Comparison of turbulence models for the axisymmetrical separated boundary layer problem

Murodil Madaliev[1,2], *Mavlonbek* Usmonov[2,*], *Oybek* Soliyev[2], *Jamshidbek* Otajonov[3], *Khasanboy* Kadirov[3], *Zamira* Otakhanova[3], and *Dilnoza* Mavlonova[3]

[1]Institute of Mechanics and Seismic Resistance of Structures of the Academy of Sciences of the Republic of Uzbekistan, Tashkent, Uzbekistan
[2]Fergana polytechnic institute, Fergana, Uzbekistan
[3]Ferghana branch of Tashkent University of Information Technology, Ferghana, Uzbekistan

Abstract. The article is devoted to the study of the influence of turbulent models on various processes in nature and technology. The paper presents the results of applying the standard one-parameter turbulence model SA, two-parameter turbulence model SST and three-parameter turbulence model v2-f in the Comsol Multiphysics 6.1 software package for the problem of an axisymmetric separated boundary layer. These results are then compared with experimental data from the NASA TMR database. Analysis of the results obtained shows that the conclusions obtained using various turbulent models are different, despite the unique formulation of the problem. This study makes a significant contribution to the field of numerical modeling of turbulent phenomena and serves as a basis for further developments in engineering practice and scientific research. Keywords: Navier–Stokes equations, separated flow, SST model, SA model, v2-f model, Comsol Multiphysics, NASA.

RETRACTED

1 Introduction

Turbulent flows play a key role in a variety of phenomena, both in natural and technical systems. Understanding and modeling them are important challenges for engineering practice and scientific research. In this article we will focus on studying the influence of various turbulence models on processes occurring both in nature and in technology [1–2]. Many engineering problems are studied using computer simulations that involve modeling turbulent flows. However, selecting an appropriate turbulence model is challenging because different models may produce different results even with the same input data. In this paper, we study three different turbulence models: the standard one-parameter SA model, the two-parameter SST model, and the three-parameter v2-f model. We will apply these models in the Comsol Multiphysics 6.1 software package to analyze an axisymmetric separated boundary layer. The purpose of our study is to compare the results obtained using various

* Corresponding author: mavlonbek5488@gmail.com

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).

turbulence models and compare them with experimental data from the NASA TMR database [3].

Our analysis will allow us to evaluate how well different turbulence models correspond to real phenomena. The results obtained are important both for engineering practice and for scientific research in the field of numerical modeling of turbulent flows. They can serve as a basis for further developments and improvements in the field, facilitating more accurate and reliable modeling of complex turbulent phenomena.

2 Mathematical model

To solve this problem, the Nava-Stokes equation was used. The Navier-Stokes equations are a system of differential equations that describe the movement of a liquid or gas. They are formulated on the basis of the law of conservation of mass and Newton's second law for a continuum medium. Here is the general form of the Navier-Stokes equations for an incompressible fluid in three dimensions:

Mass conservation equation (continuity equation):

$$\nabla \mathbf{v} = 0 \tag{1}$$

Equation of motion (Navier-Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\nabla\mathbf{v} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{v} + \mathbf{F} \tag{2}$$

Where:
- v is the fluid velocity vector,
- t - time,
- p - pressure,
- ρ - density,
- ν - kinematic viscosity,
- F - external force acting on the fluid,
- ∇ is the nabla operator that determines the gradient and divergence of the vector field.

The first equation, known as the Navier-Stokes equation of motion, formulates the dynamic change in fluid velocity as a function of time, external forces, viscosity and pressure. The second equation, the continuity equation, ensures that the mass of the fluid in the system is conserved. These equations form the key system of differential equations that describe incompressible fluid motion and are used in a wide range of scientific and engineering applications. The Navier-Stokes equations serve as the foundation for numerical methods such as Computational Fluid Dynamics (CFD), allowing for deeper study and optimization of fluid flows under different conditions. The Navier-Stokes equations for an incompressible fluid, which describe the motion of a fluid under the influence of viscous forces, are nonlinear. This means that nonlinear terms are present in these equations and depend on the derivatives of unknown functions. This creates difficulties in the analytical solution of these equations and requires the use of numerical methods to obtain approximate solutions [4].

2.1. Reynolds approach

The Reynolds approach is a method of decomposing the flow into average and fluctuation (pulsation) components. This approach is widely used in turbulent fluid dynamics for the analysis and modeling of turbulent flows. Reynolds' approach represents an important tool in the mechanics of turbulent transport in fluids. Based on the averaged Navier-Stokes equations, this method describes the motion of a turbulent medium by dividing the velocity and pressure variables into mean and pulsational values and averaging them over time and

RETRACTED

space. Thus, equations are obtained for the average values of the variables, which can be solved numerically or analytically. Reynolds' approach, although approximate, successfully describes the average behavior of a turbulent medium without going into the details of turbulent fluctuations.

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}',$$
$$p = \overline{p} + p'. \tag{3}$$

The Reynolds equation is not a closed equation. Therefore, various semi-empirical turbulence models are used to close this equation.

2.2. Turbulence models

The SA (Spalart-Allmaras) turbulence model [5] is one of the most common and simple models for predicting turbulent flows. It was developed by Frank Spalart and Eric Almaraz in 1992 and has been widely used in engineering practice. The main idea of the SA model is that it uses a single equation to predict the variable ε, which is the analogue of kinematic viscosity in the k-ε model. This equation is the transport equation for turbulent viscosity $v_t = \tilde{v} f_{v1}$, where f_{v1} is a function that corrects turbulent viscosity.

The turbulent viscosity equation in the SA model is as follows:

$$(\mathbf{U}\cdot\nabla)\tilde{v} = C_{b1}\tilde{S}\tilde{v} - C_{w1}f_w\left(\frac{\tilde{v}}{d}\right)^2 + \frac{1}{\sigma_v}\nabla\left([(v+\tilde{v})\nabla\tilde{v}\right] + \frac{1}{\sigma_v}C_{b2}\nabla\tilde{v}\nabla\tilde{v}. \tag{4}$$

Advantages of the SA model include its relative simplicity and low computational cost compared to more complex models. It works well for a wide range of flows, including boundary layers and flows around bodies. However, like any model, it has its limitations and may produce inaccurate results in some special cases, such as highly separated flows or flows with high viscosity gradients.

Menter's shear stress transfer (SST) model [6–7] is a combination of the k-ε and k-ω models. In this scheme, k-ω is used for the wall layer, while k-ε is used for the outer region. At the moment, this model is very popular and is included in many computational fluid dynamics programs. It is important to note that it provides effective tools for modeling the transfer of shear stresses and is successfully used in various packages for calculating hydrodynamic processes.

$$\begin{cases}(\mathbf{U}\cdot\nabla)k = \nabla\left[(v+\sigma_k v_t)\nabla k\right] + P - \beta^*\omega k,\\ (\mathbf{U}\cdot\nabla)\omega = \nabla\left[(v+\sigma_\omega v_t)\nabla\omega\right] + \frac{\gamma}{v_t}P - \beta\omega^2 + 2(1-F_1)\frac{\sigma_{\omega 2}}{\omega}\nabla\omega\nabla k.\end{cases} \tag{5}$$

Here k is the specific turbulent kinetic energy (m2 s−2), ω is the specific rate of turbulent dissipation (s−1). Additional data are presented in studies [6–7]. These works contain details and extended information about other meanings.

v2-f turbulence model: Near solid walls, the intensity of velocity fluctuations in the direction tangential to the wall is usually much greater than the intensity of fluctuations in the direction normal to the wall. In other words, velocity fluctuations are characterized by anisotropy. As you move away from the wall, the intensity of fluctuations in all directions becomes the same. Velocity fluctuations become uniform or isotropic. The anisotropy of turbulent fluctuations in the boundary layer is described by the v2-f turbulence model by introducing two additional equations, solved together with the equations for the kinetic energy of turbulence (k) and the rate of dissipation of kinetic energy (ε).

RETRACTED

$$\begin{cases} (\mathbf{U}\cdot\nabla)k = \nabla\left[(v+\frac{v_t}{\sigma_k})\nabla k\right] + P - \varepsilon, \\ (\mathbf{U}\cdot\nabla)\varepsilon = \nabla\left[(v+\frac{v_t}{\sigma_\varepsilon})\nabla\varepsilon\right] + \frac{1}{\tau}\left(C_{\varepsilon 1}'(\zeta,\alpha)P_k - C_{\varepsilon 2}'(k,\varepsilon,\alpha)\varepsilon\right), \\ (\mathbf{U}\cdot\nabla)\zeta = \nabla\left[(v+\frac{v_t}{\sigma_\zeta})\nabla\zeta\right] + \frac{2}{k}\left[\alpha^3 v + \frac{v_t}{\sigma_\zeta}\right]\nabla k\nabla\zeta + (1-\alpha^3)f_w + \alpha^3 f_h - \frac{\zeta}{k}P_k, \end{cases} \quad (6)$$

Turbulent eddy viscosity is calculated by: $\nu_t = C_\mu k\zeta\tau$. The remaining coefficients and functions were presented in the article [8–9].

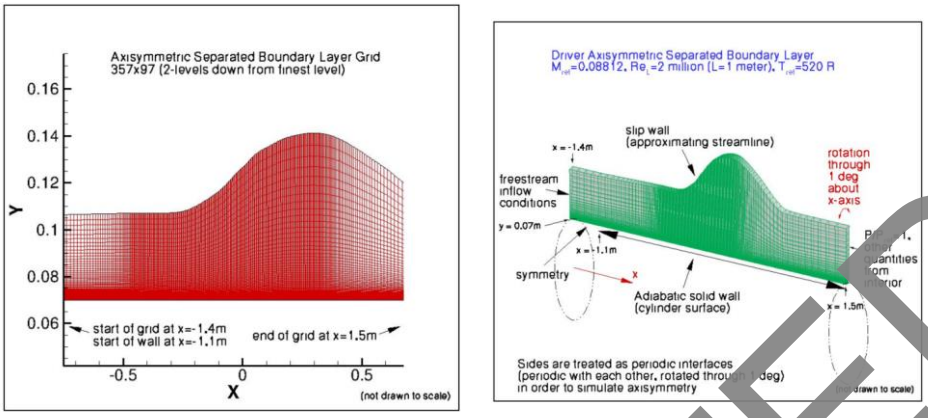
3 Solution method

COMSOL Multiphysics offers a variety of solvers to solve problems of various physics phenomena. Selecting the appropriate solver depends on the type of physics being modeled, the complexity of the problem, the required accuracy, and the available computing resources. To solve the equations of the two-phase turbulent model, a Fully Coupled method was used using the PARDISO direct solver algorithm. Newton's iterative method was used with a damping coefficient of 0.1. For the problem, the iterative process continued up to 150 iterations. The tolerance factor is set to 1 and the residual factor to 1000.

4 Physical statement of the problem

The goal here is to provide an example of testing turbulence models. Unlike verification, which aims to establish whether a model has been implemented correctly, verification compares CFD results to data to establish the model's ability to reproduce physics. If desired, a large sequence of nested meshes of the same family is provided here. Data is also provided for comparison. For this particular case of a "virtually incompressible" axisymmetric separated boundary layer (from Driver) [10], the data are taken from experiment. The experiment used a cylinder with a diameter of 0.140 m in a tunnel in which an unfavorable pressure gradient was created for part of the flow due to the divergence of the four walls of the tunnel. Each wall was deflected by as much as 0.045 m, resulting in an area expansion factor of approximately 1.6. During the experiment, the boundary layers of the tunnel side wall were thinned by suction. In Experimental Case C (used here), the unfavorable pressure gradient was so strong that the flow in the cylinder separated, resulting in a bubble approximately 0.2 m long. The experimenter provided a streamline shape well outside the boundary layer of the cylinder that can be used as inviscid surface to determine the upper boundary condition in CFD simulations. The inflow into the region is controlled such that the naturally developing turbulent boundary layer on the cylinder in the CFD solution increases to approximately 0.012 m in thickness near the position x = -0.3 m (immediately upstream of the unfavorable pressure gradient) of Fig. 1, as noted into the experiment. A short symmetrical region is introduced upstream to avoid possible incompatibilities between the freestream inflow and the BC walls. (Note that certain variants of the inflow and outflow BC can also work and produce similar results for this problem.) It is important to note that this axisymmetric case is not a two-dimensional calculation; it uses a periodic (rotated) mesh with appropriate boundary conditions on the periodic sides of the mesh.

RETRACTED

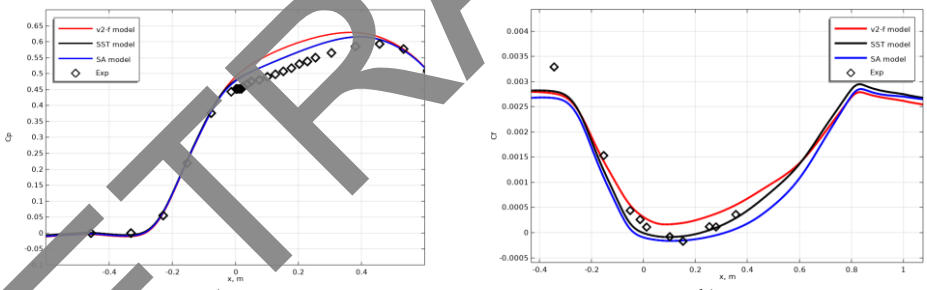


a) b)

Fig. 1. Axisymmetric Separated Boundary Layer a) computational mesh and b) boundary conditions.

5 Results and discussion

Below are comparisons of numerical results with known experimental data. Figure 2 shows the following graphs: a) dependence of the pressure friction coefficient in the lower part of the channel; b) dependence of the pressure friction coefficient in the upper part of the channel; c) friction coefficient in the lower part of the channel, as well as comparative experimental results.



a) b)

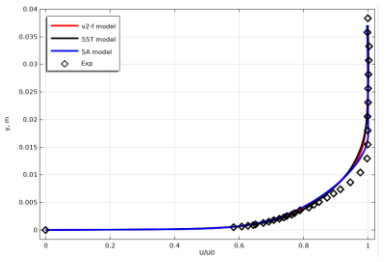
Fig. 2 a) dependence of the coefficient of friction pressure on the lower part of the channel; b) friction coefficient of the lower part of the channel as well as the results of the experiment.

It can be seen from Fig. 2a that the pressure coefficient shows similar results in the SA and SST models, while in the v2-f model it exceeds the values obtained in the experiment. This observation indicates that the v2-f model does not fit the experimental data in this case. Possible reasons for this difference may include insufficient consideration of turbulent effects in the v2-f model, as well as insufficient accuracy in modeling the physical processes occurring in this flow [11–12]. Such differences between simulation results and experimental data can be caused by various factors, such as simplifications in turbulence models not taken into account in the simulated conditions, as well as inaccuracies in the modeling of geometry and boundary conditions [13–14]. To improve the consistency of simulations with experimental results, careful calibration of model parameters or the use of more complex turbulence models that can better describe the flow physics in a given case may be required. Additional analysis of simulation results may also be required to identify and correct potential

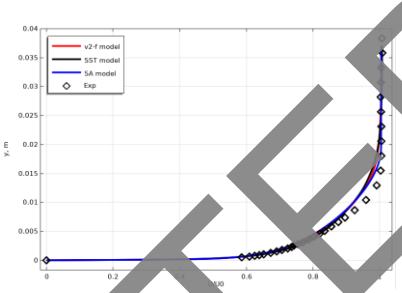
RETRACTED

causes of differences between model and experimental data. From Figure 2b, represented by the drag coefficient, it can be observed that the SST model exhibits closer results to the experimental data compared to other models.

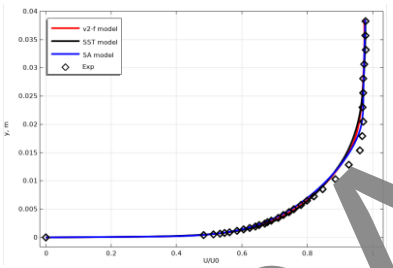
In Fig. Figures 3-4 show the profiles of the longitudinal velocity U and the turbulent stress $\overline{u'\vartheta'}$ profiles along the lower surface of the channel at different sections along the flow, respectively.



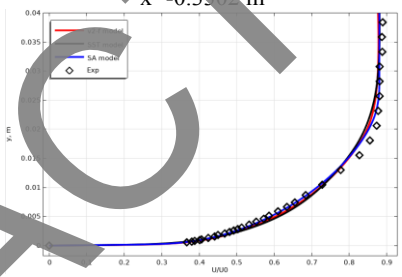
x=-0.4572 m



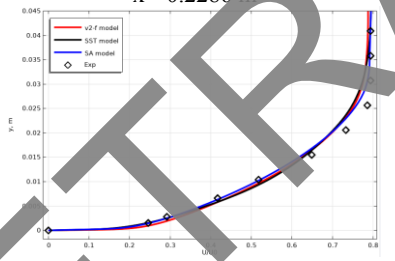
x=-0.3302 m



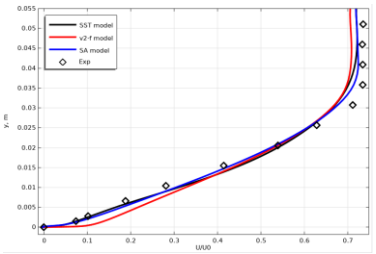
x=-0.2286 m



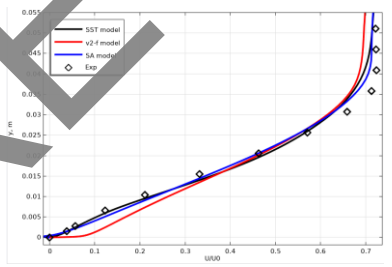
x=-0.1524 m



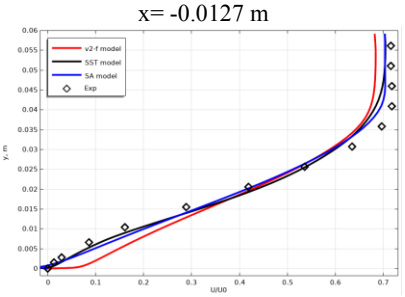
x=-0.0762 m



x= -0.0127 m

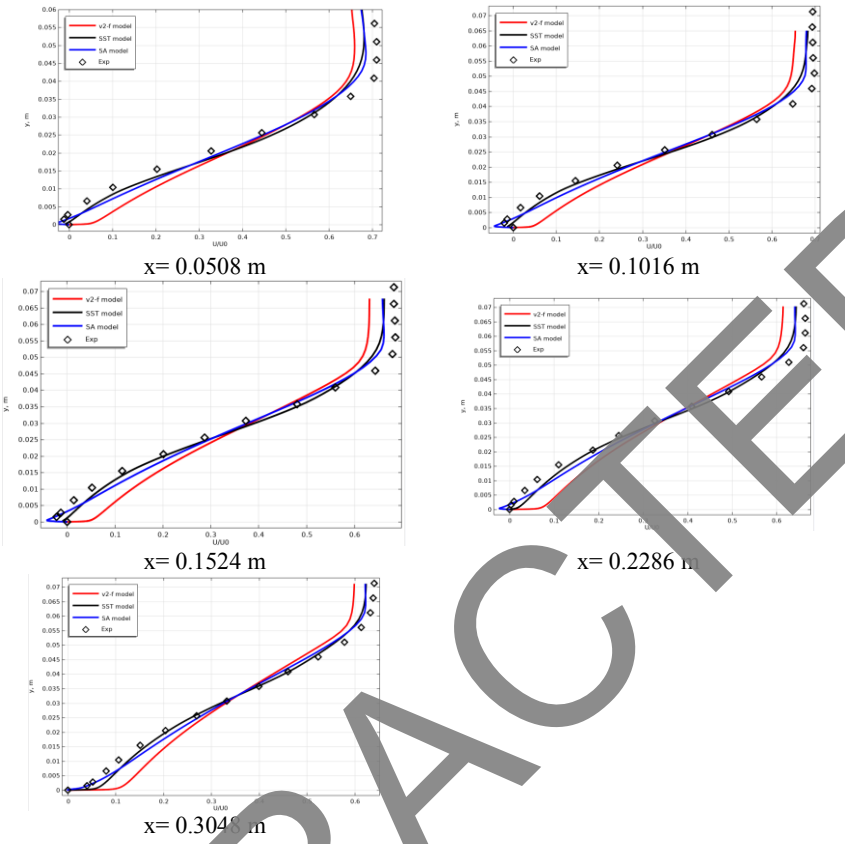


x= 0.0127 m



x= 0.0254 m

RETRACTED

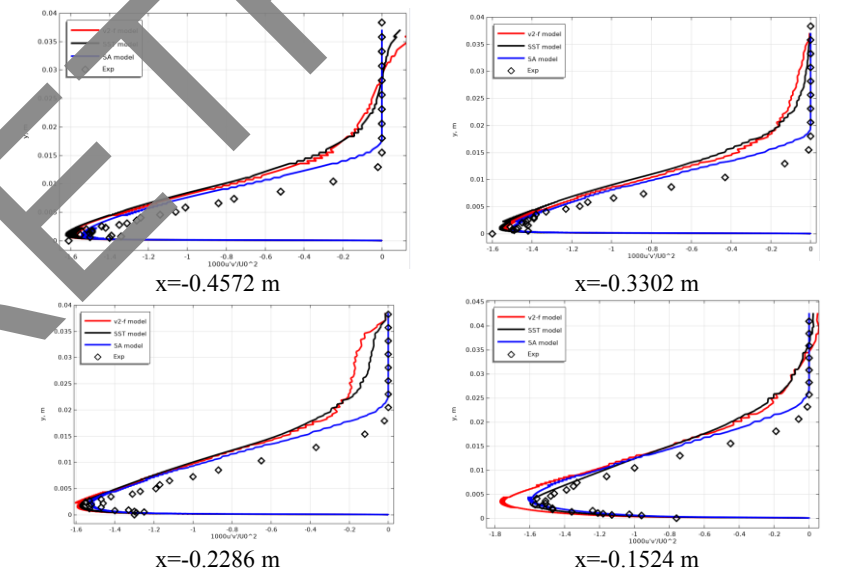


x= 0.0508 m x= 0.1016 m

x= 0.1524 m x= 0.2286 m

x= 0.3048 m

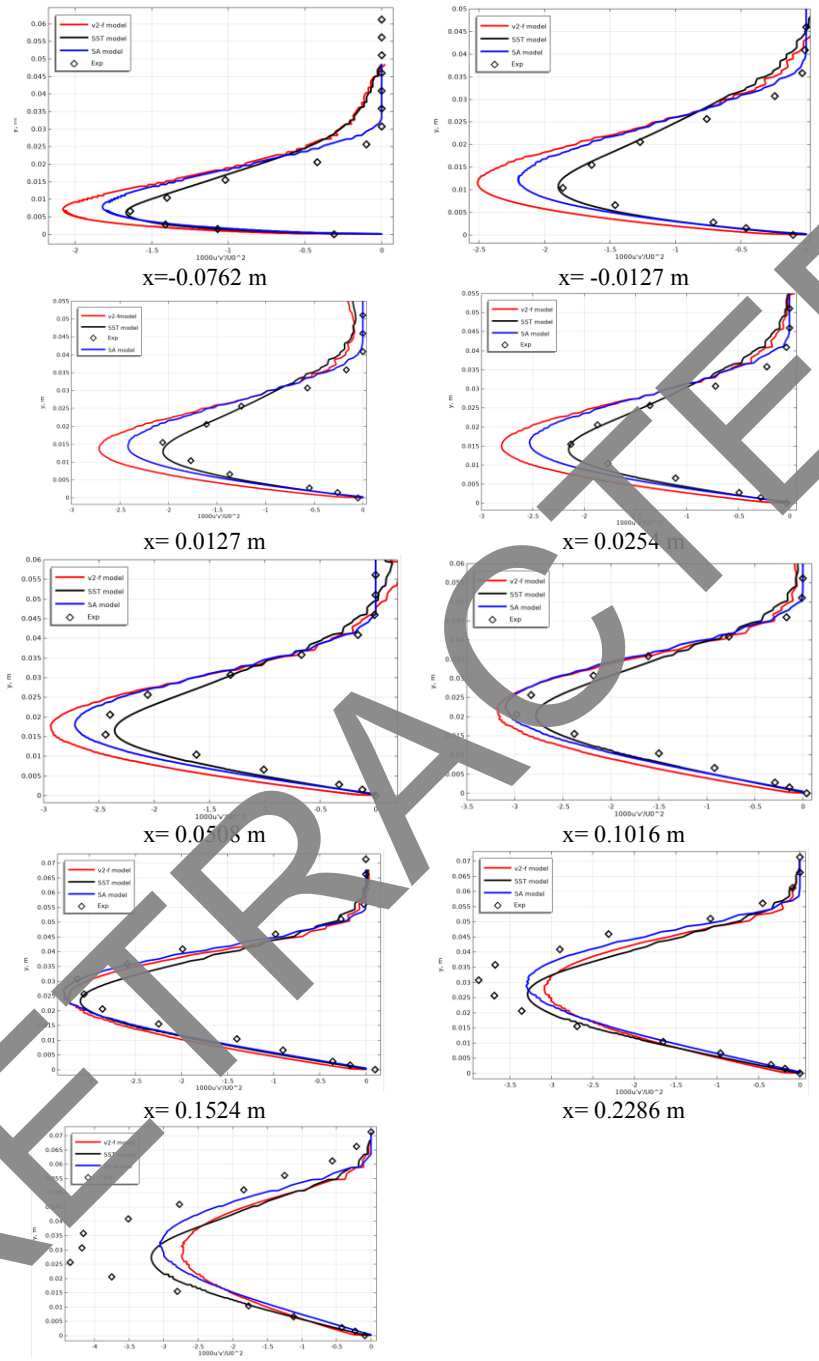
Fig. 3. Longitudinal velocity profiles on the lower surface.



x=-0.4572 m x=-0.3302 m

x=-0.2286 m x=-0.1524 m

RETRACTED



RETRACTED

Fig. 4. Reynolds stress velocity profiles on the bottom surface

From Figures 3 and 4, it appears that the results obtained using the SST model show closer agreement with the experimental data than the SA and v2-f models. This observation confirms the advantages of the SST model in this particular problem and highlights its ability to more accurately predict physical phenomena in the flow. The SST model, which combines

the features of the k-ω and k-ε models, generally provides good agreement with experimental data in various types of flows, including separated flows and flow around bodies [15–25]. However, it is important to remember that simulation results depend on many factors, including mesh selection, boundary conditions, and model parameters. In some cases, other turbulence models may also show good agreement with experiment [26–40]. Thus, based on the data presented, it can be concluded that the SST model may be more preferable for this task, but other factors must also be taken into account and additional analyzes must be carried out to make final conclusions.

6 Conclusion

In this paper, the influence of different turbulence models on the prediction of an axisymmetric separated boundary layer was investigated. The SA, SST and v2-f models were applied to simulate this phenomenon using a CFD software package. Analysis of the results showed that the SST model shows a closer fit to the experimental data compared to the SA and v2-f models. In particular, the pressure and drag coefficients obtained using the SST model showed better agreement with the experimental data. These results indicate the advantages of the SST model in this particular flow problem. The SST model, which combines features of the k-ω and k-ε models, provides a wider range of applicability and more accurate prediction of turbulent flows, including separated flows. However, it should be noted that the choice of a turbulence model should take into account the specifics of a particular task and flow characteristics. In some cases, other turbulence models can also be effective and show good agreement with experiment. Thus, the results of this study highlight the importance of proper choice of turbulence model in numerical simulation of turbulent flows and provide a basis for further research in this area.

References

1. Orozco Murillo, W., Palacio-Fernande, J. A., Patiño Arcila, I. D., Zapata Monsalve, J. S., Hincapié Isaza, J. A. (2020). Journal of Applied and Computational Mechanics, **6(Special Issue)**, 1228-1244.
2. Hadad, K., Eidi, H. R., Mokhtari, J. (2017). Journal of Applied and Computational Mechanics**, 3(3),** 171-177.
3. "Turbulence modeling Resource. NASA Langley Research Center", http://turbmodels.larc.nasa.gov.
4. Semyabov A.V., Gavrilov A.A., Dekterev A.A. Thermophysics and aeromechanics. **18:1,** 73-85. 2011.
5. P. Spalart and S. Allmaras, *"A one-equation turbulence model for aerodynamic flows,"* in 30th aerospace sciences meeting and exhibit, 1992, p. 439.
6. Menter F. R., Kuntz M., Langtry R. *"TenYears of Industrial Experience with the SST Turbulence Model".* Turbulence, Heat and Mass Transfer 4, ed: K. Hanjalic, Y. Nagano, and M. Tummers, Begell House, Inc., 2003, pp. 625 - 632.
7. Pasha, A. A. (2018). Journal of Applied and Computational Mechanics, **4(2),** 95-104.
8. P.A. Durbin, Int. J. Heat and Fluid Flow, **14**, 4, 1993.
9. K. Hanjalic, M. Popovac and M. Hadziabdic, Int. J. Heat and Fluid Flow **25**, 6, 2004.
10. Driver, D. M., *"Reynolds Shear Stress Measurements in a Separated Boundary Layer Flow,"* AIAA Paper 91-1787, from the AIAA 22nd Fluid Dynamics, Plasma

RETRACTED

Dynamics, and Lasers Conference, June 1991, Honolulu, HI, https://doi.org/10.2514/6.1991-1787.

11. Malikov, Z., Mirzoev, A., Madaliev, M., Yakhshibayev, D., Usmonov, A. (2021). *Numerical simulation of flow through an axisymmetric two-dimensional plane diffuser based on a new two-fluid turbulence model.* In 2021 International Conference on Information Science and Communications Technologies (ICISCT) (pp. 1-4). IEEE.
12. Malikov, Z. M., Mirzoev, A. A., Madaliev, M. (2022). Journal of Computational Applied Mechanics, **53(2),** 282-296.
13. Madaliev, E., Madaliev, M., Mullaev, I., Sattorov, A., Ibrokhimov, A. (2023). AIP Conference Proceedings **2612**, 1
14. Malikov, Z. M., Madaliev, M. E. (2022). Journal of Wind Engineering and Industrial Aerodynamics, **231**, 105171.
15. Malikov, Z. M., Madaliev, M. E. (2021). Herald of the Bauman Moscow State Technical University, Series Natural Sciences, **4(97),** 24-39.
16. Malikov, Z. M., Madaliev, M. E. (2021). Vestnik Tomskogo gosudarstvennogo universiteta. Matematika i mekhanika, **(72),** 93-101.
17. Mirzoev, A. A., Madaliev, M., Sultanbayevich, D. Y. (2020). *Numerical modeling of non-stationary turbulent flow with double barrier based on two liquid turbulence model.* In 2020 International Conference on Information Science and Communications Technologies (ICISCT) (pp. 1-7). IEEE.
18. Madaliev, E., Madaliev, M., Adilov, K., Pulatov, T. (2021). E3S Web of Conferences **264**, 01009
19. Rasulov, R., Mahkamova, D. (2024). AIP Conference Proceedings **3004**, 1
20. Hayotov, A. R., Rasulov, R. G. (2020). Lhomat, **34(11),** 3835-3844.
21. Muminov, K. K., Gafforov, R. A. (2024). Journal of Mathematical Sciences, **278(4),** 623-632.
22. Sharipov, A., Topvoldiyev, F. (2023). Conference Proceedings **2781**, 1
23. Hayotov, A., Bozarov, B. (2021). AIP Conference Proceedings **2365**, 1, 020022
24. Shadimetov, K., Daliyev, B. (2021). AIP Conference Proceedings **2365**,1, 020025
25. Artykbaev, A., Madaliyev, B.M. (2023). Lobachevskii Journal of Mathematics, **44(4),** 1251-1255.
26. Madaliev, M., Yunusaliev, E., Usmanov, A., Usmonova, N., Muxammadyoqubov, K. (2023). E3S Web of Conferences **365**, 01011
27. Tojiev, R., Yunusaliev, E., Abdullaev, I. (2021). E3S Web of Conferences **264**, 02044
28. Madaliev, M., Usmonov, M., Kadyrov, K., Abdullajonov, N., Mavlonova, D., Otakhanova, Z., Muminov, K. (2024) E3S Web of Conferences **508**, 06005
29. Madaliev, M., Usmonov, M., Otajonov, J., Bilolov, I., Otakhanova, Z., Rajabova, K., Israilov, S. (2024) E3S Web of Conferences **508**, 06003
30. Ibrokhimov, A., Orzimatov, J., Usmonov, M., Otakulov, B., Mirzababayeva, S. (2024). BIO Web of Conferences **84**, 02026
31. Abdulkhaev, Z., Abdujalilova, S., Usmonov, M., Askarov, K., Nazirova, R. (2024). BIO Web of Conferences **84**, 05040
32. Bilolov, I., Otajonov, J., Isroilov, S., Mavlonova, D., Abdurakhmonov, S., Aliev, I. (2024). E3S Web of Conferences **508**, 05005

RETRACTED

33. Madaliev, M., Abdulkhaev, Z., Otajonov, J., Kadyrov, K., Bilolov, I., Israilov, S., Abdullajonov, N. (2024). E3S Web of Conferences **508**, 05007
34. Salomov, U., Madaliyev, M., Kuchkarov, A. (2024). BIO Web of Conferences **84**, 02024
35. Salomov, U. R., Chiavazzo, E., Asinari, P. (2014). Computers & Mathematics with Applications, **67(2),** 393-411.
36. Salomov, U. R., Chiavazzo, E., Fasano, M., Asinari, P. (2017). International Journal of Hydrogen Energy, **42(43),** 26730-26743.
37. Salomov, U., Abduraxmonov, S., Urishev, O., Juraev, N. (2024). BIO Web of Conferences **84**, 05028
38. Lesnikova, E. P., Jakhongirov, I. J. O., Sadykova, K. V., Zakharova, T. I., Santalova, M. S. (2021). *Management of innovative working behavior*. In Modern Global Economic System: Evolutional Development vs. Revolutionary Leap II (pp. 1008-1016). Springer International Publishing.
39. Obrenovic, B., Gu, X., Wang, G., Godinic, D., Jakhongirov, I. (2024). *Generative AI and human–robot interaction: implications and future agenda for business, society and ethics.* AI & SOCIETY, 1-14.
40. Usarov, M., Ayubov, G., Usarov, D., Mamatisaev, G. (2022). *Spatial Vibrations of High-Rise Buildings Using a Plate Model*. In Proceedings of MPCPE 2021: Selected Papers (pp. 403-418). Cham: Springer International Publishing.

RETRACTED