Formalization of modular algorithms for terrain modeling

Vladimir Georgievich Kulikov*

Moscow State University of Civil Engineering, 26, Yaroslavskoye shosse, Moscow, 129337, Russia

Abstract. Theoretical and practical issues of creating analog-discrete terrain models and their use for solving engineering problems are presented. With their help, it is possible to simulate, for example, areas of flooding, calculation of solar radiation intake, planning the construction of infrastructure facilities and other tasks. Various variants and interpretations of the construction of analytical, analog, graphical and discrete terrain models are considered. Block modules of pseudocodes of algorithms for constructing, processing and visualizing analog terrain models are given. Structured data sources for their creation are given. A method of step-by-step terrain modeling based on the use of logical-mathematical and matrix methods for processing large data arrays is proposed
Keywords. Local coordinate system, terrain, medium-planning surface, analytical relief model, vectorization, approximation.

Introduction

The modules of algorithms for analytical dependencies of digital models of territories are a means of analytical and digital representation of three–dimensional spatial objects of surfaces or reliefs in the form of data arrays forming a set of elevation marks, depth marks and other values of applications - the Z coordinates of the local coordinate system (LSC) in nodes of a regular or irregular network.

This is relevant when solving many engineering problems: 1) creating maps and displaying the land relief on them 2) tasks of vertical planning of the territory for the needs of construction; 3) hydrological tasks, land reclamation; 4) environmental tasks, etc. The main provisions of the methods for solving these problems are set out in [1-5].

The author proposed analytical-digital modeling [6-13] in the form of separate algorithmic modules [14] when creating a digital terrain model based on nonlinear schemes for interpolating marks between the points under study.

The purpose of the study:

1. To develop block algorithms for analytical modeling of the terrain relief surface (AMPR) for the "zero" balance of soil development of the considered area of the territory.

Research objectives:

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1. To develop an algorithm for determining the average planning mark of the territory under consideration;
2. Formalize the analytical dependencies of the initial data structures and visualize the results of the study.

Materials and methods

Let's use the local coordinate system (LKS), in the form of a " the right three -local coordinate system (0,X,Y,Z)».
We will look for an analytical-digital model of the relief surface (AMRP), and an analytical model of the medium-planning surface (MPS), considered in Fig.1 of the site. as a function of two variables:

$$AMRP\ \{0Z\};\ MPS\ \{0Z\} = f\ \{№1[0X],\ №2\ [0У]\} \quad (1)$$

where: the black marks indicated in the two-dimensional array [ere] along the "0X" axis are local variable No. 1; the black marks along the "0Y" axis are local variable No. 2 of the desired analytical function.

What is the meaning of searching for analytical expressions of relief models and planning surfaces in the form of functions from black marks in two directions?

A joint, systematic solution of the obtained analytical expressions of the corresponding surfaces with each other will make it possible to algorithmize the finding of the analytical expression of the line of their intersection with each other, i.e., the analytical finding of the so-called "zero" line of work during the development of the ground under the "zero" balance, and its subsequent optimization.

We use the results of in-house processing of the situational topographic plan of the area in the form of calculated "black" marks [English- existing relief elevation (ere)], shown in Fig.1.

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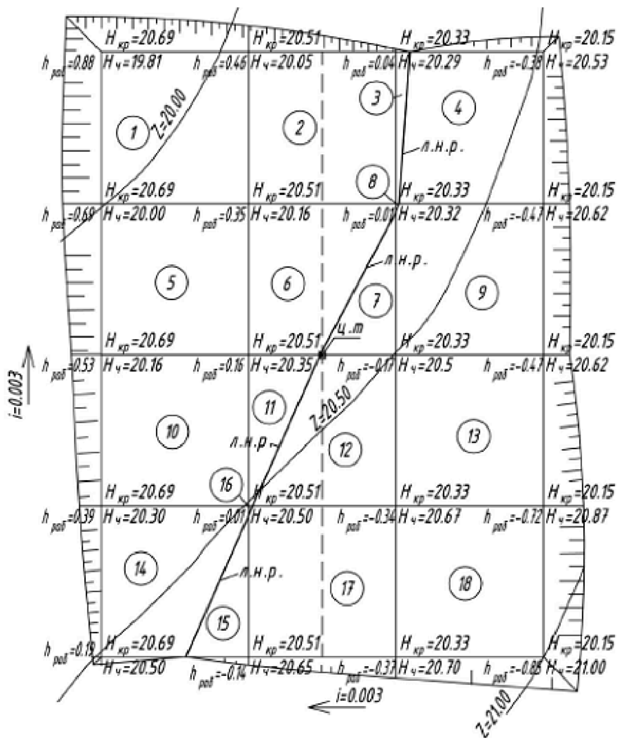

Fig. 1. Variant of the topographic plan for works under the "zero" balance.

Figure 1 shows the grid marking in the directions of the axes "0X" and "0U" in both directions equal to 50m,

Research results 1. Let's combine the information about the "black" marks shown in Fig.1 into a common table, presenting them as a two-dimensional data array [ere] (5x4). The data structure in the [ere] array will be ordered in accordance with the local coordinate system - the "right triple (0XYZ)", as shown in Fig.2. For the beginning of the counts (LKS), we will take the upper left corner of the site considered in Fig.1.

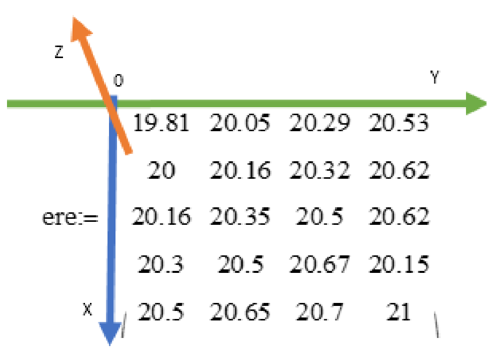

Fig2 "Binding" of the local coordinate system to the data of a two-dimensional array $[ere]_{(5x4)}$.

Development of an algorithm for determining the average planning mark. Figure 3 shows the sequential processing of array elements [ere].

With the help of auxiliary variables e(1,2,3), corresponding addresses to the encoded and structured data of the array [ere(i,j)], the instructions given, the value of the elevation of the average planning surface "Hm" of the site under consideration is calculated:

$$e1 := ere_{0,0} + ere_{0,3} + ere_{4,0} + ere_{4,3}$$

$$e2 := ere_{0,1} + ere_{0,2} + ere_{1,0} + ere_{1,3} + ere_{2,0} + ere_{2,3} + ere_{3,0} + ere_{3,3} + ere_{4,1} + ere_{4,2}$$

$$e3 := ere_{1,1} + ere_{1,2} + ere_{2,1} + ere_{2,2} + ere_{3,1} + ere_{3,2}$$

$$n := 12$$

$$Hm := \frac{(1 \cdot e1 + 2 \cdot e2 + 4 \cdot e3)}{4 \cdot 12} = 20.394$$

Fig.3 Pseudocode of the algorithm for calculating the average planning mark "Hm" of the plot in Fig.1. (n is the number of grid squares in Fig.1)

Research results 2. Thus, the average planning mark for the site in question, based on the data given, is 20.394m. This value represents the altitude mark "Hz" of the horizontal plane in the accepted (LKS). Consequently, this is such a medium-planning surface, relative to which the "large" black relief marks will be aligned with the concept of "excavation", and the "smaller" - the concept of "embankment" when developing the soil for a "zero" balance.

Formalization of the output of the analytical model (AMPR) of the relief surface of the site under consideration.

Let's imagine a function approximating the landscape of the site in question with a sixth-degree polynomial as the most suitable approximation of the data given in the [ere] array.

The approximation functions of the values of the black marks (LSC) "on the X axis" and "on the Y axis" Fig.1, we will also look for in the form of corresponding polynomials of the sixth degree p(x) and p(y).

The superposition of the functions of the black marks [p(x) and p(y)] is denoted as the "relief" marks of the surface of the site under consideration approximated by the desired polynomial.

From the analysis of the data structure of the array [ere], the black marks corresponding to the given markup on the axes "0X" and "0Y" (LSK) differ from each other not by a constant, but by some random variable deviation. Finding the "average" values of deviations is possible by calculating their "geometric mean". This value corresponds most accurately to the assessment of the central trend.

Research results 3. The analytical expression (AMPR) in the form of a sixth-degree polynomial from the specified superpositions of sixth-degree polynomials p(x) and p(y) is represented as:

$$p(x, y) = \sqrt[2]{p(x) \bullet p(y)} \qquad (2)$$

Representation of the system of equations for p(x) along the "0X" axis, in which the coefficients of the polynomial p(x) of degree six are unknown, and the free terms of the elements of the array [ere] (5x4) structured in Fig. 2 are shown in Fig.3:

$$p(x) = a\cdot x^6 + b\cdot x^5 + c\cdot x^4 + d\cdot x^3 + e\cdot x^2 + f\cdot x + g$$

$$Ax := \begin{pmatrix} 0^6 & 0^5 & 0^4 & 0^3 & 0^2 & 0^1 & 1 \\ 1^6 & 1^5 & 1^4 & 1^3 & 1^2 & 1^1 & 1 \\ 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 1 \\ 3^6 & 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 1 \\ 4^6 & 4^5 & 4^4 & 4^3 & 4^2 & 4^1 & 1 \end{pmatrix} \qquad Bx := \begin{pmatrix} 19.81 \\ 20 \\ 20.16 \\ 20.3 \\ 20.5 \end{pmatrix} \qquad Kx := \text{lsolve}(Ax, Bx)$$

Fig.3 Pseudocode of the polynomial search algorithm p(x)

where: "Ax" is the matrix of coefficients in the polynomial under consideration, "Bx" is the vector of free terms of the polynomial, - as a sample from the array [ere](5x4) along the axis "0X", "Kx" is the result of solving a system of equations, - the vector of values of the found unknowns.

The derivation of the equation of the polynomial p(x) is carried out using the function "vectorization " of the results of solving the system of equations in Fig.3. Let's show the transformations performed with the vectors "Kx" and "C1" in Fig. 4:

$$Kx = \begin{pmatrix} -1.952\times 10^{-3} \\ 0.017 \\ -0.043 \\ 3.425\times 10^{-3} \\ 0.077 \\ 0.137 \\ 19.81 \end{pmatrix} \qquad C1 := \begin{pmatrix} x^6 \\ x^5 \\ x^4 \\ x^3 \\ x^2 \\ x^1 \\ 1 \end{pmatrix} \qquad p(x) := \begin{pmatrix} -1.952\times 10^{-3} \\ 0.017 \\ -0.043 \\ 3.425\times 10^{-3} \\ 0.077 \\ 0.137 \\ 19.81 \end{pmatrix} \cdot \begin{pmatrix} x^6 \\ x^5 \\ x^4 \\ x^3 \\ x^2 \\ x \\ 1 \end{pmatrix}$$

Fig.4 Pseudocode of the operation of vectorization of the polynomial p(x).

After the transformations, we obtain the desired analytical expression for the polynomial p(x) of the following form:

$$p(x) = 0,137\cdot x^1 + 0,077\cdot x^2 + 0,003425\cdot x^3 - 0,043\cdot x^4 + 0,017\cdot x^5 -- 0,001952\cdot x^6 + 19.81 \qquad (3)$$

The system of equations p(y) along the "0Y" axis, in which the coefficients of the polynomial p(y) of degree six are unknown, is shown in Fig. 5:

$$p(y) = a \cdot y^6 + b \cdot y^5 + c \cdot y^4 + d \cdot y^3 + e \cdot y^2 + f \cdot y + g$$

$$Ay := \begin{pmatrix} 0^6 & 0^5 & 0^4 & 0^3 & 0^2 & 0^1 & 1 \\ 1^6 & 1^5 & 1^4 & 1^3 & 1^2 & 1^1 & 1 \\ 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 1 \\ 3^6 & 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 1 \end{pmatrix} \quad By := \begin{pmatrix} 19.81 \\ 20.05 \\ 20.29 \\ 20.53 \end{pmatrix} \quad Ky := \mathrm{lsolve}(Ay, By)$$

Fig.5 Pseudocode of the polynomial search algorithm p(y)

where - Ay, is the matrix of coefficients in the polynomial under consideration, By is the vector of free terms of the polynomial, is a sample from the array [ere] parallel to the axis "0Y", Ky is the result of solving the specified system, is the vector of values of the found unknowns.

The derivation of the polynomial equation p(y) is also feasible using the function of "vectorization" of the results of solving the system of equations. Transform the vector "Ky" and "C2", Fig.6:

$$Ky = \begin{pmatrix} 0.011 \\ -0.044 \\ 6.852 \times 10^{-3} \\ 0.059 \\ 0.094 \\ 0.114 \\ 19.81 \end{pmatrix} \quad C2 := \begin{pmatrix} y^6 \\ y^5 \\ y^4 \\ y^3 \\ y^2 \\ y^1 \\ 1 \end{pmatrix} \quad p(y) := \begin{pmatrix} -1.952 \times 10^{-3} \\ 0.017 \\ -0.043 \\ 3.425 \times 10^{-3} \\ 0.077 \\ 0.137 \\ 19.81 \end{pmatrix} \cdot \begin{pmatrix} y^6 \\ y^5 \\ y^4 \\ y^3 \\ y^2 \\ y \\ 1 \end{pmatrix}$$

Fig.6 Pseudocode of the operation of vectorization of the polynomial p(y).

As a result, we also get an analytical expression of the equation of the polynomial p(y) of the sixth degree along the "0Y" axis, which in general will be:

$$p(y) = a \bullet y^1 + b \cdot y^2 + c \cdot y^3 - d \cdot y^4 + e \cdot y^5 - f \cdot y^6 + g \qquad (4)$$

and after the pseudocode calculations in Fig. 6:

$$p(y) = 0,137 \cdot y^1 + 0,077 \cdot y^2 + 0,003425 \cdot y^3 - 0,043 \cdot y^4 + 0,017 \cdot y^5 -- 0,001952 \cdot y^6 + 19.81 \qquad (5)$$

Graphically, we construct the desired function of the approximation polynomial modeling the relief surface of the area in question in the stated form (2), - as functions of superpositions of variables p(x) and p(y).

Let's transform the [ere](5x4) data array into a matrix representation form of type "D" with a size of (5x4) as follows:

$$D = matrix(5, 4, p) \quad (6)$$

In Fig.7, we present the traditional form of representation of the obtained results of modeling the surface relief in plan, in the traditional form - with the help of isolines corresponding to a given landscape, Fig.7a.

Figure 7b shows a graphical representation of the developed model (AMP) in the form of an analytical function of superpositions p(x) and p(y) (2):

a

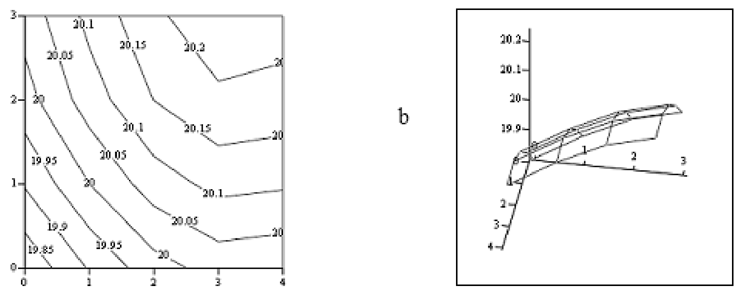

Fig.7(a, b) a) The developed relief model of the site in question is presented in the traditional form using equipotential level surfaces specified by the [ere] data array (5x4); b) also, in the form of a model volumetric surface of a given landscape.

We show the process of deriving the analytical function of the mid-planning plane for a previously defined mark «Hm» = 20.394м.

Generically, the average-planning surface is also represented by a polynomial of the sixth degree. Functionally, as the square root of the superpositions of functions p0(x) and p0(y). To do this, alternately, along the axes of the LSK "0X" and "0Y", we approximate the obtained results of calculating the average planning mark.

The system of polynomial equations passing through the maximum number of specified topographic points, the coordinates of the array [ere] along the axis "0X", in general form, we represent as follows:

$$p0(x) = a \bullet x^1 + b \cdot x^2 + c \cdot x^3 - d \cdot x^4 + e \cdot x^5 - f \cdot x^6 + g \quad (7)$$

Let's write down a system of equations along the "0X" axis, in which the unknowns are represented by the coefficients of the polynomial p0(x), as shown in Fig. 8

$$Ax0 := \begin{pmatrix} 0^6 & 0^5 & 0^4 & 0^3 & 0^2 & 0^1 & 1 \\ 1^6 & 1^5 & 1^4 & 1^3 & 1^2 & 1^1 & 1 \\ 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 1 \\ 3^6 & 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 1 \\ 4^6 & 4^5 & 4^4 & 4^3 & 4^2 & 4^1 & 1 \end{pmatrix} \quad Bx0 := \begin{pmatrix} 20.394 \\ 20.394 \\ 20.394 \\ 20.394 \\ 20.394 \end{pmatrix} \quad Kx0 := \mathrm{lsolve}(Ax0, Bx0)$$

$$Kx0 = \begin{pmatrix} 2.279 \times 10^{-14} \\ -2.105 \times 10^{-13} \\ 6.28 \times 10^{-13} \\ -5.678 \times 10^{-13} \\ -2.258 \times 10^{-13} \\ 3.423 \times 10^{-13} \\ 20.394 \end{pmatrix} \quad C10 \rightarrow \begin{pmatrix} x^6 \\ x^5 \\ x^4 \\ x^3 \\ x^2 \\ x \\ 1 \end{pmatrix} \quad p0(x) := \begin{pmatrix} 2.279 \times 10^{-14} \\ -2.105 \times 10^{-13} \\ 6.28 \times 10^{-13} \\ -5.678 \times 10^{-13} \\ -2.258 \times 10^{-13} \\ 3.423 \times 10^{-13} \\ 20.394 \end{pmatrix} \cdot \begin{pmatrix} x^6 \\ x^5 \\ x^4 \\ x^3 \\ x^2 \\ x \\ 1 \end{pmatrix}$$

Fig.8 Pseudocode of the algorithm for calculating the polynomial p0(x).

where Ax0, is the matrix of coefficients of the mean-planning surface, Bx0 is the vector of free terms of the polynomial, Kx0 is the result of solving the specified system, is the vector of values of the found unknowns. By performing "vectorization" and transformations with vectors "X0" and "C10" we get:

$$p0(x) = 3,423 \cdot 10^{-13} \bullet x^1 - 2,258 \cdot 10^{-13} \bullet x^2 - 5,678 \cdot 10^{-13} \bullet x^3 + \\ + 6,28 \cdot 10^{-13} \bullet x^4 - 2,105 \cdot 10^{-13} \bullet x^5 + 2,279 \cdot 10^{-13} \bullet x^6 + 20,394 \quad (8)$$

Let's write down a system of equations along the "0Y" axis, in which the unknowns are represented by the coefficients of the polynomial p0(y), - an array of size (4x7), Fig.9:

$$Ay0 := \begin{pmatrix} 0^6 & 0^5 & 0^4 & 0^3 & 0^2 & 0^1 & 1 \\ 1^6 & 1^5 & 1^4 & 1^3 & 1^2 & 1^1 & 1 \\ 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 1 \\ 3^6 & 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 1 \end{pmatrix} \quad By0 := \begin{pmatrix} 20.394 \\ 20.394 \\ 20.394 \\ 20.394 \end{pmatrix} \quad Ky0 := \mathrm{lsolve}(Ay0, By0)$$

$$Ky0 = \begin{pmatrix} 7.059 \times 10^{-14} \\ -3.561 \times 10^{-13} \\ 4.023 \times 10^{-13} \\ 1.345 \times 10^{-13} \\ -6.13 \times 10^{-14} \\ -2.004 \times 10^{-13} \\ 20.394 \end{pmatrix} \quad C20 \rightarrow \begin{pmatrix} y^6 \\ y^5 \\ y^4 \\ y^3 \\ y^2 \\ y \\ 1 \end{pmatrix} \quad p0(y) := \begin{pmatrix} 7.059 \times 10^{-14} \\ -3.561 \times 10^{-13} \\ 4.023 \times 10^{-13} \\ 1.345 \times 10^{-13} \\ -6.13 \times 10^{-14} \\ -2.004 \times 10^{-13} \\ 20.394 \end{pmatrix} \cdot \begin{pmatrix} y^6 \\ y^5 \\ y^4 \\ y^3 \\ y^2 \\ y \\ 1 \end{pmatrix}$$

Fig.9 Pseudocode of the algorithm for calculating the polynomial p0(y).

where Ay(0) is a matrix of coefficients in the polynomial under consideration, By0 is a vector of free terms of the polynomial, is a sample from the matrix (ere) parallel to the axis "0Y", Cy0 is the result of solving the specified system, is a vector of values of the found unknowns
As a result, we get:

$$p0(y) = 2,004 \cdot 10^{-13} \bullet y^{1} - 6,13 \cdot 10^{-14} \bullet y^{2} + 1,345 \cdot 10^{-13} \bullet y^{3} + \\ + 4,023 \cdot 10^{-13} \bullet y^{4} - 3,561 \cdot 10^{-13} \bullet y^{5} + 7,059 \cdot 10^{-14} \bullet y^{6} + 20,394 \quad (9)$$

Research results 4. We obtain a polynomial function from the calculated variables for constructing the average planning surface of the site under consideration, as the surface of "zero" works in the form of a function of two variables p0(x) and p0(y):

$$p0(x, y) = \sqrt[2]{p0(x) \bullet p0(y)} \quad (10)$$

Based on the results of (10), we will construct the desired surface of the "zero" works of the site under consideration, Fig.10:

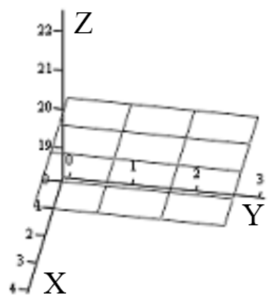

Fig.10 Model of the mid-planning surface under the "zero" balance at the mark 20,396м

Conclusion 1.

Four stages of algorithms of the derivation of analytical dependencies of models of the relief surface and the average planning surface of the territory under consideration for the "zero" balance of soil development of the considered area of the territory were created in the work, Fig. 11.

At the fourth stage, the joint system solution of the above dependencies allows you to automatically match the relative values corresponding to the elevation marks of the surface relief models and the average planning surface - the values of the volumes of the "excavation" and "embankment" of the soil under the "zero" balance.

The joint analytical solution of the obtained analytical dependencies allows us to obtain an exact algebraic solution of the line of "zero" works.

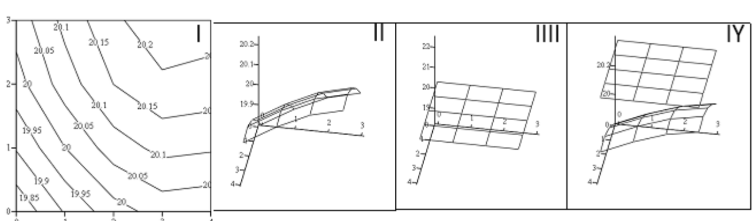

Fig.11 Visualization of the stages of algorithms of terrain modeling under the "zero" balance.

Where I,II,III and IY are the stages of algorithms.

Conclusion 2.

Thus, all the goals and objectives of the study set in the work have been achieved.

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