Digital simulation of load-bearing structures with movable cargo carts

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Abstract. The process of digital simulation of the design parameters of load-bearing structures when moving a cargo cart on them is developed on the example of a load-bearing beam of a crane (bridge, gantry, ...). This process includes three main stages. The first stage is the creation of appropriate mathematical multiparametric equations for automated simulation of: the values of reactions in the supports of the crane load-bearing beam; bending moments; transverse forces; displacements; beam deflection angles and maximum stresses at different positions of the cargo cart on the crane beam. The second stage is the creation of appropriate computer models in the Mathcad system using an effective functional programming language built into the system with a graphical representation of the simulation results depending on the location of the cargo cart on the load-bearing beam of the crane. The third stage is the animation simulation of the studied parameters, both for each parameter individually and all together.

Keywords: load-bearing structures, digital double of products, cargo carts, design parameters, digital models, mathematical and computer simulation.

1 Introduction

GOST R 57700.37—2021. Computer models and simulation. Digital twins of products. General provisions. Effective date 01.01.2022 [1]. It establishes the general concept of a digital twins of products, as well as general provisions and requirements for the development and application of digital twins of product. The digital twins involves the creation of mathematical and computer models for all stages of the product life cycle. In this paper, an attempt is made to digitally simulate the girder structures of cranes when a cargo cart moves along them at the early stages of their design. The widespread Mathcad system is used as computer simulation software.

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2 Materials and Methods

2.1 Formulation of tasks

To carry out digital simulation (mathematical and computer) of the internal forces of load-bearing structures of cranes with movable cargo carts with different input parameters (multi parametric). We will limit ourselves to digital simulation of girder load-bearing structures of cranes (gantry, bridge, ...) with known loads on each support \(F_1\) and \(F_2\) of the cargo cart. The general scheme of the action of loads on the beam load-bearing structure of the crane is shown in Fig. 1.

Fig. 1. The general scheme of the action of loads on the beam load-bearing structure of the crane during the movement of the cargo cart

Create multi parametric digital models and simulation internal forces with different parameters of the load-bearing structure of the crane and the mobile cargo cart:
- time and location of the wheels of the mobile cargo cart on the load-bearing beam;
- values of reactions in the supports of the cargo beam at different positions of the cargo cart;
- bending moments at different positions of the cargo cart on the beam: in analytical-software, numerical and graphical forms;
- transverse forces at different positions of the cargo cart on the beam: in analytical, software, numerical and graphical forms;
- movement of the load-bearing beam and its deflection angles;
- maximum stresses in the load-bearing beam.

2.2 Mathematical simulation

2.2.1 Simulation of the time of movement of the cargo cart \(T\) along the beam and the step of its movement in time \(\Delta t\), s

\[
T = \frac{L - L_t}{V}; \quad \Delta t = \frac{T}{S}
\]  

(1)

where:

- \(L\) is the length of the cargo beam, \(m\);
- \(L_t\) is the distance between the axles of the wheels of the cargo cart, \(m\);
- \(V\) is the speed of movement of the cargo cart, \(m/s\);
- \(S\) is the number of positions of the cargo cart on the beam.

2.2.2 Simulation of the positions of the 1st and 2nd supports of the wheels of the cargo cart on the beam at time – \(t\).

\[
LF_1(t) = V \cdot t; \quad LF_2(t) = LF_1(t) + L_t;
\]  

(2)

2.2.3 Simulation of the time of occurrence of the calculated position of the cargo cart \(s = 0,1, \ldots, S\) in time \(t\) and length \(x\):

\[
t = s \cdot \Delta t; \quad x = V \cdot t
\]  

(3)

2.2.4 Simulation of analytical expressions of the reactions of the load-bearing beam supports at different positions of the cargo cart depending on the time \(RA\) and \(RB\):
- the analytical expression of the line of influence of the reference reaction $RB$ at the reference point $B$ is determined from the sum of the moments relative to the reference point $A$

$$\sum M_A(x) = F_1 \cdot LF_1(t) + F_2 \cdot LF_2(t) - RB \cdot L = 0,$$

from where

$$RB = \frac{F_1 \cdot LF_1(t) + F_2 \cdot LF_2(t)}{L} = \frac{F_1 \cdot V \cdot t + F_2 \cdot (V \cdot t + Lt)}{L}.$$  \hspace{1cm} (4)

- the analytical expression of the line of influence of the reference reaction $RA$ at the reference point $A$ is determined from the sum of the moments relative to the reference point $B$

$$\sum M_B = F_1 \cdot (L - LF_1(t)) + F_2 \cdot (L - LF_2(t)) - RA \cdot L = 0,$$

from where

$$RA = \frac{F_1 \cdot (L - LF_1(t)) + F_2 \cdot (L - LF_2(t))}{L} = \frac{F_1 \cdot (L - V \cdot t) + F_2 \cdot (L - (V \cdot t + Lt))}{L}.$$ \hspace{1cm} (5)

### 2.2.5 Simulation of the bending moment in the beam at different positions of the cargo cart, kN·m.

The bending moment in the beam $M$ at a different position of the cargo cart - $x$ is equal to the sum of the moments of all forces acting on the beam for a given section on the beam:

$$M = MRA + MF_1 + MF_2.$$ \hspace{1cm} (9)

### 2.2.6 Simulation of transverse forces in the beam at different positions of the cargo cart, kN.

The transverse force in the beam $Q$ at a different position of the cargo cart - $x$ is equal to the sum of all the transverse forces acting on the beam:

$$Q = QRA + QF_1 + QF_2.$$ \hspace{1cm} (13)

### 2.2.7 Mathematical simulation of the displacements of the cargo beam and its deflection angles at different positions of the cargo cart by the method of initial parameters.

The movement of the beam at different positions of the cargo cart - $x$ is equal to the sum of the movements of the beam, depending on the action of all forces in a given section of the beam:

$$E_0 = E \cdot J \cdot u_0 + E \cdot J \cdot \theta_0;$$ \hspace{1cm} (14)

where: $E$ is Young's modulus, MPa. $E = 2 \cdot 10^5$ MPa;

$J$ is the moment of inertia, $cm^4$.

$$ERA = RA \cdot \frac{x^3}{6} \text{ if } x \geq 0$$ \hspace{1cm} (15)

where: $RA$ is the reaction in support $A$, N;

$LA$ is the length of the cargo to the support $A$, m.
\[ ERB = RB \cdot \frac{(x-L)^3}{6} \quad \text{if } x \geq L \]  - from the reaction RB in support B; (16)

where: RB is the reaction in support B, N;
L is the length of the cargo beam to the support B, m.

\[ EF1 = F1 \cdot \frac{(x-LF1(t))^3}{6} \quad \text{if } x \geq LF1(t) \]  - from the force F1 in the 1st support of the cart; (17)

where: F1 is the load transmitted through the 1st axle to the load beam, N;
LF1 is the distance from the beginning of the beam to the first axle of the cargo cart, m.

\[ EF2 = F2 \cdot \frac{(x-LF2(t))^3}{6} \quad \text{if } x \geq LF2(t) \]  - from the force F2 in the 2nd support of the cart (18)

where: F2 is the load transmitted through the 2nd axle to the load beam, N;
LF2 – distance from the beginning of the beam to the second axis of the cargo cart, m

\[ E = E0 + ERA + ERB + EF1 + EF2 \] ;

\[ u(x) = \frac{EJu(x)}{E \cdot J} \quad \text{if } x \geq 0 \] ;

2.2.8 Simulation of maximum stresses in the cargo beam at different positions of the cargo cart.

\[ \sigma_{\text{max}}(t) = \frac{M(x_{\text{max}})}{W} \] ; 

(21)

2.3 Computer simulation in the Mathead system
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Input data:
- \( L \): the length of the cargo beam, \( m \)
- \( V \): the speed of movement of the cargo cart, \( m/s \)
- \( F1, F2 \): loads on each support, \( N \)
- \( S \): the number of positions of the cargo cart on the beam
- \( L_t \): the distance between axes of the wheels of the cargo cart

Young's modulus, moment of inertia and moment of resistance of the cargo beam section:
- \( E = 2 \cdot 10^3 \text{ MPa} \)
- \( I = 20000 \text{ cm}^4 \)
- \( W = 2000 \text{ cm}^3 \)

Computer modeling:
1. The time of movement of the cargo cart \( T \) along the beam and the step of its movement in time \( \Delta t \), \( s \)

\[
T = \frac{L - L_t}{V} = 2.5 \text{ s} \\
\Delta t = \frac{T}{S} = 0.5 \text{ s}
\]

2. Positions of the 1st and 2nd supports of the wheels of the cargo cart on the beam at time \( t \):

\[
LF1(t) = V \cdot t \\
LF2(t) = LF1(t) + L_t
\]

3. The time of occurrence of the calculated position of the cargo cart \( s = 0, 1, \ldots S \) in time \( t \) and length \( x \):

\[
\delta = \text{FRAME} \\
\tau = s \cdot \Delta t \\
x = V \cdot t
\]

4. Reactions of the load beam supports at different positions of the cargo cart depending on the time, \( N \):

\[
RB = -\frac{F1\cdot V \cdot t + F2 \cdot (V \cdot t + L_t)}{L} = 6.667 \times 10^3 N \\
RA = -\frac{F1 \cdot (L - V \cdot t) + F2 \cdot (L - (V \cdot t + L_t))}{L} = 7.333 \times 10^4 N
\]

5. Bending moments and transverse forces in the beam at different positions of the cargo cart, \( kN \cdot m \):

\[
M(x) = \begin{cases} 
MRA & \text{if } x \geq 0 \\
MF1 & \text{if } x \geq LF1(t) \\
MF2 & \text{if } x \geq LF2(t) \\
M & = MRA + MF1 + MF2
\end{cases}
\]

\[
Q(x) = \begin{cases} 
QRA & \text{if } x \geq 0 \\
QF1 & \text{if } x \geq LF1(t) \\
QF2 & \text{if } x \geq LF2(t) \\
Q & = QRA + QF1 + QF2
\end{cases}
\]

6. Modeling of the displacements of the cargo beam and its deflection angles at different positions of the cargo cart by the method of initial parameters:

\[
EJh(x, u_0, \theta_0) = \begin{cases} 
E0 & = E \cdot J \cdot u_0 + E \cdot J \cdot \theta_0 \cdot x \\
ERA & = RA \cdot \frac{x^3}{6} & \text{if } x \geq 0 \\
ERB & = RB \cdot \frac{(x - L)^3}{6} & \text{if } x \geq L \\
EF1 & = F1 \cdot \frac{(x - LF1(t))^3}{6} & \text{if } x \geq LF1(t) \\
EF2 & = F2 \cdot \frac{(x - LF2(t))^3}{6} & \text{if } x \geq LF2(t) \\
E & = E0 + ERA + ERB + EF1 + EF2
\end{cases}
\]
2.4 Animation simulation in Mathcad system

For animation simulation:
- click on the Tools item in the main menu of the Mathcad system, and on the Animation item in the drop-down menu, and then on the Record item in the pop-up menu;
- set the Animation Recording in the dialog box that appears, in the section For FRAME in the fields With, By and Frame Rate/seconds, respectively: 0, 6 and 1;
- select the simulation graph with the mouse frame;
- click the Animate button in the Animation Recording dialog box. At the end of the recording process, the Play Animation window will appear Fig. 2.
- click the **Playback** button in the **Play Animation** dialog box. The process of showing frames of an animation file with the results of simulation movements, maximum stresses and internal forces in graphic form in the crane beam at different positions (cross section on the beam) of the cargo cart will begin. In the **Animation Recording** dialog box, the **Save As** button is activated;
3 Results

As a result of the conducted research, an interconnected complex of digital parametric mathematical, computer and animation models has been developed that allow automated determination of the values of reactions in the crane load beam supports; bending moments; transverse forces; displacements; beam deflection angles and maximum stresses at various positions of the cargo cart on the crane beam with the possibility of issuing simulation results in graphical forms, reducing by the order of the time of the research. This approach can be widely used in conducting similar studies and for other products in other industries.

References