Deformation and fracture of a thick-walled pipeline made of nonlinearly elastic material

Vladimir Mavzovin,[1] *Vadim* Karakhanyan[2,*], *Igor* Ovchinnikov[3]

[1] Moscow State University of Civil Engineering, 129337, Yaroslavskoye Shosse 26, Moscow, Russia
[2] Yuri Gagarin State Technical University of Saratov, Department of Transport Facilities, 410054, Polytechnicheskaya st 77, Saratov, Saratov Oblast, Russia
[3] Tyumen Industrial University, 625000, Volodarskogo st 38, Tyumen, Russia

Abstract. The problem of calculating the stress-strain state of a thick-walled pipeline subjected to the joint action of force load, temperature and corrosion wear is considered. The influence of temperature on the mechanical properties of the pipeline material, as well as on the rate of corrosion wear are taken into account. Such relations of material deformation model and corrosion wear model are used, which allowed to obtain an analytical solution. The analytical solution will allow to perform a timely and correct analysis of the stress state of a thick-walled pipeline, to determine the value of the ultimate pressure for a given pipe wall thickness, to determine the ultimate service life of the pipeline at known initial parameters, limit condition and corrosion wear regularities.

Key words: thick-walled pipeline, temperature effect, corrosion wear, pipeline durability, analytical solution

1 Introduction

The wall thickness of the pipeline depends on the internal pressure, but corrosion wear can vary and therefore this effect must be taken into account when calculating the pipeline. Corrosion wear leads to thinning of pipeline walls and therefore stresses in pipeline walls increase, and the use of linear physical relations may be incorrect, as a result, the calculation requires the use of physical relations accounting for the non-linearity of the material.

The problems of calculating the stress state and durability of pipeline and plate elements of structures subjected to corrosion wear using numerical methods were considered in [1,2,3].

Here we deal with the problem of constructing an analytical solution for determining the stress state of a pipeline made of nonlinear elastic material subjected to corrosive wear. The analytical solution will allow us to perform timely and correct analysis of the stress state of a thick-walled pipeline, to determine the value of the ultimate pressure for a given pipe wall

* Corresponding author: vadimkarakhanyan@mail.ru

thickness, or to determine the ultimate service life of the pipeline at known initial parameters, limit condition and corrosion wear regularities.

2 Calculations

Consider a thick-walled pipeline, whose inner and outer radii are respectively $r_{вн}$ and $r_{нар}$, is under pressure: inner $P_{вн}$ and outer $P_{нар}$. The pipeline material is nonlinear-elastic, and the relationship between stress σ_u and strain ε_u intensities is described by a dependence containing four parameters:

$$\sigma_u = a\varepsilon_u^k - b\varepsilon_u^m.$$

Under such conditions, there are no tangential stresses in the pipeline, and the relations between the components of stress and strain tensors are as follows:

$$\varepsilon_r = \frac{\sigma_r - \nu\left(\sigma_f + \sigma_z\right)}{\psi\left(\varepsilon_u\right)}, \quad \varepsilon_f = \frac{\sigma_f - \nu\left(\sigma_r + \sigma_z\right)}{\psi\left(\varepsilon_u\right)}, \quad \varepsilon_z = \frac{\sigma_z - \nu\left(\sigma_r + \sigma_f\right)}{\psi\left(\varepsilon_u\right)}.$$

Here *r, f, z* are cylindrical coordinates; $\varepsilon_r, \varepsilon_f, \varepsilon_z$ - components of the strain tensor; $\sigma_r, \sigma_f, \sigma_z$ - components of the stress tensor; ν - transverse deformation coefficient; $\psi\left(\varepsilon_u\right) = \frac{\sigma_u}{\varepsilon_u} = a\varepsilon_u^{k-1} - b\varepsilon_u^{m-1}$ - strain intensity function.

In this case, the stress and strain intensities will be determined by the expressions:

$$\sigma_u = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_r - \sigma_f\right)^2 + \left(\sigma_f - \sigma_z\right)^2 + \left(\sigma_z - \sigma_r\right)^2},$$

$$\varepsilon_u = \frac{\sqrt{2}}{3}\sqrt{\left(\varepsilon_r - \varepsilon_f\right)^2 + \left(\varepsilon_f - \varepsilon_z\right)^2 + \left(\varepsilon_z - \varepsilon_r\right)^2}.$$

and the average strain is equal to:

$$\varepsilon_0 = \frac{\left(\varepsilon_r + \varepsilon_f + \varepsilon_z\right)}{3}.$$

If the pipeline is pinched in the ground or at the ends, there are no longitudinal movements in the pipeline, hence:

$$\varepsilon_z = 0.$$

Assuming also that the pipeline material is incompressible, i.e. there are no volumetric deformations, we write:

$$\varepsilon_0 = 0.$$

Taking into account (5), (6) and (7), we obtain:

$$\varepsilon_f = -\varepsilon_r .$$

Substituting here the expressions ε_r and ε_f from (2) we have:

$$\sigma_z = \frac{\sigma_r(1-\nu)}{2\nu} + \frac{\sigma_f(1-\nu)}{2\nu}.$$

Substituting (9) into (3), we obtain:

$$\sigma_u = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_r - \sigma_f\right)^2 + \left(\frac{\sigma_f(3\nu-1)}{2\nu} - \frac{\sigma_r(1-\nu)}{2\nu}\right)^2 + \left(\frac{\sigma_r(3\nu-1)}{2\nu} - \frac{\sigma_f(1-\nu)}{2\nu}\right)^2}.$$

Under the hypothesis of incompressibility, $\nu = 0.5$, and hence (10) is transformed to the form:

$$\sigma_u = \frac{\sqrt{3}}{2}\left(\sigma_r - \sigma_f\right).$$

From (4), taking into account (6) and (8), we obtain:

$$\varepsilon_u = \frac{2}{\sqrt{3}}\varepsilon_r .$$

The equilibrium of the pipeline element is described by Eq:

$$\frac{d\sigma_r}{dr} + \frac{\left(\sigma_r - \sigma_f\right)}{r} = 0,$$

and the equation of continuity of deformations takes the form:

$$\frac{d\varepsilon_f}{dr} = \frac{\left(\varepsilon_r - \varepsilon_f\right)}{r}.$$

Since given (8) $\left(\varepsilon_r - \varepsilon_f\right) = -2\varepsilon_f$, then (13) takes an integrable form:

$$\frac{d\varepsilon_f}{dr} = -\frac{2\varepsilon_f}{r}.$$

Integrating, we obtain $\varepsilon_f = \frac{C}{r^2}$, where C is an arbitrary constant. Taking into account (8) $\varepsilon_r = -\frac{C}{r^2}$, and

$$\varepsilon_u = -\frac{2C}{r^2\sqrt{3}}.$$

Equation (13) is transformed to the form:

$$\frac{d\sigma_r}{dr} + \frac{2\sigma_u}{r\sqrt{3}} = 0.$$

Let us use the nonlinear relation (1), assuming that the exponents "k" and "m" behave as odd numbers.

Substituting expression (16) into (1), we obtain:

$$\sigma_u = -a\frac{2^k C^k}{r^{2k} 3^{\frac{k}{2}}} + b\frac{2^m C^m}{r^{2m} 3^{\frac{m}{2}}}.$$

Finally, substituting (18) into (17) and introducing the notations:

$$A = \frac{a2^{k+1}}{3^{\frac{k+1}{2}}}, \quad B = \frac{b2^{m+1}}{3^{\frac{m+1}{2}}},$$

we obtain the following differential equation with respect to σ_r:

$$\frac{d\sigma_r}{dr} = \frac{AC^k}{r^{2k+1}} - \frac{BC^m}{r^{2m+1}}.$$

integrating which we find:

$$\sigma_r = -\frac{AC^k}{2kr^{2k}} + \frac{BC^m}{2mr^{2m}} + D.$$

Here D is also an arbitrary constant. The arbitrary constants C and D are defined through the boundary conditions:

r = $r_{вн}$, $P = -P_{вн}$; $r = r_{нар}$, $P = -P_{нар}$.

If in (1) we put $a = E$, $k = 1$, $b = 0$, we obtain Hooke's law, for which expression (21) takes the form:

$$\sigma = \frac{P_1\left[1-\left(\frac{r}{r_0}\right)^2\right] - P\left[1-\left(\frac{r}{r_1}\right)^2\right]}{\left(\frac{r}{r_0}\right)^2\left[1-\left(\frac{r_0}{r}\right)^2\right]},$$

coinciding with the Lame formula, proves the correctness of the expressions obtained above.

In case of corrosive wear of pipeline walls, it can be written

$$r_0(t) = r_0 + \delta_0(t), \quad r_1(t) = r_1 + \delta_1(t),$$

and the stress intensity has the form:

$$\sigma_u(t) = \frac{G}{\left[1-\left(\frac{r_0(t)}{r_1(t)}\right)^{2k}\right]^{\frac{1}{k}}},$$

where

$$G = \left(3k^2\right)^{\frac{1}{2k}}\left(P_1 - P_0\right)^{\frac{1}{k}} A^{\frac{k-1}{k}}$$

If we take the fracture condition in the form of $\sigma_u(t) = \sigma_{пред}$, we obtain:

$$\frac{r_0 + \delta_0(t)}{r_1 + \delta_1(t)} = \gamma,$$

where $\gamma = \left[1-\left(\frac{G}{\sigma_{пред}}\right)^k\right]^{\frac{1}{2k}}$

In the case of corrosive wear on the inside of the pipeline, the time to failure can be found $\delta_0(t) = \alpha t^s$ from (26) using the steppe model:

$$t_{\text{DEST}} = \left[\frac{\gamma\left(r_1 - r_0\right)}{\alpha}\right]^{\frac{1}{s}}$$

Let us now study the corrosion-mechanical behavior of a thick-walled pipeline made of nonlinear-elastic material, non-uniformly heated along the wall thickness, subjected to corrosion wear, the rate of which depends on the temperature level. The outer radius of the pipeline is r_2, the inner radius is r_1, the temperature of the outer surface of the pipeline is T_2, the inner surface is T_1, the inner pressure is P_1, the outer pressure is P_2.

Assuming that the heat flow is steady-state from the heat conduction equation

$$\frac{d^2T_r}{dr^2}+\frac{1}{r}\frac{dT_r}{dr}=0$$

under the boundary conditions $T_{r_2}=T_2$, $T_{r_1}=T_1$, we find:

$$T_r=T_1+\left(T_2-T_1\right)\frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)},$$

where T_r is the temperature at the points of the pipeline with coordinate ($r_1 < r < r_2$).

Corrosive wear causes the outer radius of the pipeline to decrease and the inner radius to increase:

$$r_2(t)=r_2^0-\delta(t),\quad r_1(t)=r_1^0+\delta(t),$$

and therefore, even at steady heat flow the law of temperature distribution along the wall thickness changes.

Further we will assume that corrosion wear of pipeline walls is described by the model:

$$\frac{d\delta_j}{dt}=\Phi\left(t,T,\sigma_u\right)=V_j\left(1+\gamma\sigma_u\right)e^{b\left(T_j-T_0\right)},$$

where $j = 1$ corresponds to the internal surface of the pipeline, $j = 2$ – to the external surface of the pipeline.

We will assume that the deformation diagram of the pipeline material depends on the temperature:

$$\sigma_u=\left(A\varepsilon_u^k-B\varepsilon_u^n\right)\varphi(T),$$

where A, k, B, n are coefficients, $\varphi(T)$ - function of temperature influence.

Deformations depend on stresses and temperature as follows:

$$\varepsilon_r = \frac{\sigma_r - \nu\left(\sigma_f + \sigma_z\right)}{\psi\left(\varepsilon_u, T\right)} + \alpha T,$$

$$\varepsilon_f = \frac{\sigma_f - \nu\left(\sigma_r + \sigma_z\right)}{\psi\left(\varepsilon_u, T\right)} + \alpha T,$$

$$\varepsilon_z = \frac{\sigma_z - \nu\left(\sigma_r + \sigma_f\right)}{\psi\left(\varepsilon_u, T\right)} + \alpha T,$$

where, $\psi\left(\varepsilon_u, T\right) = \frac{\sigma_u}{\varepsilon_u} = \left(A\varepsilon_u^{k-1} - B\varepsilon_u^{n-1}\right)\varphi(T)$, ν is the transverse deformation coefficient, where:

$$\nu(T) = \frac{1}{2} - \frac{\left(1 - 2\nu_0\right)\psi\left(\varepsilon_u, T\right)}{2E_0},$$

α is the coefficient of linear expansion.

Determining from the third equation (33) σ_z and substituting into the other two, we obtain:

$$\varepsilon_r = \frac{1}{\psi}\left(1 - \nu^2\right)\left(\sigma_r - \frac{\nu\sigma_f}{(1-\nu)}\right) - \nu e + \alpha T(1+\nu),$$

$$\varepsilon_f = \frac{1}{\psi}\left(1 - \nu^2\right)\left(\sigma_f - \frac{\nu\sigma_r}{(1-\nu)}\right) - \nu e + \alpha T(1+\nu),$$

$$\varepsilon_z = \nu\left(\sigma_r + \sigma_f\right) + \psi e - \alpha T\psi,$$

where $e = \varepsilon_z$ is the axial deformation.

Expressions for stress and strain intensities are expressed by form (3) and (4).

Equations of equilibrium and continuity of deformations have the form (13) and (14), and geometrical relations:

$$\varepsilon_r = \frac{dw}{dr}; \quad \varepsilon_f = \frac{w}{r}; \quad \varepsilon_z = e = const.$$

Substituting expressions (35) into (14) and considering (13) and (34), after some transformations we obtain Eq:

$$\frac{d^2\sigma_r}{dr^2}+\xi\frac{d\sigma_r}{dr}+\eta\sigma_r=\lambda,$$

in which:

$$\xi=\frac{3}{r}-\frac{2\nu\nu'}{\left(1-\nu^2\right)}-\frac{\psi'}{\psi},$$

$$\eta=-\frac{(1+4\nu)\nu'+\frac{\psi'}{\psi}\left(1-\nu-2\nu^2\right)}{r\left(1-\nu^2\right)},$$

$$\lambda=\frac{\psi\left[e\nu'-\nu'\alpha T-(1+\nu)\left(T\alpha+\alpha T'\right)\right]}{r\left(1-\nu^2\right)}.$$

The dash denotes differentiation by radius.

The force boundary conditions for equation (36) are written as:

$$r=r_1,\quad \sigma_r=-P_1;\quad r=r_2,\quad \sigma_r=-P_2.$$

To determine the value of longitudinal deformation e, the condition of equilibrium of the pipeline in the longitudinal direction is used, which has the form:

$$N=2\pi\int_{r_1}^{r_2}\sigma_z r dr=2\pi\int_{r_1}^{r_2}\left[\nu\left(\sigma_r+\sigma_f\right)+\psi e-\alpha T\psi\right]r dr.$$

From here we find:

$$e=\frac{\frac{N}{2\pi}-\int_{r_1}^{r_2}\nu\left(\sigma_r+\sigma_f\right)r dr+\int_{r_1}^{r_2}\alpha T\psi r dr}{\int_{r_1}^{r_2}\psi r dr}.$$

To solve equation (36), we apply the method of successive approximations, with the zero-approximation obtained from the linear solution:

$$\sigma_r=I+\frac{J}{r^2},\quad \sigma_f=I-\frac{J}{r^2},$$

where notations are introduced:

$$I = \frac{P_1 r_1^2 - P_2 r_2^2}{r_2^2 - r_1^2}; \quad J = \frac{\left(P_2 - P_1\right) r_1^2 r_2^2}{r_2^2 - r_1^2}.$$

3 Conclusion

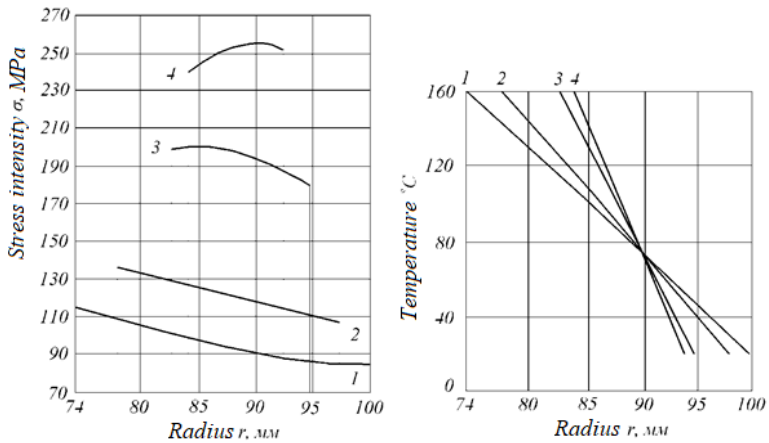

Fig.1. Stress and temperature intensity diagrams along the pipeline thickness: 1 - t = 0; 2 - t = 1 year; 3 - t = 2 years; 4 - t = 2.27 years

Using the obtained relations, the problem on the stress-strain state of a thick-walled pipeline made of nonlinear-elastic material subjected to corrosion wear in a non-uniform temperature field was solved for the following parameter values: $k = 1$;

$n = 3$; $b = 0.003$; $V_1 = V_2 = 0.2$ *mm/year*; $\gamma = 0.1$; $T_0 = 20°C$; $T_2 = 20°C$; $T_1 = 160°C$; $r_2^0 = 100$ *mm*; $r_1^0 = 74$ *mm*; $P_1 = 0$;

$P_2 = 25$ *MPa*; $A(T) = 7{,}27 \cdot 10^4 + 22 \cdot T - 0{,}37 \cdot T^2$, *MPa*; $B(T) = 8{,}87 \cdot 10^8 - 5{,}73 \cdot 10^4 \cdot T - 344 \cdot T^2$, *MPa*. The calculation results in the form of stress intensity σ_u and temperature T distribution diagrams along the pipeline thickness at different moments of time are shown in Fig. 1. At the moment of limit state onset, the pipeline dimensions were as follows: $r_2 = 94$ *mm* and $r_1 = 85$ *mm*.

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