Numerical and Asymptotic Solutions for Multi Particle Bridging in Filtration Process

Galina Leonidovna Safina*

Moscow State University of Civil Engineering, 26, Yaroslavskoye shosse, Moscow, 129337, Russia

Abstract. The preservation of cultural and historic buildings is an important task for society. Historical buildings are prone to the appearance of numerous cracks and damage, therefore they require strengthening of their foundations and soils. Strengthening the foundations of historic buildings using injection methods is a technique used to improve and restore the foundations of old or damaged buildings. This method involves injecting special injection materials into the soil or foundation structure to improve its bearing capacity, stability and durability. The study of suspension flow with suspended solids in a porous medium is an integral part of ensuring the effectiveness of injection techniques. The paper considers a classical filtration model with a nonlinear filtration function. As a concentration function, a fifth-degree polynomial is used, describing the dimensional mechanism of particle capture in combination with the formation of arched partitions consisting of three or five particles blocking the pores. The considered problem with nonlinear filtration function and concentration function does not have an analytical solution, the paper presents its numerical solution using finite difference schemes. In addition to the numerical solution, expressions of analytical asymptotic solutions are obtained, which approximate the numerical solutions quite well even at sufficiently large values of time.

Keywords: porous medium, deep bed filtration, nonlinear filtration function and concentration function, arched bridging, asymptotic solution, numerical solution

1 Introduction

Historic buildings are part of a society's cultural heritage. They reflect the history, style, architecture and technical achievements of past eras. The preservation of such buildings helps to understand and recreate the past and pass it on to future generations.

Strengthening the foundations of historic buildings is an important process of preserving and maintaining the integrity of old historic structures by ensuring their stable and safe foundations. Such works may be required due to various factors such as ageing materials, natural disasters, geological changes, inadequate foundation design and so on.

It is important to take into account that historic buildings have their cultural and historical value, so careful and gentle methods must be applied when strengthening soils in order to preserve the authenticity of their appearance and structure. Currently, there are

*Corresponding author: minkinag@mail.ru

numerous examples of strengthening the foundations of historic buildings. These include, for example, the strengthening of foundations of old Tbilisi buildings [1], reconstruction of St. John's Church in Tartu [2] and the Church of Saint George in Proszkow [3], soil strengthening of the St. Nicholas Gate of Kiev Fortress [4] and many others.

The following methods exist to reinforce the soils of historic buildings:

1. injection methods involve the introduction of special materials (e.g. cement or resin solutions) into the soil to improve its strength and stability [5-6];
2. geotextiles: geotextile layers can be placed under foundations or around buildings to reinforce soils and prevent erosion [7];
3. micropiles and piles: rods driven or drilled into the ground to create a stable foundation for a building. Micropiles and piles can be used to reinforce foundations and raise a structure [8];
4. wall reinforcement: sometimes it may be necessary to reinforce the walls of a building by adding reinforcing elements or special reinforcing materials [9];
5. drainage systems: installing drainage systems can help manage soil moisture around the building, which helps to strengthen foundations [10].

It should be noted that when carrying out soil reinforcement works on historic buildings, it is necessary to cooperate with engineers specialising in restoration and conservation of cultural heritage in order to find the best balance between preserving and reinforcing the original building elements.

Injection methods of soil reinforcement are among the most common methods used today, with slurry filtration playing an important role in injected soil reinforcement methods such as cement mortar or polymer injections. This process involves the penetration of a liquid or semi-thick mixture (slurry) into the soil to reinforce its properties.

The main significance of slurry filtration lies in the following aspects:

1. penetration and uniform distribution. The slurry is injected into the ground using injection syringes or special pumping systems. Filtration allows controlling the rate of penetration of the mixture and uniform distribution of the injection material in the soil. This is important to ensure a uniform reinforced zone and prevent the formation of voids or uneven compaction;
2. reduction of soil permeability. Injection materials such as cement mortars or polymers have the ability to harden or polymerise. Once the mixture is injected into the soil, the filtration process allows the material to penetrate the pores and cracks of the soil, which ultimately reduces its permeability. This helps to strengthen the soil and improve its bearing properties and stability;
3. creating a barrier to water and soil particles. Slurry filtration can help create a barrier to groundwater and soil particle movement. This is important, for example, to prevent soil from being washed away by water, which can lead to cavities or landslides;
4. strengthening the soil and increasing its mechanical properties. Filtration and subsequent curing of the injection materials leads to the formation of cohesive elements in the soil. This strengthens the soil and increases its mechanical strength, which can be particularly important during construction or reconstruction;
5. displacement prevention. Proper slurry filtration helps prevent ground shifts and ensures the stability of structures on the surface.

In general, slurry filtration in soil reinforcement injection techniques plays a role in providing effective and sustainable soil reinforcement, as well as preventing potential problems associated with soil movement, water movement and cavity formation.

The process of deep bed filtration is considered in the paper. It involves passing the fluid through a porous medium, typically a bed of granular materials such as sand and gravel. This medium acts as a filter, trapping the particles as the fluid passes through it.

The particle capture in porous media can occur by different mechanisms depending on the characteristics of the particles, pores and physicochemical interactions. These include such mechanisms as size exclusion mechanism [11], sedimentation [12], pore surface adhesion [13], multi particle bridging [14], etc. Here the size exclusion mechanism, when large particles get stuck in small pores, and freely pass through pores with diameters larger than the particle size, together with multi particle bridging mechanism, when three or five particles form arched bridging (Fig. 1), are investigated. When multi particle bridging, arched partitions are formed, thus blocking the pores.

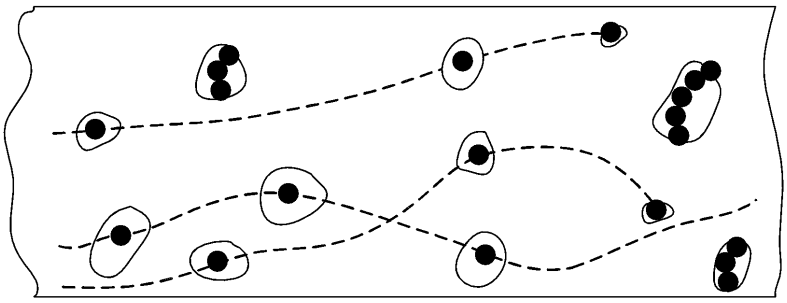


Fig. 1. Combined mechanism of particle capture.

The process of deep bed filtration is modelled using a pair of partial derivative equations, the first of which describes the mass balance of suspended and retained particles and the second determines the formation of the deposition. The problem with the formation of five-particle arch partitions is solved for the first time. This paper is a continuation of the study of [15], where the three particles arched bridging was investigated.

2 Materials and Methods

1.1 The problem statement

Let us $C(x,t)$ is the concentration of suspended particles, $S(x,t)$ is the concentration of retained particles, $\Lambda(S)$ is the filtration function and $F(C)$ is the concentration function. We will solve the problem of deep bed filtration in the domain $\Omega = \{0 \le x \le 1,\ t \ge 0\}$. It is defined by the system of equations

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial S}{\partial t} = 0 \quad , \tag{1}$$

$$\frac{\partial S}{\partial t} = \Lambda(S) F(C) \quad . \tag{2}$$

At the boundary $x = 0$ the concentration of suspended particles is zero, i.e.

$$C(0,t) = 1 \quad , \tag{3}$$

and at the initial moment of time $t = 0$ the concentrations of suspended and retained particles are zero

$$C(x,0) = 0,\ S(x,0) = 0 \quad . \tag{4}$$

The line $t = x$ is called the concentration front of suspended and retained particles. Under this line, i. e. in the subdomain $\Omega' = \{0 < x < 1,\ 0 < t < x\}$, the concentrations $C(x,t)$ and $S(x,t)$ have zero values. Above the concentration front, i.e. in the subdomain $\Omega'' = \{0 < x < 1,\ t > x\}$ the concentrations $C(x,t)$ and $S(x,t)$ have positive values. The concentration of retained particles $S(x,t)$ is zero at the concentration front, it has no breaks everywhere. The concentration of suspended particles $C(x,t)$ has a break along the concentration front.

As a filter function we will use the same cubic function that was used in the paper [16]:

$$\Lambda(S) = \lambda_0 + \lambda_1 S + \lambda_2 S^2 + \lambda_3 S^3 \quad , \tag{5}$$

it is a decreasing function, and there exists such a value of $S_0 > 0$ that $\lambda_0 + \lambda_1 S_0 + \lambda_2 S_0^2 + \lambda_3 S_0^3 \equiv 0$.

Consider the concentration function

$$E(C) = (1 - \alpha - \beta) \quad + \alpha \quad ^3 + \beta \quad ^5 \quad , \tag{6}$$

which corresponds to the combined mechanism of particle capture.

Substituting (5)-(6) into (1)-(2), we obtain the system of equations

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \left(\lambda_0 + \lambda_1 S + \lambda_2 S^2 + \lambda_3 S^3\right)\left((1 - \alpha - \beta) \quad + \alpha \quad ^3 + \beta \quad ^5\right) = 0 \quad , \tag{7}$$

$$\frac{\partial S}{\partial t} = \left(\lambda_0 + \lambda_1 S + \lambda_2 S^2 + \lambda_3 S^3\right)\left((1 - \alpha - \beta) \quad + \alpha \quad ^3 + \beta \quad ^5\right) \tag{8}$$

with boundary and initial conditions (3)-(4).

The solution $S_0(0,t)$ *at the filter inlet.*

Since at the filter inlet $C(0,t) = 1$, then the solution $S_0(0,t)$ is determined by equation (8):

$$\frac{\partial S}{\partial t} = \left(\lambda_0 + \lambda_1 S + \lambda_2 S^2 + \lambda_3 S^3\right) \quad .$$

Then the solution $S_0(0,t)$ at the filter inlet is determined from the general integral of the differential equation

$$\int_0^{S(0,t)} \frac{dS}{\lambda_0 + \lambda_1 S + \lambda_2 S^2 + \lambda_3 S^3} = t \quad . \tag{9}$$

The solution $C_0(x,t)$ *at the concentration front.*

Turning to the characteristic variables $T = t - x, \quad x = x$ allows to rewrite equation (7) in the form

$$\frac{\partial C}{\partial x} + \left(\lambda_0 + \lambda_1 S + \lambda_2 S^2 + \lambda_3 S^3\right)\left((1-\alpha-\beta) \quad + \alpha \quad {}^3 + \beta \quad {}^5\right) = 0 \quad . \tag{10}$$

At $T = 0$ the solution $S(x,T)$ is zero, then we can rewrite equation (10) in the form

$$\frac{\partial C}{\partial x} + \lambda_0 \left((1-\alpha-\beta)C + \alpha C^3 + \beta C^5\right) = 0 \quad ,$$

and find the solution using the expression

$$\int_{C_0(x)}^{1} \frac{dc}{(1-\alpha-\beta)C + \alpha C^3 + \beta C^5} = \lambda_0 x \quad .$$

Thus, the solution $C_0(x,t)$ at the concentration front $t = x$ is given in implicit form: Thus, the solution $C_0(x,t)$:

$$\frac{C_0^{\frac{1}{1-\alpha-\beta}} \left(\alpha + \sqrt{(\alpha+2\beta)^2 - 4\beta} + 2\beta \quad {}_0^2\right)^{\frac{\alpha - \sqrt{(\alpha+2\beta)^2-4\beta}}{4(1-\alpha-\beta)\sqrt{(\alpha+2\beta)^2-4\beta}}}}{\left(\alpha - \sqrt{(\alpha+2\beta)^2 - 4\beta} + 2\beta C_0^2\right)^{\frac{\alpha + \sqrt{(\alpha+2\beta)^2-4\beta}}{4(1-\alpha-\beta)\sqrt{(\alpha+2\beta)^2-4\beta}}}} =$$

$$= \frac{\left(\alpha + \sqrt{(\alpha+2\beta)^2 - 4\beta} + 2\beta\right)^{\frac{\alpha - \sqrt{(\alpha+2\beta)^2-4\beta}}{4(1-\alpha-\beta)\sqrt{(\alpha+2\beta)^2-4\beta}}}}{\left(\alpha - \sqrt{(\alpha+2\beta)^2 - 4\beta} + 2\beta\right)^{\frac{\alpha + \sqrt{(\alpha+2\beta)^2-4\beta}}{4(1-\alpha-\beta)\sqrt{(\alpha+2\beta)^2-4\beta}}}} e^{-\lambda_0 x}. \tag{10}$$

1.2 Asymptotic solution of the problem

Let us find the asymptotic solutions near the concentration front $t = x$ in the form:

$$C(x,t) = C_0(x) + x_1(x)(C - x) + t\frac{1}{2} x_2(x)(x - t)^2 + \ldots, \tag{11}$$

$$S(x,t) = S_1(x)(t-x) + \frac{1}{2} S_2(x)(t-x)^2 + \frac{1}{6} S_3(x)(t-x)^3 \ldots \tag{12}$$

Note that at the concentration front $C_0(x)$ is determined by the expression (10) and $S_0(x)=0$.

In expressions (11)-(12) we restrict ourselves to the first two summands to avoid cumbersome calculations, i.e., we search for asymptotic solutions in the form:

$$C(x,t)=C_0(x)+x_1(x)(t-x), \tag{13}$$

$$S(x,t)=S_1(x)(t-x)+\frac{1}{2}S_2(x)(t-x)^2. \tag{14}$$

Differentiate (13) by x and t and (11) by t. After substituting the obtained partial derivatives and expansions (13)-(14) into the system of equations (7)-(8), we group the summands having the same degrees of the multiplier $(t-x)$, and equate the expansion coefficients at the same degrees:

$(t-x)^0$:

$$C_0'(x)=-\lambda_0(1-\alpha-\beta)C_0(x)-\alpha\lambda_0C_0^3(x)-\beta\lambda_0C_0^5(x),$$

$$S_1'(x)=\lambda_0(1-\alpha-\beta)C_0(x)+\alpha\lambda_0C_0^3(x)+\beta\lambda_0C_0^5(x);$$

$(t-x)^1$:

$$C_1'(x)=-\lambda_0(1-\alpha-\beta)S_1(x)-\lambda_1(1-\alpha-\beta)C_0(x)C_1(x)-3\alpha\lambda_0C_0^2(x)S_1(x)C_1(x)-\alpha\lambda_1C_0^3(x)C_1(x)-$$
$$-5\beta\lambda_0C_0^4(x)C_1(x)-\beta\lambda_1S_1(x)C_0^5(x)$$

$$S_2'(x)=\lambda_0(1-\alpha-\beta)C_1(x)S_1+\lambda_1(1-\alpha-\beta)C_0(x)C_1(x)+3\alpha\lambda_0C_0^2(x)S_1(x)C_1(x)+\alpha\lambda_1C_0^3(x)C_1(x)+$$
$$+5\beta\lambda_0C_0^4(x)C_1(x)+\beta\lambda_1S_1(x)C_0^5(x).$$

Omitting cumbersome calculations, we obtain the solution of the recurrent system of equations:

$$S_1'(x)=\lambda_0(1-\alpha-\beta)C_0(x)+\alpha\lambda_0C_0^3(x)+\beta\lambda_0C_0^5(x);$$

$$C_1(x)=\lambda_1C_0(x)\left(C_0(x)-1\right)\left(1-\alpha-\beta+\alpha C_0^2(x)+\beta C_0^4(x)\right);$$

$$S_2'(x)=6\beta^2\lambda_0\lambda_1C_0^{10}(x)-5\beta^2\lambda_0\lambda_1C_0^9(x)+10\alpha\beta\lambda_0\lambda_1C_0^8(x)-8\alpha\beta\lambda_0\lambda_1C_0^7(x)+$$
$$+\left(4\alpha^2\lambda_0\lambda_1+8(1-\alpha-\beta)\beta\lambda_0\lambda_1\right)C_0^6(x)-\left(3\alpha^2\lambda_0\lambda_1+6(1-\alpha-\beta)\beta\lambda_0\lambda_1\right)C_0^5(x)+$$
$$+6\alpha(1-\alpha-\beta)\lambda_0\lambda_1C_0^4(x)-4\alpha(1-\alpha-\beta)\lambda_0\lambda_1C_0^3(x)+2(1-\alpha-\beta)^2\lambda_0\lambda_1C_0^2(x)-(1-\alpha-\beta)^2\lambda_0\lambda_1C_0(x).$$

Then the asymptotic solutions are

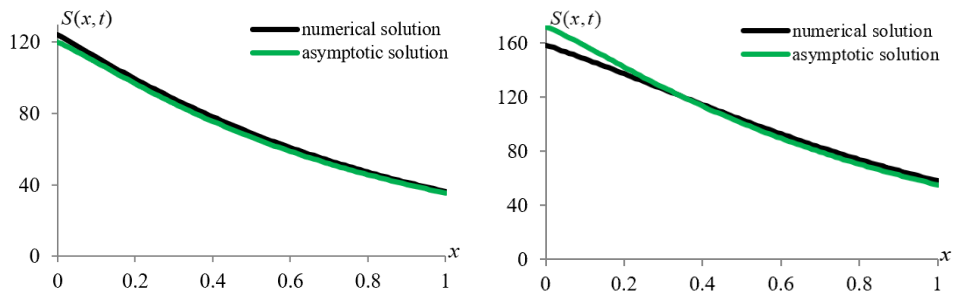
$$C(x,t) = C_0(x) + \lambda_1 C_0(x)\left(\lambda_0 C_0(x) - 1\right)\left(1 - \alpha - \beta + \alpha C_0^2(x) + \beta C_0^4(x)\right)(t - x);$$

$$S(x,t) = \left(\lambda_0 C_0(x)(1 - \alpha - \beta) + \alpha\lambda_0 C_0^3(x) + \beta\lambda_0 C_0^5(x)\right)(t - x) +$$
$$+\frac{1}{2}\Big(6\beta^2\lambda_0\lambda_1 C_0^{10}(x) - 5\beta^2\lambda_0\lambda_1 C_0^9(x) + 10\alpha\beta\lambda_0\lambda_1 C_0^8(x) - 8\alpha\beta\lambda_0\lambda_1 C_0^7(x) +$$
$$+\left(4\alpha^2\lambda_0\lambda_1 + 8(1-\alpha-\beta)\beta\lambda_0\lambda_1\right)C_0^6(x) - \left(3\alpha^2\lambda_0\lambda_1 + 6(1-\alpha-\beta)\beta\lambda_0\lambda_1\right)C_0^5(x) +$$
$$+6\alpha(1-\alpha-\beta)\lambda_0\lambda_1 C_0^4(x) - 4\alpha(1-\alpha-\beta)\lambda_0\lambda_1 C_0^3(x) +$$
$$+2(1-\alpha-\beta)^2\lambda_0\lambda_1 C_0^2(x) - (1-\alpha-\beta)^2\lambda_0\lambda_1 C_0(x)\Big)(t-x)^2.$$

2 Results

Due to cubic filtration function and concentration function in the form of polynomial of the fifth degree, the problem has no solution found analytically, so the problem was solved numerically using finite differences. Steps of the reference grid along the axes and were chosen equal to $h_t = h_x = 0.01$, thus Courant condition is fulfilled [17]. As coefficients of the filtration function $\Lambda(S)$ we will use the values obtained experimentally in the paper [18]: $\lambda_0 = 1.551$, $\lambda_1 = -3.467 \cdot 10^{-3}$, $\lambda_2 = -1.16 \cdot 10^{-6}$, $\lambda_3 = -1.16 \cdot 10^{-7}$. Let $\alpha = 0.1$ and $\beta = 0.05$, such a choice of values corresponds to the fact that the formation of arch slabs from three particles is more probable than from five.

Figure 2 shows the graphs of retained particle concentrations for different times: $t = 100$ (Figure 2a) and $t = 150$ (Figure 2b). The graphs show that the numerical and asymptotic solutions are very close to each other. Note that for $t = 100$ the relative error of the asymptotic solution reaches a maximum at the filter inlet and is equal to 3.1 % and for $t = 150$ the maximum of the relative error is observed at the filter inlet too and is equal to 8.5 %.



a) $t = 100$ b) $t = 150$

Fig. 2. Numerical and asymptotic solutions of retained particles concentrations for different times

The following figure shows the dependences similar to Figure 2 for suspended particle concentrations. Figure 3a corresponds to the time moment $t = 100$, Figure 3b corresponds to the time moment $t = 150$. In the first case the maximum value of the relative error of the asymptotic solution is equal to 3.9 % at $x = 0.6$; in the second case the maximum value of the relative error of the asymptotic solution is reached at the filter outlet and is equal to 13.6 %.

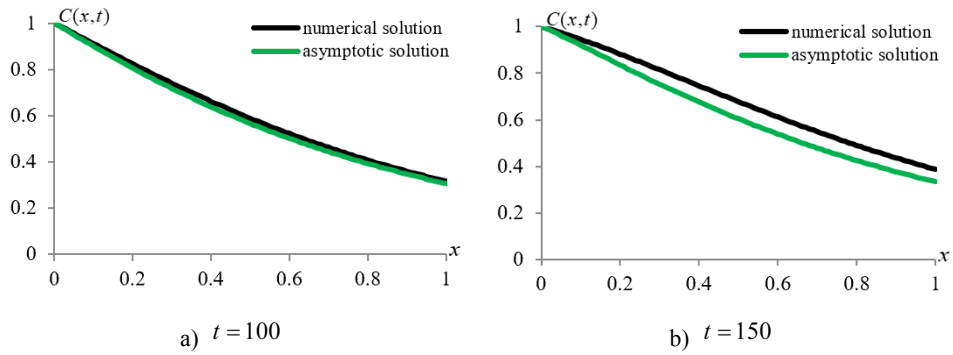


Fig. 3. Numerical and asymptotic solutions of suspended particles concentrations for different times

In Figure 4 the plots of relative error of the asymptotic solution at different time values for retained particles (Figure 4a) and suspended particles (Figure 4b) are presented.

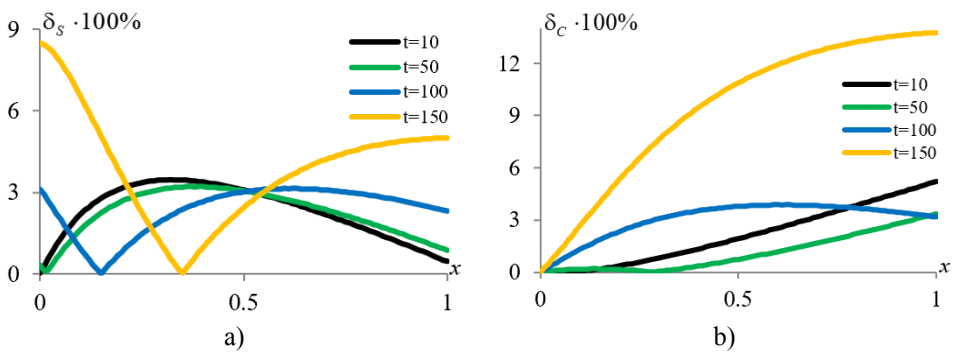
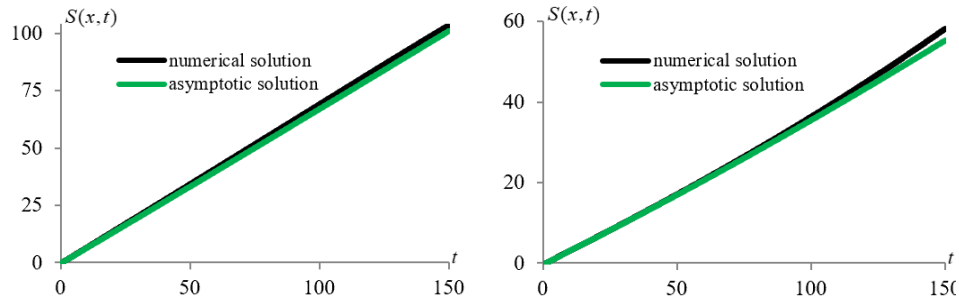


Fig. 4. Relative error of the asymptotic solution for retained (a) and suspended (b) particles

Figure 4 shows that with increasing time, the error of the asymptotic solution for the concentration of suspended particles is more than 1.5 times larger than the error of the asymptotic solution for the concentration of retained particles.

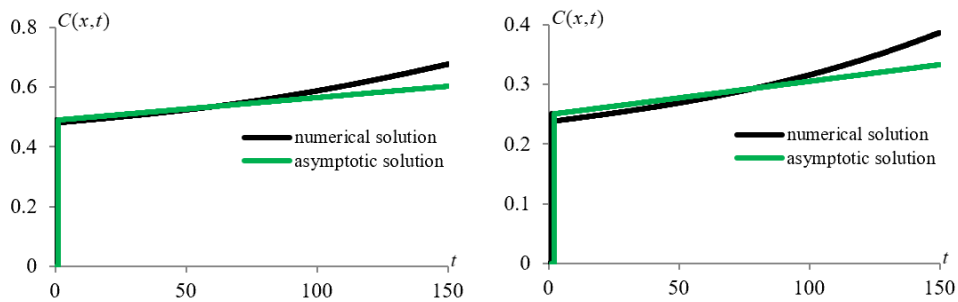
Figure 5 shows the time dependences of the concentrations of retained particles for $x = 0.5$ (Figure 5a) and $x = 1$ (Figure 5b). At $x = 0.5$ the maximum value of the relative error of the asymptotic solution is reached under the concentration front and is equal to 4.8 %; at $x = 1$ the maximum value of the relative error of the asymptotic is reached at finite time $t = 150$ and is equal to 5 %.



a) $x = 0.5$ b) $x = 1$

Fig 5. Numerical and asymptotic solutions of retained particles concentrations for different *x*

Figure 6 is similar to Figure 5, but for the concentration of suspended particles. Figure 6a corresponds to the case $x = 0.5$, Figure 6b corresponds to the case $x = 1$. In both cases, the maximum of the relative error of the asymptotic solution is observed at the finite time value $t = 150$: at $x = 0.5$ is equal to 10.9 %, at $x = 1$ is equal to 13.8 %.



a) $x = 0.5$ b) $x = 1$

Fig 6. Numerical and asymptotic solutions of suspended particles concentrations for different *x*

3 Discussion

The paper deals with the problem of deep bed filtration. The peculiarity is the use of a fifth degree polynomial as a concentration function, which defines a combined particle entrapment mechanism: along with the size exclusion mechanism, when suspended particles get stuck in pores of smaller diameter than the particle size, thus blocking the pores, arch partitions consisting of three or five particles are formed, which also block free pores.

As a continuation of [15], in addition to the numerical solution of the problem by the finite difference method, i.e., obtaining the concentration values of suspended and retained particles at each node of the rectangular grid, the asymptotic solution of the problem near the concentration front is found by representing the functions and in the form of degree series. The obtained asymptotic solutions are sufficiently close to numerical ones even at large time intervals. This allows us to state that the use of asymptotics can significantly reduce the calculation time of deep bed filtration process and at the same time obtain sufficiently accurate values of suspended and precipitated particle concentrations. It is also worth noting that the asymptotic solutions for the cubic concentration function in [15] give

a smaller relative error than for the case of the concentration function in the form of a fifth degree polynomial, which is considered in this paper. This can be explained by the fact that the values of constants at the discarded terms in the expansions (11)-(12) for the fifth degree polynomial have larger values, so their exclusion leads to a larger error.

4 Conclusion

The problem with nonlinear filtration function and concentration function does not have an exact solution, the paper presents its numerical solution by the finite difference method. The concentration function, set by a polynomial of the fifth degree, describes a complex filtration process that combines the mechanical and geometric capture of particles, in which suspended particles freely pass through pores of larger diameter and get stuck in the necks of those whose diameter is smaller than the particle size, and the mechanism of formation of arched bridging consisting of three or five solid particles that block the pores and thereby prevent the suspension from flowing inside. The paper proposes an asymptotic approximation of the numerical solution, which is constructed near the concentration front and is limited to two expansion terms due to cumbersome calculations. The presented graphs of numerical solutions of retained particle concentrations and their asymptotics showed that the relative error of the approximation, even with a sufficiently large time $t = 150$, is about 8.5 %, while the asymptotics of suspended particle concentrations gives a large relative error, for example, at the outlet of the filter at a time $t = 150$ it is about 13.6 %. In real conditions when studying the filtration process, the measurement error should not exceed 10 % [19], so moment of time $t = 135$ is the limit value, for which the obtained formulas of asymptotic solutions can be used.

References

1. N. Mskhiladze, I. Iremashvili, L. Pipia, Construction of optimized energy potential **10(2)**, 89-97 (2021), DOI: 10.17512/bozpe.2021.2.11
2. K. Avellan: Limit state design for strengthening foundations of historic buildings using pretested drilled spiral piles with special reference to St. John's Church in Tartu (Acta Universitatis Ouluensis, 2011)
3. E. Kokocinska-Pakiet, D. Bajno, Acta Scientiarum Polonorum. Architectura **17(2)** , 97-106 (2017), DOI: 10.22630/ASPA.2018.17.2.18
4. A. Severino, A. Wahrhafti, O. Tiutkin, V. Gubashova, L. Neduzha, Buildings **12(3)**, 368 (2022), DOI: 10.3390/buildings12030368
5. R. M. Al-Khadaar, M. D. Ahmed, Journal of Engineering **29(7)**, 48-70 (2023), DOI: 10.31026/j.eng.2023.07.04
6. A. Andriansyah, L. Afriani, D. I. Kusumastuti, E. P. Wahono, Journal of Sustainability Science and Technology **1(2)**, 72-79 (2021), DOI: 10.23960/josst.v1i2.9
7. S. Jaiswal, A. Srivastava, V. B. Chauhan, Lecture Notes in Civil Engineering **152**, 709-720 (2021)
8. S. Hemeda, Pali radice (Root pile, Micro pile) Technology for Preservation of Historical Islamic Monuments in Cairo Egypt (Mistral Service sas Via U. Bonino, Italy, 2015)

9. Zh. Wang, J. Liu, L. Zhu, Yu. Xiao, Highlights in Science, Engineering and Technology **28**, 305-309 (2022)
10. J. Wang, Q. Shen, Sh. Yuan, X. Wang, J. Shu, J. Zheng, H. Sun, Applied Sciences **13(6)**, 3633 (2023), DOI: 10.3390/app13063633
11. Yu. Osipov, N. Kotov, IOP Conference Series Materials Science and Engineering **365(4)**, 042006 (2018), DOI: 10.1088/1757-899X/365/4/042003
12. E. Nazirova, A. Nematov, R. Sadikov, I. Nabiyev, Lecture Notes in Computer Science **12615**, 353-360 (2021), DOI: 10.1007/978-3-030-68449-5_35
13. R. Li, Ch. Wei, Ch. Wei, H. Cheng, H. Cheng, G. Chen, Progress in Adhesion and Adhesives **5**, 417-460 (2020), DOI: 10.1002/9781119749882.ch13
14. L. Kuzmina, Yu. Osipov, IOP Conference Series Materials Science and Engineering **661(1)**, 012122 (2019), DOI: 10.1088/1757-899X/661/1/012122
15. G. Safina, International Journal for Computational Civil and Structural Engineering **18(1)**, 129-140 (2022), DOI: 10.22337/2587-9618-2023-19-2-14-30
16. S. Yang, T. Russell, A. Badalyan, U. Schacht, M. Woolley, P. Bedrikovetsky, Journal of Natural Gas Science and Engineering **65**, 108-124 (2019), DOI: 10.1016/j.jngse.2019.02.005
17. Yu. Osipov, G. Safina, Yu. Galaguz, MATEC Web of Conferences **251**, 04021 (2018), DOI: 10.1051/matecconf/201825104021
18. Z. You, Y. Osipov, P. Bedrikovetsky, L. Kuzmina, Chemical Engineering Journal **258**, 374-385 (2014), DOI: 10.1016/j.cej.2014.07.051
19. G.K. Dev Burman, P.K. Das, The Hydrological Basis for Water Resources Management **197**, 19-30 (1990)