Calculation of deflections of bent reinforced concrete elements on the basis of a nonlinear deformation model taking into account the forces of adhesion of reinforcement with concrete

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Abstract. The purpose of this work is to numerically implement the calculation of deflections of bent reinforced concrete elements based on a nonlinear deformation model and taking into account the characteristics of reinforcement coupling with concrete. Most of the existing engineering methods for calculating bendable elements are based on approaches based on the determination of integral stiffness characteristics of the section by simplified methods. This problem acquires new relevance when using the current norms, when it is necessary to take into account the features of nonlinear deformation of materials. The article considers the application of a nonlinear deformation model in the calculation of deflections and taking into account the coupling forces of reinforcement with concrete.When solving systems of equilibrium equations of a nonlinear deformation model, the iteration method is used. In the areas between cracks, the stresses in the reinforcement are corrected by adding the coupling forces of the reinforcement with concrete, which are included in the equilibrium equations of the normal section and are further refined. The proposed method makes it possible to use the resources of reinforced concrete structures for both the first and second groups of limit states in the calculations of anchoring, crack opening width and deflections.

Keywords. nonlinear deformation model, iteration method, reinforcement coupling with concrete

Introduction

Calculations according to the current regulatory methodology do not take into account the characteristics of the coupling of reinforcement with concrete explicitly and are accepted in the form of separate empirical coefficients [2,3]. At the same time, currently metallurgical plants produce rolled products with a sufficiently high controlled coupling parameter.

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This circumstance makes it possible to use the resources of reinforced concrete structures for both the first and second groups of limit states in the calculations of anchoring, crack opening width and deflections. This problem is especially relevant when using the current norms, when it is necessary to take into account the features of nonlinear deformation of materials and the transfer of forces from reinforcement to concrete. The problems of coupling rebar fittings are considered in some detail in [5-20].In the proposed work, the calculation of deflections of reinforced concrete elements is considered on the basis of a nonlinear deformation model taking into account the coupling forces of reinforcement with concrete. To solve the equations in a physically nonlinear formulation, the iteration method was used in combination with Microsoft Excel and VBA [1,4,21].

Methods

The bent element is divided into separate sections, on each of which the stiffness coefficients calculated using a nonlinear deformation model are calculated. The dimensions of the element along the length in the first approximation should be such that an integer number of sections are located at a length equal to the distance between the cracks. At each site in its middle, an equation of the equilibrium of forces in concrete and reinforcement materials, internal forces from external load is compiled. Moreover, if the site is located between the cracks, then the coupling forces are added to the equilibrium equation. The calculation scheme is given in Figure 1.

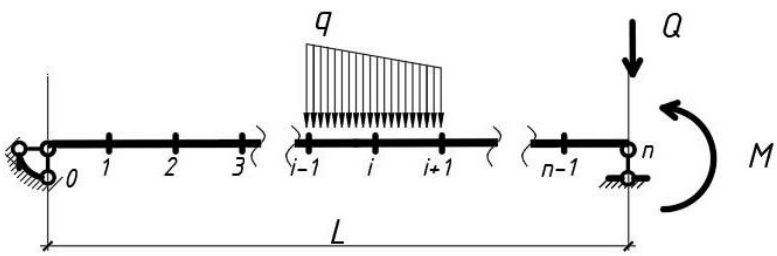

Fig. 1. Calculation scheme

When solving systems of equilibrium equations of a nonlinear deformation model, the iteration method is used. In the section where the crack has formed, the system of equilibrium equations (1)-(2) is solved:

$$D_{11}\frac{1}{\rho_x}+D_{13}\varepsilon_0=M_x, \qquad (1)$$

$$D_{31}\frac{1}{\rho_x}+D_{33}\varepsilon_0=N, \qquad (2)$$

$\frac{1}{\rho_x}$ - the curvature of the beam in the section under consideration,

ε_0 - relative axial deformation under compression (tension),

D_{11}, D_{13}, D_{33} – coefficients determined by the method [2,21],
The solution at each step can be written:

$$\left(\frac{1}{\rho_x}\right)_j^{(k)} = \frac{M_{x,j}}{D_{11,j}^{(k-1)}} - \left(\frac{D_{13,j}^{(k-1)}}{D_{11,j}^{(k-1)}}\right)\left(\varepsilon_0\right)_j^{(k-1)}, \quad (3)$$

$$\left(\varepsilon_0\right)_j^{(k)} = \frac{N}{D_{33,j}^{(k-1)}} - \left(\frac{D_{31,j}^{(k-1)}}{D_{33,j}^{(k-1)}}\right)\left(\frac{1}{\rho_x}\right)_j^{(k-1)}, \quad (4)$$

(k)- is the iteration number,
j- is the upload step.
The convergence of the iterative process should be determined from the condition:

$$\left|D_{ii}\right| \geq \left|D_{i1}\right| + \ldots + \left|D_{i,i-1}\right| + \left|D_{i,i+1}\right| + \ldots + \left|D_{in}\right|, \quad (5)$$

for all i, or at least for one i.
The calculation is carried out in the following order:
-the dimensions, concrete class, reinforcement are set,
-the cross section of the rod is divided into separate sections,
-the areas of concrete and reinforcement sections are set, A_{bij}, A_{sij}
--the origin of coordinates (for example, at the top) and the coordinates of the center of gravity of each section are set x_{ij}, y_{ij},
-the initial curvature, relative deformations and the coordinate of the neutral layer are assigned (usually from the conditions of elastic work of materials). For example, for a rectangular section in the first iteration step:

$$\left(\frac{1}{\rho_x}\right)^{(0)} = \frac{M_x}{E_b \cdot J_{b,x}}, \quad (6)$$

$$\left(\varepsilon_0\right)^{(0)} = \frac{N}{E_b \cdot A_b}, \quad (7)$$

$$z_{y0j}^{(0)} = -\frac{h}{2}, \quad (8)$$

$$z_{x0i}^{(0)} = -\frac{b}{2}, \quad (9)$$

$J_{b,x}$, $J_{b,y}$ - moments of inertia of the section (on concrete),

E_b -modulus of elasticity,

A_b - cross-sectional area (for concrete),

h - height of the rectangular section,

b - width of rectangular section,

z_{y0j}, z_{x0i} -position of the neutral layer,
-relative deformations on the sections are calculated

$$\varepsilon_{ij}^{(0)} = \left(\varepsilon_0\right)^{(0)} + \left(\frac{1}{\rho_x}\right)^{(0)} \cdot \left(y_{ij} + z_{y0j}^{(0)}\right), \quad (10)$$

-the stresses and secant modules of concrete and steel are calculated according to the laws

$$\sigma_{bij}\left(\varepsilon_{ij}^{(0)}\right), \ \sigma_{sij}\left(\varepsilon_{ij}^{(0)}\right), \quad (11)$$

$$E_{b,redij}^{(0)} = \frac{\sigma_{bij}\left(\varepsilon_{ij}^{(0)}\right)}{\varepsilon_{ij}^{(0)}}, \quad (12)$$

$$E_{s,redi,j}^{(0)} = \frac{\sigma_{si,j}\left(\varepsilon_{i,j}^{(0)}\right)}{\varepsilon_{i,j}^{(0)}}, \quad (13)$$

-the new position of the neutral layer is determined

$$z_{x0i}^{(0)} = -\frac{\sum_{j=1}^{m}\left(A_{bij} \cdot E_{b,redij}^{(0)} + A_{sij} \cdot E_{s,redij}^{(0)}\right) \cdot x_{i,j}}{\sum_{j=0}^{m}\left(A_{bij} \cdot E_{b,redij}^{(0)} + A_{sij} \cdot E_{s,redij}^{(0)}\right)}, \quad (14)$$

$$z_{y0j}^{(0)} = -\frac{\sum_{i=1}^{n}\left(A_{bij} \cdot E_{b,redij}^{(0)} + A_{sij} \cdot E_{s,redij}^{(0)}\right) \cdot y_{ij}}{\sum_{i=0}^{n}\left(A_{bij} \cdot E_{b,redij}^{(0)} + A_{sij} \cdot E_{s,redij}^{(0)}\right)}, \quad (15)$$

-coefficients of the system of equilibrium equations are calculated

$$D_{11}^{(0)} = \sum_{i=1}^{n}\sum_{j=1}^{m}\left(A_{bij} \cdot E_{b,redij}^{(0)} + A_{si,j} \cdot E_{s,redij}^{(0)}\right) \cdot \left(y_{ij} + z_{y0j}^{(0)}\right)^2, \quad (16)$$

$$D_{13}^{(0)} = D_{31}^{(0)} = \sum_{i=1}^{n}\sum_{j=1}^{m}\left(A_{bij} \cdot E_{b,redij}^{(0)} + A_{sj} \cdot E_{s,redij}^{(0)}\right) \cdot \left(y_{ij} + z_{y0j}^{(0)}\right), \quad (17)$$

$$D_{33}^{(0)} = \sum_{i=1}^{n}\sum_{j=1}^{m}\left(A_{bij} \cdot E_{b,redij}^{(0)} + A_{si,j} \cdot E_{s,redij}^{(0)}\right), \quad (18)$$

-new values of curvature and axial deformation are calculated at the next iteration

$$\left(\frac{1}{\rho_x}\right)^{(1)} = \frac{M_x}{D_{11}^{(0)}} - \left(\frac{D_{13}^{(0)}}{D_{11}^{(0)}}\right) \left(\varepsilon_0\right)^{(0)}, \quad (19)$$

$$\left(\varepsilon_0\right)^{(1)} = \frac{N}{D_{33}^{(0)}} - \left(\frac{D_{31}^{(0)}}{D_{33}^{(0)}}\right) \left(\frac{1}{\rho_x}\right)^{(0)}, \quad (20)$$

-calculation by formulas (10)-(20) continues until the conditions are met

$$\left|\frac{\left(\frac{1}{\rho_x}\right)^{(k)} - \left(\frac{1}{\rho_x}\right)^{(k-1)}}{\left(\frac{1}{\rho_x}\right)^{(k-1)}}\right| \le \delta, \quad (21)$$

$$\left|\frac{(\varepsilon_0)^{(k)}-(\varepsilon_0)^{(k-1)}}{(\varepsilon_0)^{(k-1)}}\right| \le \delta \quad , \qquad (22)$$

δ - the accuracy of the calculation.

The solution of the system of equations (3)-(4) is considered in more detail in [21]. An example of the section breakdown is given in Figure 2.

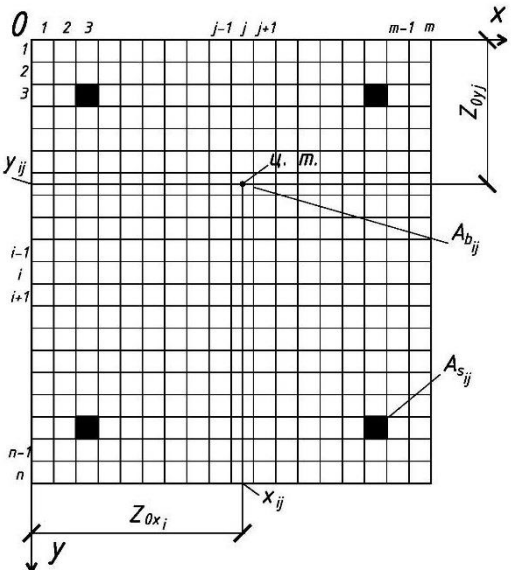

Fig. 2. Calculation of the normal cross section

In the area between the cracks, when calculating the stiffness coefficients, equation (13) will take the form

$$E_{s,red\,i,j}^{(i)} = \frac{\sigma_{si,j}\left(\varepsilon_{i,j}^{(i)}\right) - \dfrac{\tau_{c,j}^{(i)} \cdot U \cdot l_j}{A_s}}{\varepsilon_{i,j}^{(i)}} \quad , \qquad (23)$$

$\tau_{c,j}^{(i)}$ - clutch stresses,

U - the sum of the perimeters of reinforcing bars,

A_s - the sum of the areas of reinforcing bars:

$$\tau_c^{(i)} = B\frac{\ln(1+\alpha(u_{j,s}^{(i)} - u_{j,b}^{(i)}))}{1+\alpha(u_{j,s}^{(i)} - u_{j,b}^{(i)})} \quad , \qquad (24)$$

B, α - coefficients determined experimentally,

$u_{i,s}$ - elongation of reinforcing steel in the middle of the section

$u_{i,b}$ - elongation of stretched concrete in the middle of the site,

l_j - length of the th section,

(i) - iteration number.

The distribution of rebar displacements relative to concrete is described by the exponential function

$$(u_{j,s}{}^{(i)} - u_{j,b}{}^{(i)}) = P^{(i)} \cdot \lambda \cdot e^{-\lambda x_j}, \qquad (25)$$

P - the parameter of the shear force in the armature, determined experimentally,

λ - the coupling parameter of reinforcement with concrete, determined experimentally,

x_i - the coordinate of the middle of the area between the cracks.

Additionally, we calculate the unloading moment from the coupling forces in the cross section.

After determining the stiffness coefficients in each section, the displacement is calculated by integrating the values in the sections of unit moments and the calculated curvature by the numerical method of trapezoids. It is possible to calculate deflections by the method of initial parameters.

Results

For the numerical implementation of the methodology, we determine the deflection of the floor slab of a civil building (example 44 [2]) of rectangular cross-section with dimensions h = 20 cm, b = 100 cm; h0 = 17.3 cm; span l = 5.6 m; heavy concrete of class B15 (Eb = 24000 MPa; Rb, ser = 11 MPa, Rbt,ser = 1.1 MPa); stretched reinforcement of class A400 (Es = MPa) with a cross-sectional area of As = 7.69 cm2 (5Ø14); determine residual deformations from constant and prolonged loads ql = 3.25 kN/m; Diagrams are accepted according to Appendix [3].

The values of the limiting relative deformations under the short-term action of the load were assumed to be equal to: εb0=-0.002, εb2=-0.0035, εbt0=0.0001, εbt2=0.00015. Under the prolonged action of the load, the values of the limiting relative deformations are: εb0,l=-0.0034, εb2,l=-0.0048, εbt0,l=0.00024, εbt2,l=0.00031. The deformation diagram for steel is adopted according to the bilinear law. The distances between the cracks are assumed to be equal to 400 mm.

The displacement of the reinforcement relative to the concrete is taken according to the exponential function shown in Figure 3.

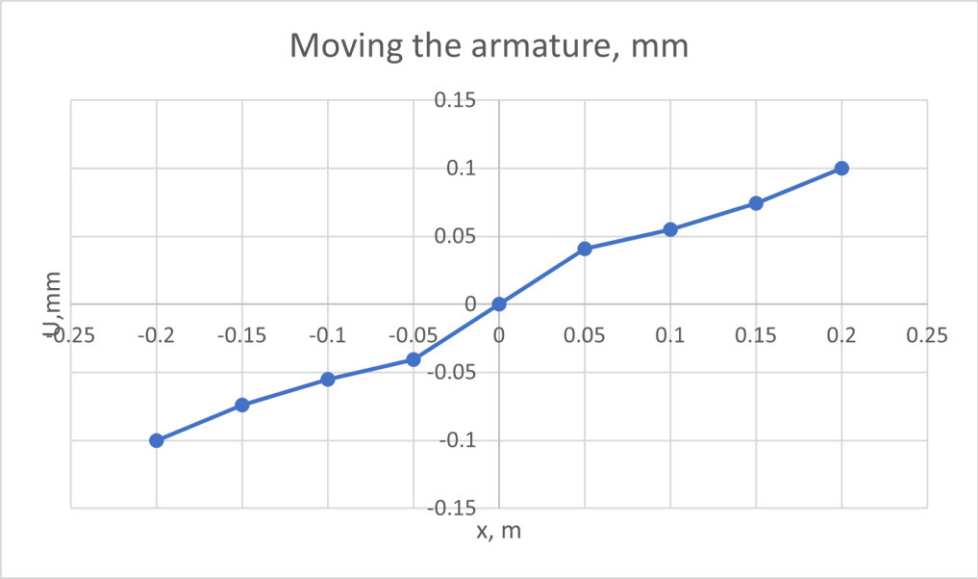

Fig. 3. Displacement of reinforcement relative to concrete

An example of the distribution of contact voltages is given in Figure 4.

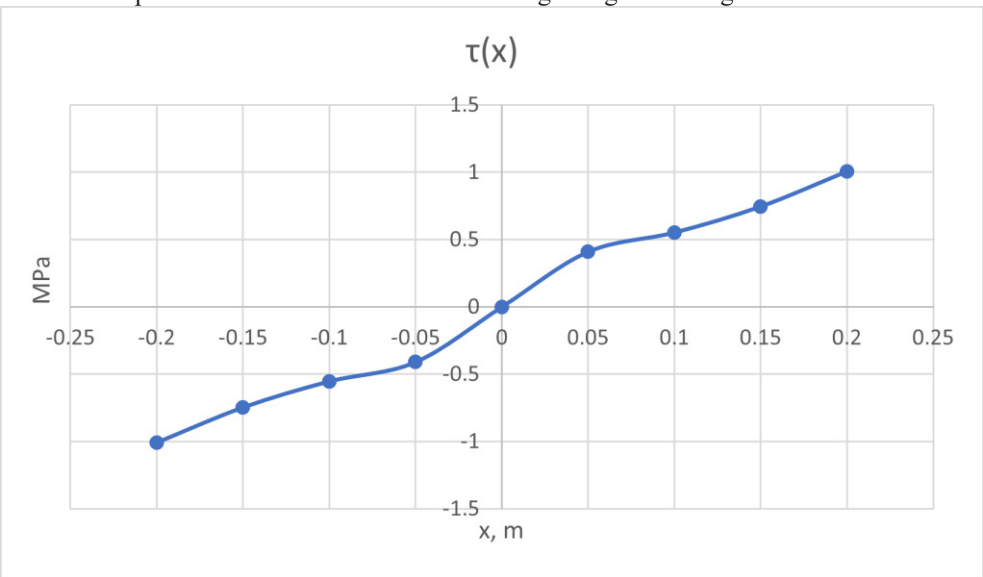

Fig. 4. Distribution of contact voltages

To automate the calculation, a program was written in the VBA language. According to the compiled program, we build the dependence of the load on the movements $q(v)$. The dependency was constructed for numerical analysis $q(v)$ based on nonlinear finite element calculations with the same diagrams of materials. The calculation results are given in Figure 5 and in Table 1.

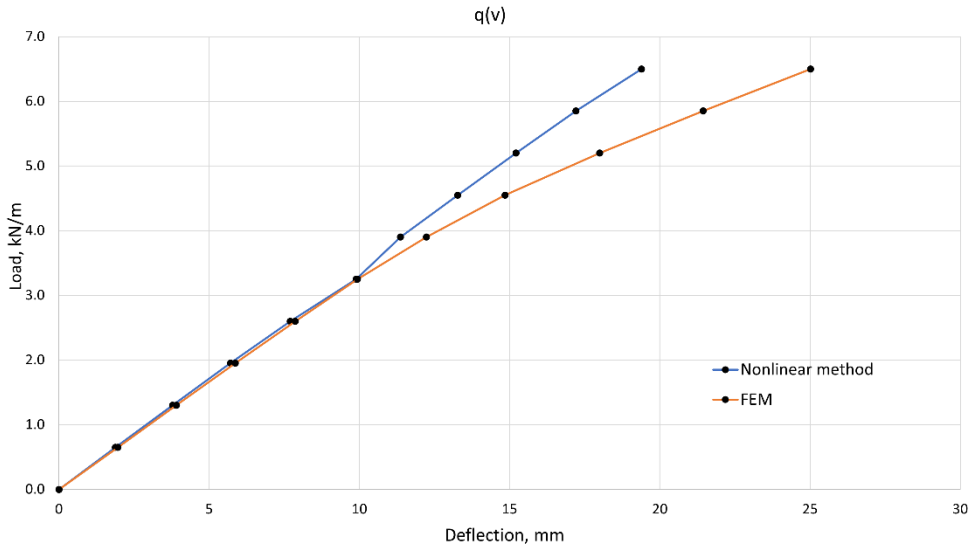

Fig. 5. Deflections of the beam

Table 1. Comparison of calculation results

Calculation method	Deflection, mm
SoR 63.13330.2018	26,34
FEM	25,02
Nonlinear method	19,39

Discussion

Calculations have shown that taking into account the increased rigidity makes it possible to identify reserves of rigidity of the bent elements. With the help of the developed methodology, it is possible to solve the practical problem of determining deflections, taking into account the physical nonlinearity and the actual characteristics of the coupling of the reinforcement. It is important to note that the step of splitting the structure into sections and taking into account the uneven stresses between cracks in the calculation of stiffness is of great importance.

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