

Accurate model for simply supported plates

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Abstract. The reliability of a model in ABAQUS depends mainly on the experience of the modeler. For the problem of elastic buckling of a simply supported thin plate in compression, except for the example in ABAQUS's user manual, which only applies to rectangular plates that are symmetrical in both directions, the current proposed models are personalized and untested. Therefore, the purpose of this paper is to verify the currently used models and thereby propose a general model for the problem. For this purpose, different models are created in ABAQUS, and their reliability will be asserted by comparing the results with the theoretical solution of Timoshenko. The critical stresses and buckling coefficients of models are the values to be obtained and used for comparison. The results show that the model outlined in ABAQUS's user manual and the model proposed in the paper match the theoretical solution, while other models have significant differences. The proposed model is more general than that of ABAQUS's manual, and it is applied to an example of a trapezoidal plate. The knowledge obtained from the proposed model can be applied to the modeling of other forms of structure.

Keywords. ABAQUS, modeling, buckling, simply supported, rectangular plate, compression

1 Introduction

With the great development of structural analysis software, many complicated structural problems can be simulated and solved with high accuracy. Therefore, the software can be used to validate the analytical solution. However, to be verification data, the results obtained from the software must be reliable, in which the user's experience in modeling is one of the decisive factors.

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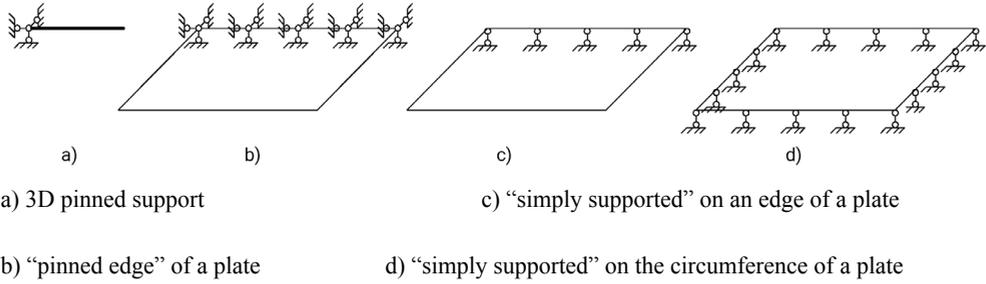


Fig. 1. Types of support

The calculation model needs to be accurate and consistent with the actual structure. Models for construction structures mainly use elements connected only through nodes, so they are comparatively easy to manage. For the plate and solid element problem using ABAQUS, the boundary condition is more complicated because it involves edges and faces. For example, the 3D translational constraint of a node in Fig. 1a, when applied to an edge in Fig. 1b, will produce a "pinned edge", similar to a fixed spindle with infinity flexural stiffness, which allows the plate to rotate around this edge, but the edge cannot display any deformation, even in the plane of the plate. This is inconsistent with the "simply supported edge" in flexural plate theories, with the exact model shown in Fig. 1c, where the "simply supported edge" would be prevented from out-of-plane displacement, while in-plane displacements are free. However, if this "simply supported edge" is applied to Fig. 1d, boundary conditions are insufficient and the plate is unstable in its plane. Therefore, how to properly model restraints as well as loads in complex ABAQUS models is a frequently asked question by structural engineers. For the problem of elastic stability of thin plates, ABAQUS, in its manual [1], conducted a verification and validation problem with a square plate under compression simply supported around the circumference. However, the applicability of this example is very limited due to the requirement that the plate be doubly axially symmetrical in terms of geometry, boundary conditions, and applied loads. Except for this guide, models established by engineers and researchers are personalized and unverified.

In order to provide more reliable experiences for engineers and researchers while modeling in ABAQUS, this paper will model the buckling problem of a thin plate simply supported around the circumference under compressive loads applied on two opposite edges, then compare the outcome with the theoretical result conducted by Timoshenko [2], thereby making comments and proposing an accurate model.

2 Some solutions for modeling the buckling problem of a thin plate simply supported around the circumference under compressive loads applied on two opposite edges

1.1 ABAQUS

ABAQUS has tested the problem of a square thin plate simply supported around the circumference under uniaxial compressive loads and presented it in *Section 1.2.4. Buckling of a simply supported square plate* of the part "Analysis Test" of the "Abaqus Benchmarks Manual" [1]. The length of the edge of the plate is taken as 2, the thickness of the plate is 0.01, and the slenderness of the plate is equal to 200. Thus, it can be considered a thin plate. Since the plate is doubly symmetrical, only one-quarter of the plate is modeled (upper-right quadrant of Fig. 2). The boundary conditions on the model are:

- On edge $x = 0$ $u_x = r_y = r_z = 0$
- On edge $y = 0$ $u_y = r_x = r_z = 0$
- On edge $x = b/2$ $u_z = r_x = 0$
- On edge $y = b/2$ $u_z = r_y = 0$

where the z axis is normal to the plane of the plate.

Loads are applied to the plate using two different methods: uniform edge loads and thermal loads.

Calculation results using many element types have been considered, such as S8R5, S8R, S9R5, STRI65, S4R5, S4R, S4, S3R, and STRI3. Results have been compared with those calculated by the theory of thin plates presented by Timoshenko [2]. Comparisons show that using different element types will not give completely consistent results. The results when using elements S8R5 and S9R5 are the most accurate; using S3R is the most misleading; with other element types, the error is acceptable.

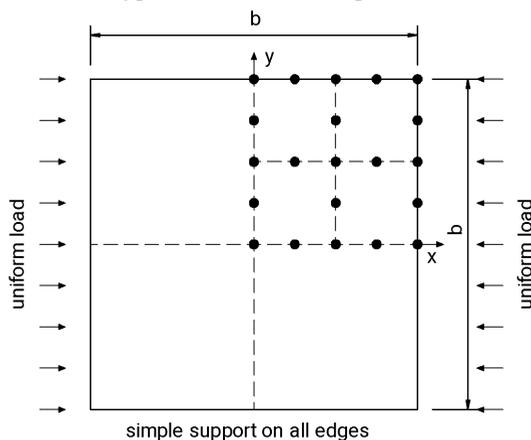


Fig. 2. Calculation diagram in the example of ABAQUS [1]

This model is only used for plates with axial symmetry because at the axis of symmetry, the in-plane displacement perpendicular to the axis is zero; thus, supports that prevent in-plane displacement can be assigned to edges associated with the axes of symmetry. Therefore, the model is not general and is not applicable to plates of any shape.

1.2 Study of Yanli Guo and Xingyou Yao on the buckling of thin plates

In the study on buckling of thin plates [3], Yanli Guo and Xingyou Yao used ABAQUS software to model those with round and rectangular holes. The outer edges are set up to have no out-of-plane displacement, while in-plane displacement and rotation are free. The two midlines passing through the hole are assigned supports that disallow in-plane displacements perpendicular to the lines, as shown in Fig. 3.

Similar to the ABAQUS example, this model also takes advantage of the feature that at symmetry axes, the in-plane displacements perpendicular to the axes are zero, so that assigns supports that restrain these displacements. Obviously, this model is again only applicable to doubly symmetrical plates.

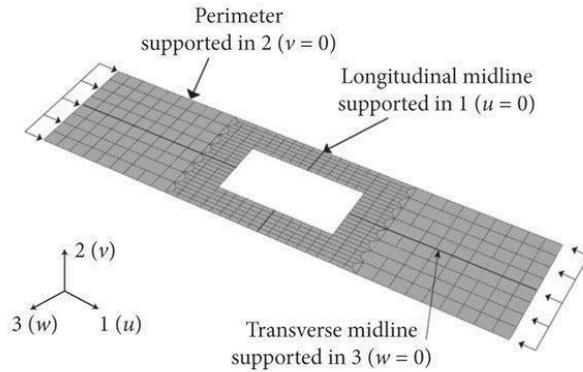


Fig. 3. Calculation diagram in the example of Yanli Guo and Xingyou Yao [3]

1.3 Study of Kanta Prajapat, Samit Ray-Chaudhuri, and Ashwini Kumar

In the study of the buckling of thin plates with edges partially restrained for in-plane movement [4], Kanta Prajapat, Samit Ray-Chaudhuri, and Ashwini Kumar used ABAQUS to model a rectangular plate with the following boundary conditions:

Loaded edge: $u \neq 0, v = 0$

The edge opposite the loaded edge: $u = 0, v = 0$

Other edges: $u \neq 0, v = 0$

where u and v are the in-plane displacements along and perpendicular to the direction of the load.

The element used in the model is S8R5, with a size of 1/40 of the plate width.

1.4 Other models

In Ronald Wagner's "ABAQUS tutorial" [5], the presenter shows how to use ABAQUS to calculate the critical stress of a thin steel plate of size 100x100 and 0.1 thick with four simply supported edges. The supports in the model are of two types: supports prohibiting out-of-plane displacement and freeing in-plane displacement as well as rotation are assigned to the four sides of the plate; supports that prevent in-plane displacement are assigned to two adjacent nodes (nodes A and B in Fig. 4). "Edge Load" is applied to two opposite sides of the plate. Ronald Wagner's model is shown in Fig. 4.

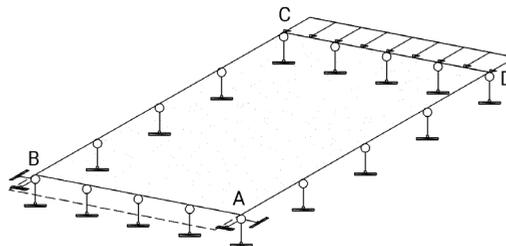


Fig. 4. ABAQUS model of Ronald Wagner

2 Methods

2.1 Buckling coefficient determination

The models in the above cases will be studied on a thin plate with dimensions $axb = 1000 \times 1000$ mm, where b is the width of the plate (the length of the loaded edge) and a is the length of the plate (the length of the non-loaded edge). The plate thickness t is 10 mm. The material is steel, with the elastic modulus $E = 210000$ MPa and the Poisson ratio $\nu = 0.3$.

According to Timoshenko [2], the critical stress of a flat plate is obtained from the formula:

$$\sigma_{cr} = \frac{N_{cr}}{t} = k \frac{\pi^2 D}{b^2 t} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \tag{1}$$

where, D is the stiffness of the plate

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{2}$$

in which k is the buckling coefficient and is tabulated with respect to the ratio of the length and the width of the plate a/b .

From Eq. 1

$$k = \frac{12(1-\nu^2)}{\pi^2 E} \left(\frac{b}{t}\right)^2 \sigma_{cr} \tag{3}$$

Eq. 3 is used to calculate the coefficient k for the studied cases, and the results will be compared with the theoretical value of Timoshenko. For a square plate, the coefficient k , according to the theoretical analysis, has a value of 4.0.

2.2 Model selection

In addition to the three models of ABAQUS, Yan Li Guo, and Ronald Wagner, five more models were used for comparison.

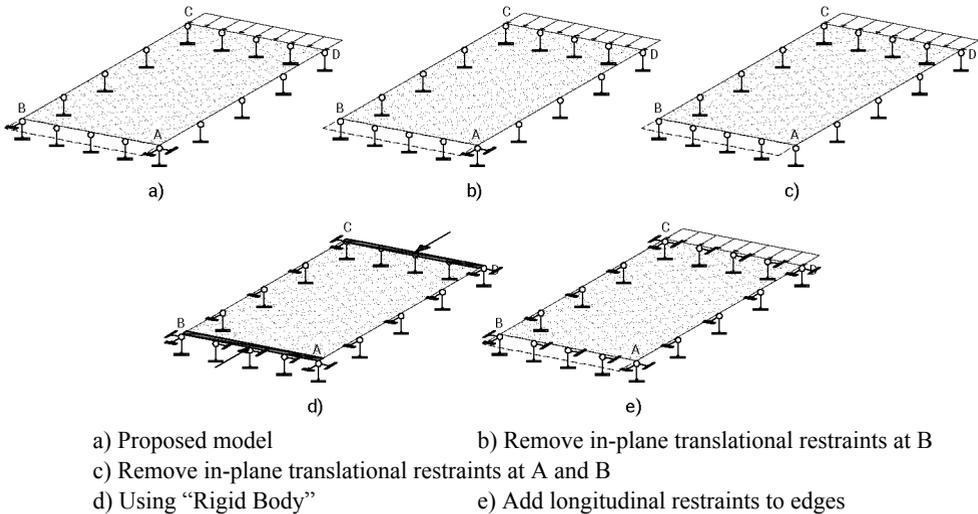


Fig. 5. Investigated models

The first model proposed in this paper is based on the model of Ronald Wagner, with some modifications. This model keeps the edge supports and the bidirectional in-plane restraint at corner A while adjusting the support at corner B from preventing displacement in two directions to preventing displacement in just one direction (see Fig. 5a).

The second and third models are shown in Figs. 5b and 5c. These two models are derived from the “Proposed” model, but some supports at corners are removed. These two forms of support do exist in practice, e.g., a plate lays freely on a square rim, but their computational models lack the in-plane boundary conditions required for model stability.

The fourth model uses the “Constraint-Rigid Body” function of ABAQUS. This feature is popularly used in ABAQUS to substitute a distributed load on a cross-section for a concentrated force or moment, or to assign restraint to the whole cross-section. As can be seen in Fig. 5d, the load uniformly distributed over the edge of the plate is replaced by a concentrated force. Through the “Rigid body” definition for the edge of the plate, this concentrated force will be evenly distributed over the edge of the plate, similar to the edge load.

The final model adds longitudinal restraints to the edges of the slab (see Fig. 5e). These restraints do not in principle affect the rotation of the edges.

3 Results

Models are established based on the ABAQUS User's Manual [6]. The calculation results of the critical stress σ_{cr} and buckling coefficient k from the studied models are listed in Table 1.

Because the models shown in Figs. 5b and 5c are unstable, their results are not included in Table 1.

Table 1. The critical stress σ_{cr} and buckling coefficient k

Vars	Units	ABAQUS	Yan Li Guo	Ronald Wagner	Proposed	Rigid Body	Longit Rst
σ_{cr}	MPa	75.96	75.93	75.39	75.93	161.25	103.38
k		4.00	4.00	3.97	4.00	8.50	5.45

Note:

Columns “ABAQUS”, “Yan Li Guo”, and “Ronald Wagner” are the results of the models of ABAQUS and the corresponding authors.

The “Proposed” column is the result of the proposed model, as shown in Fig. 5a.

The column “Rigid Body” is the result of the model using ABAQUS's “Rigid Body” feature (Fig. 5d)

The column “Longit Rst” is the result of the model in Fig. 5e.

4 Discussions

The buckling coefficient k obtained from the model of ABAQUS, YanLi Guo, and the proposed model has a value of 4.0 and is consistent with the theoretical value. In fact, the value of k is not exactly equal to 4 (k of ABAQUS is 4.00195 and of the other two models is 4,00068), due to the fineness of the mesh. That explains why the critical buckling stress of the ABAQUS model is slightly higher than that of the other two models. The larger the number of elements, the smaller the size of the elements, and the closer the value of k is to the theoretical solution. It can be concluded that these three models match the theoretical

model of the problem of the buckling of a rectangular thin plate with simple supports around the circumference subjected to a uniform unidirectional compressive load.

The buckling mode shapes corresponding to these three models are shown in Figs. 6a to 6c. The buckling forms of YanLi Guo and the proposed model in Figs. 6b and 6c are in half-wave form, which is consistent with the theoretical model. The buckling form of ABAQUS in Fig. 6a is a quarter-wave form because the ABAQUS model is built only in one quadrant.

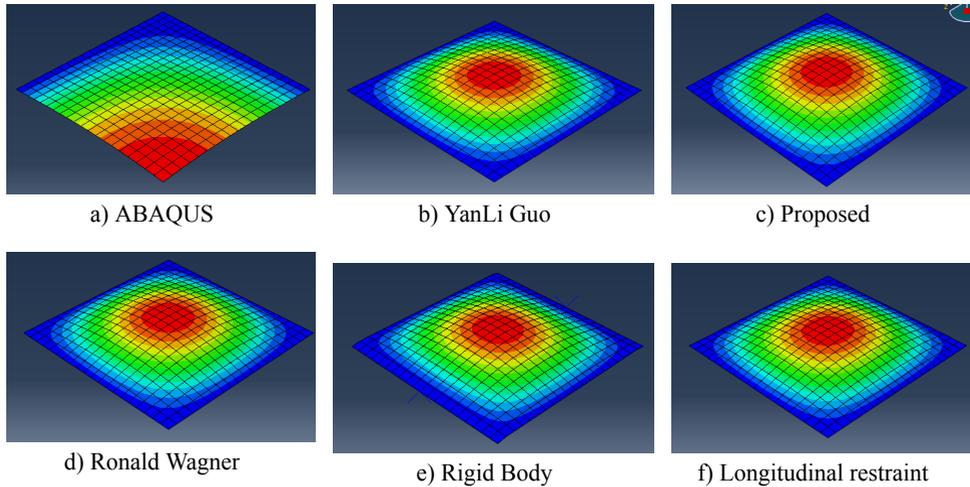
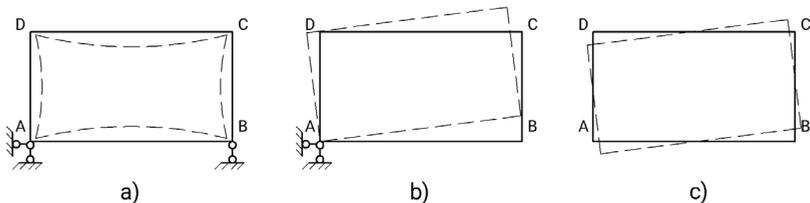


Fig. 6. Buckling mode shapes of the investigated models

As pointed out above, the models of ABAQUS and YanLi Guo require the plate to be symmetrical about two axes, so they cannot be applied to any shape or loading type. Thus, the proposed model in Fig. 5a is the most accurate and general. This model commits the boundary conditions in the normal direction of the plate, similar to those in Fig. 5c. In the plane of the plate, the supports are installed as a simple beam, with one pinned and one roller support, ensuring the minimum number of boundary conditions for the plate to be in-plane stable, as shown in Fig. 7a. This modeling approach ensures that the edges do not move in the out-of-plane direction while they can rotate freely around the edge; therefore, it matches the requirement for a simple support on the circumference of the plate. Meanwhile, in the plane of the plate, every point on the edge can freely move, not only in the direction perpendicular to edges but also in the longitudinal direction of edges, as shown by the dashed line in Fig. 7a.



a) Proposed model b) Support at B is removed c) Supports at B and A are both removed

Fig. 7. Models and corresponding in-plane deformations or movements

Compared to the proposed model, Ronald Wagner's model in Fig. 4 has an excessive constraint at the corner B in the direction of edge AB, which prevents deformation of edge AB in the longitudinal direction. As a result, the critical stress σ_{cr} and buckling coefficient k of this model are smaller than those of the proposed and theoretical models. The difference

is insignificant because the additional boundary condition affects primarily the AB edge rather than the whole plate.

As for the models lack of restraints as shown in Figs. 5b and 5c, ABAQUS still proceeds, but the critical buckling stresses are not reliable as the differences from the theoretical solution are magnificent, and the buckling form is not the out-of-plane half-wave but is in-plane rigid body translation and/or rotation (the model in Fig. 5b rotates around point A as presented by the dashed line in Fig. 7b, and the model in Fig. 5c rotates around the center of the plate as presented by the dashed line in Fig. 7c). Therefore, these models are dismissed.

Using a Rigid body similar to the model shown in Fig. 5d has the advantage of assigning a distributed load to an edge of the plate. But at the same time, the approach makes the edge a rigid, undeformable edge that is only able to translate or rotate. The in-plane displacements of the points on the edge are constrained, analogous to adding excessive constraints to the model in Fig. 5e. The results show that the critical stress σ_{cr} and the buckling coefficient k are both much higher than the theoretical solution, meaning that these two models are incorrect.

It should be noted that, although the models of Ronald Wagner, the Rigid Body, and adding longitudinal restraints are incorrect, their buckling forms in Figs. 6d to 6f are still half-wave shapes, similar to those of the proposed model (Fig. 6c). The difference in the buckling form of these models is so small that it is difficult to distinguish. Therefore, it is necessary to correctly assign the boundary conditions and loads from the beginning of modeling because it cannot rely on deformation to find modeling errors, which is the method often used when checking the accuracy of the model in practice.

5 Application of the proposed model

The proposed model is applied to solve the buckling problem of an isosceles trapezoidal steel plate simply supported around the circumference and subjected to compression on two opposite sides. The dimension of the plate is (900–1100) x 1000 mm (see Fig. 8a). The plate thickness t is 10 mm. Boundary conditions and loads assigned to the plate are shown in Fig. 8b. The four sides of the plate are assigned supports to prevent out-of-plane displacement (along the z-axis). The front corner of the plate is assigned an additional constraint that disallows displacement in the x and y axes (in the plane of the plate). The left corner of the plate is also assigned an additional constraint, but it only prevents displacement in the y-axis. The load on the right edge of the plate (side length 1100 mm) has a value of 1 N/mm, while the load on the left edge (edge length 900 mm) has a value of 1 N/mm x 1100 mm/900 mm = 1.222 N/mm. Elastic modulus of steel $E = 210000$ MPa and Poisson ratio $\nu = 0.3$.

Calculation results by ABAQUS are shown in Fig. 8c.

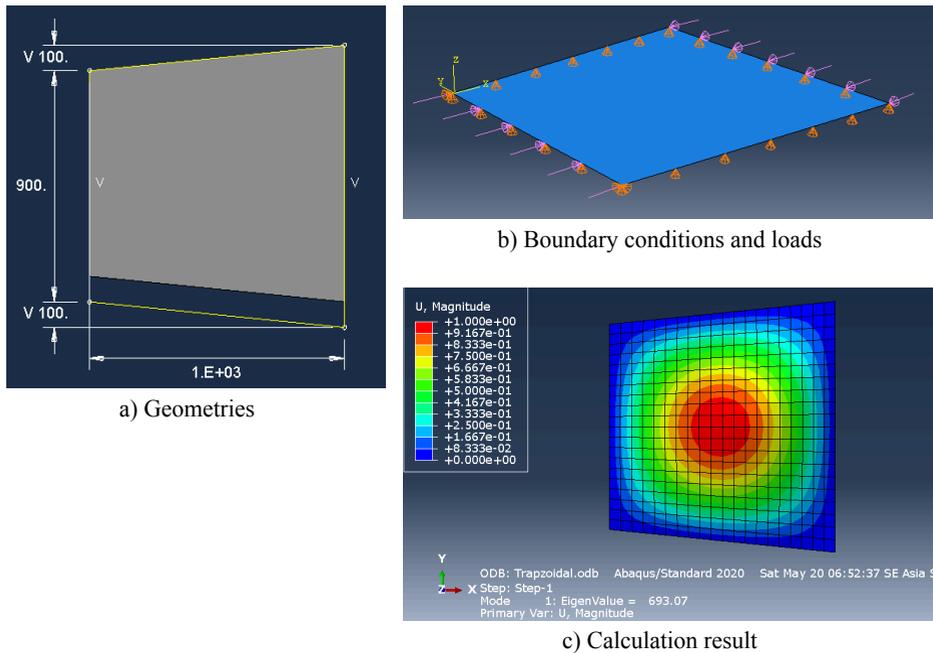


Fig. 8. Dimensions, calculation scheme, and calculation result of the trapezoidal plate

The eigenvalue determined by ABAQUS has a value of 693.07. The greatest stress is on the left side of the plate (the bottom of the trapezoid, size 900 mm). The critical stress can be determined by the stress on this edge:

$$\sigma_{cr} = \frac{N_{cr}}{t} = \frac{1.222 \times 693.07}{10} = 84.71 \text{ MPa} \quad (4)$$

The critical stress of a 1000-mm-square steel plate is 75.93 MPa (see Table 1). To compare with the square plate, the average stress at the midpoint of the trapezoidal plate will be determined. The length of the median of a trapezoid is $b_{avg} = (900 + 1100) / 2 = 1000$ mm. The internal force on the median is equal to $N_{cr,avg} = 1 \text{ N/mm} \times 1100 \text{ mm} / 1000 \text{ mm} = 1.1 \text{ N/mm}$. The average critical stress is:

$$\sigma_{cr,avg} = \frac{N_{cr,avg}}{t} = \frac{1.1 \times 693.07}{10} = 76.24 \text{ MPa} \quad (5)$$

As can be seen, the critical stress of the trapezoidal steel plate (76.24 MPa) is greater than that of the corresponding square plate (75.93 MPa).

Therefore, the mean buckling coefficient of the trapezoidal plate must be greater than 4.0.

$$\begin{aligned} k_{trap} &= \frac{12(1-\nu^2)}{\pi^2 E} \left(\frac{b_{avg}}{t} \right)^2 \sigma_{cr,avg} \\ &= \frac{12(1-0.3^2)}{3.14^2 \times 210000} \left(\frac{1000}{10} \right)^2 \times 76.24 = 4.017 \end{aligned} \quad (6)$$

6 Conclusions

Research shows that the modeling of a plate is much more complicated than that of a line element. Even small changes in the assignments of restraints and loads can result in a large error in the calculation results.

For the problem of buckling of a thin plate under uniform compression on two opposite sides with simple supports on the circumference, the accurate model is shown in Fig. 5a, conforming to the following principles:

- Constraints that resist out-of-plane translational displacement of the plate are arranged over the entire circumference of the plate.
- In the plane of the plate, arrange a point where translation is prevented in both directions and another point that is prohibited from displacement in one direction, similar to the principle of modeling a simply supported beam.
- An external force is applied on the edge of the plate on the principle that it does not interfere with the deformation of the plate or its edges; e.g., for a distributed load, the ABAQUS's edge load can be used.

Because it is difficult to use the deformed shape to evaluate the accuracy of the model, the modeling process needs to be elaborately conducted, and at the same time, the constraints as well as the loads must be determined exactly while modeling.

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