,ult} \end{aligned}$$

The calculation of normal sections, taking into account individual deformation curves, is carried out according to a nonlinear deformation model based on the equations of equilibrium of external and internal forces, the hypothesis of flat sections, as well as the equations of distribution of deformations over the section [10] according to the formulas:

$$\begin{aligned} \tilde{M}_x &= \sum \tilde{\sigma}_{bi} \cdot \tilde{A}_{bi} \cdot \tilde{Z}_{bxi} + \sum \tilde{\sigma}_{sj} \cdot \tilde{A}_{sj} \cdot \tilde{Z}_{sxj} \\ \tilde{M}_y &= \sum \tilde{\sigma}_{bi} \cdot \tilde{A}_{bi} \cdot \tilde{Z}_{byi} + \sum \tilde{\sigma}_{sj} \cdot \tilde{A}_{sj} \cdot \tilde{Z}_{syj} \\ \tilde{N} &= \sum \tilde{\sigma}_{bi} \cdot \tilde{A}_{bi} + \sum \tilde{\sigma}_{sj} \cdot \tilde{A}_{sj} \\ \tilde{\varepsilon}_{bi} &\approx \tilde{\varepsilon}_0 + \frac{1}{\tilde{r}_x} \cdot \tilde{Z}_{bxi} + \frac{1}{\tilde{r}_y} \cdot \tilde{Z}_{byi} \\ \tilde{\varepsilon}_{sj} &\approx \tilde{\varepsilon}_0 + \frac{1}{\tilde{r}_x} \cdot \tilde{Z}_{bxj} + \frac{1}{\tilde{r}_y} \cdot \tilde{Z}_{byj} \end{aligned}$$

To calculate the reliability of the design, various mathematical methods are used (the method of two moments, the linearization method, the hot spot method, the enumeration

method, the statistical modeling method, etc.). In calculations on a PC, the most applicable method is statistical modeling, which consists in artificially modeling the normal or other distribution of all random parameters in order to calculate the safety margin according to the established deterministic dependencies. The probability of obtaining a negative result of the safety margin determines the probability of failure of the structure, its reliability and durability. To calculate the durability of a reinforced concrete element, it is enough to determine the reliability for each time period and compare it with the normalized indicator.

## Results and discussion

To implement the proposed method of taking into account corrosion damage in the calculation of a nonlinear deformation model, a program for the PC VNORM was developed in the VBA language. The program allows you to calculate the reliability and bearing capacity of bent reinforced concrete elements and uses the library of built-in MS Excel functions in its work. The program takes into account the differential application of the load to carry out nonlinear calculations of the bearing capacity of the element based on the nonlinear deformation model. Modeling the distribution of random parameters is performed according to the normal law using a random number generator from the standard library of VBA functions.

Taking into account all the above hypotheses in the VNORM program, calculations were made for a cantilever beam with a section of 250x500 mm and an overhang of 2.5 m, loaded with a uniformly distributed load. The beam is reinforced with frames with working reinforcement of a periodic profile of class A500C. The beam along the entire perimeter of the section is under the influence of 1% chloride-containing medium during the entire period of operation. To calculate the reliability of bent reinforced concrete elements, the statistics of the distribution of all random parameters of structures is given. When determining the coefficients  $\nu$  of the concrete deformation curve, experimental data obtained from testing samples aged in chloride-containing media [4] were used.

To calculate the reliability and durability of a reinforced concrete bending element, taking into account the development of corrosion processes, the following algorithm is used:

1. Performing a static calculation of the structure, determining the forces in the dangerous section of the beam.
2. Dividing the beam section into sections in height and width into fragments. The reinforcement in the section is modeled without division into sections.
3. Conducting a nonlinear calculation of the cross section with a differential load application for the accepted sample of random design parameters. Determination of stresses and strains in each section of the section. Comparison of deformations in concrete and reinforcement with limiting ones.
4. Conducting repeated nonlinear calculations for the selected array of samples of random parameters using the method of statistical modeling. Accumulation of statistical information about the margin of safety of the element. Calculation of the initial reliability of the element  $Pf_0$  without taking into account corrosion processes by the number of design failures.
5. Establishment of the time step for the calculation of corrosion processes. Determination of the concentration of an aggressive medium in each section of the section at each moment of time.

6. If the concentration of an aggressive medium reaches a critical value at the location of the reinforcing bars at a given point in time, the corrosion depth and the residual cross-sectional area of the reinforcing bar are calculated.

7. Repetition of the nonlinear calculation with a selected array of random structural parameters and the development of corrosion processes in concrete and reinforcement.

8. Plotting graphs for the decrease in the reliability of the element over time, taking into account the development of corrosion processes. Determination of the durability of a reinforced concrete element with a decrease in the level of reliability below the standard indicator.

Based on the results of calculating the reliability of a bent reinforced concrete element, taking into account the impact of an aggressive environment, a graph of the dependence of reliability on the time of operation of the structure was plotted (Figure 3). It has been established that the reliability of the element, which initially has a significant margin, decreases to the minimum allowable level  $P_f=0.9973$  within 14-20 years of operation, depending on the diameter of the reinforcing bars. The concentration of the aggressive environment reaches a critical level on the surface of the reinforcing bars after eight years of operation, which contributes to the onset of the development of corrosion processes in the reinforcement.

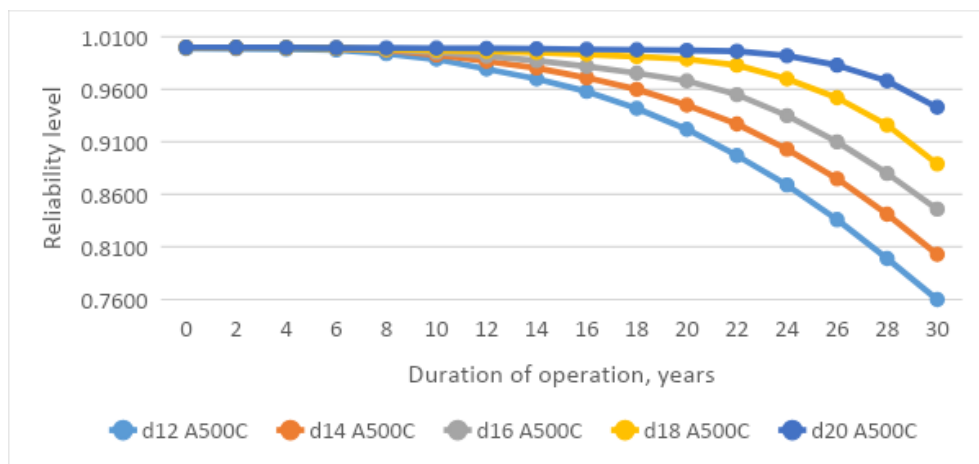


Figure 3. Reliability of cantilevered reinforced concrete element

## Conclusions

The proposed method for assessing the reliability and durability of bent reinforced concrete elements makes it possible to calculate the numerical value of reliability at any time during the operation of the structure in an aggressive environment. For the studied reinforced concrete element, the impact of an aggressive environment leads to the initialization of the corrosion processes of reinforcing bars after 8 years of operation. Further corrosion of the reinforcement and a decrease in the effective diameter of the rods, as well as the accumulation of corrosion damage in the compressed concrete, leads to a decrease in the level of reliability of the reinforced structure after 14-20 years of operation below the recommended value.

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