

# Estimation of Probability Distribution Function and Wind Energy Potential for Higher Heights

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**Abstract:** This paper focuses on analyzing wind characteristics at Maulana Azad National Institute of Technology. Wind characteristics are site dependents, so it is reasonable that different wind regimes may have different wind speed distributions. Distributions appeared in the literature are revisited and a comparison between these distributions is carried out. Modified maximum likelihood method in Weibull distribution has shown better results over other. Justus and Mikhail method is used for vertical extrapolation of wind power density for higher heights. As a conclusion, the highest wind energy potential value was found in the month of June while the lowest value was encountered in November.

**Keywords:** Probability distribution functions; weibull distribution; wind power potential; numerical methods

## 1. Introduction

Wind is an old established source of energy which is clean, renewable, and abundant, in principle, accessible at no cost [8]. After the industrial revolution, Energy created its path in determining the country development, from then it is viewed as an indicator for the development of a region. Harnessing energy from the wind is a longstanding pursuit that dates back to ancient times, starting with the use of wind to propel ships, operate grain

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mills, and power threshing machines. The serious consideration of developing 'high speed' wind turbines for electric power generation only began at the start of the last century. Wind speed fluctuation is the major criterion for the energy capture by the turbine. Therefore, detailed knowledge about wind characteristics is a crucial parameter and has a particular importance in selecting wind energy conversion system [1]. Choice of probability distribution function which fits best can provide accurate value of wind energy potential of a region for a given period of time.

Nurulkamal Masseran et al. (2013) utilized nine distinct statistical distributions, namely Log Normal, Weibull, Rayleigh, Exponential, Gamma, Inverse Gamma, Burr, and Inverse Gaussian, to fit the mean hourly wind speed data in East Malaysia. Their findings indicated that the Gamma and Burr distributions provided the best fit for all regions in East Malaysia. In a separate study, Abul Kalam Azad et al. (2013) conducted a statistical diagnosis of Weibull parameter estimation methods. They compared seven different models and performed a statistical analysis using six statistical tools for wind speed data collected at kuakata, Kutubdia, and Sitakunda, with the aim of assessing wind power for agricultural applications. Their results demonstrated that the method of moments and maximum likelihood methods were efficient in determining the Weibull parameters in these regions. Z.R. Shu et al. (2013) investigated the wind energy potential in Hong Kong based on six years of observation data from three meteorological stations. They observed seasonal variations in the Weibull parameters, with the lowest value of the scale parameter occurring in August. The variations in wind potential were found to align well with the mean speed variation. Lastly, Levent Bilir et al. (2013) conducted an analysis on the seasonal and yearly wind speed distribution and wind power potential based on wind speed data collected from June 2012 to June 2013 at Atilium University. They calculated the seasonal and yearly parameters of the Weibull distribution using hourly average wind speed data and employed five different methods for their calculations. They found variations of wind potential with seasons; maximum value of wind power density was observed in winter and lowest in autumn. F.C.Odo, Gerald Ugwu Akubue [6] made comparison between three weibull parameters estimation methods for wind energy applications in Nigeria. They found that Moment method suits best for that location whereas graphical method does not perform well.

### *1.1. Case description*

MANIT is situated in the central state of India, Madhya Pradesh, positioned at 23012'25" N Latitude and 77024'29" E Longitude, with an average elevation of 537 m above sea level. It is located in the state capital, Bhopal, which experiences a humid subtropical climate characterized by mild, dry winters, hot summers, and a humid monsoon season. The hottest months are May and June, while January is the coldest month. The hourly time series wind data of speed and direction of the year 2004-2005, measured and recorded (under the sponsored project 'Development of Wind Energy Laboratory' of MHRD, GOI, scheme of thrust areas in Technical education, grant No.F. 27/2003) from NRG systems Anemometer Direction Sensor and Data Logger on 10m and 25m above ground level at MPSTEP, MANIT, Bhopal, India.

## **2. Methodology**

### *2.1. Probability distribution function*

As wind speed is a random variable, efficient determination of probability distribution function (PDF) is the major criterion for the estimation of wind power generation characteristics. Hence, it is imperative to possess adequate understanding of the probabilistic distributions of wind speed when evaluating the wind energy potential for a particular location. A variety of PDFs have been appeared in the literature to fit wind speed

frequency distributions. Efficient PDF can reduce the uncertainties in the wind energy potential estimation so PDF estimation is a worthy step. Wind characteristics are site dependents, so it is reasonable that different wind regimes may have different wind speed distributions. Although Weibull distribution is well known in literature, it is a purely empirical form, with no base in probability theory so, distributions appeared in the literature are selected that are Exponential (EX), Gamma (GA), Inverse gamma (IGA), Log normal (LN), Rayleigh (RY), Weibull (WE) distributions are revisited. Table 1, lists the probability density function with their respective cumulative distribution functions.

## 2.2. Maximum likelihood estimator

Given the varying lengths of the wind data records, the disparities between parameter estimation methods are not as significant as the disparities between distributions. Our preference lies with maximum likelihood estimators (MLE) as they typically result in lower mean square errors (MSE) compared to alternative methods. Nevertheless, we opt for the Method of Moments when estimating parameters for GA and IGA to mitigate any potential complexities [5,7]. Efficiency of the numerical methods is judged with: Root mean square error (RMSE), Variance ( $R^2$ ) and chi – square ( $X^2$ ) [4].

Table 1. List of probability density function [5]

Model	Probability density function
EX	$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$
GA	$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{(\alpha-1)} e^{-\left(\frac{x}{\beta}\right)}$
IGA	$f(x) = \frac{\beta^\alpha}{\Gamma(p)} x^{(-\alpha-1)} e^{-\left(\frac{\beta}{x}\right)}$
LN	$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\left(\frac{(\ln x - \mu)^2}{2\sigma^2}\right)}$
RY	$f(x) = \frac{x}{\sigma^2} e^{-\left(\frac{x^2}{2\sigma^2}\right)}$
WE	$f(x) = \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\left(\frac{x}{\alpha}\right)^\beta\right)}$

Table 2. Parameter Estimator for all theoretical distributions

Model	Maximum Likelihood Estimator (MLE)
EX	$\theta = \bar{x}$
GA	$\beta = \frac{s^2}{\bar{x}}, \alpha = \frac{\bar{x}^2}{s^2}, s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$
IGA	$\alpha = 2 + \frac{(E(x))^2}{V(x)}, \beta = E(x)(\alpha - 1)$ Where $E(x), V(x)$ mean and variance respectively.
LN	$\mu = \frac{\sum_{i=1}^n \ln x_i}{n}, \sigma = \frac{\sum_{i=1}^n (\ln x_i - \mu)^2}{n}$

RY	$\sigma = \sqrt{\frac{\sum_{i=1}^n x_i}{2n}}$
WE	$\beta = \left[ \left( \sum_{i=1}^n x_i^\beta \cdot \ln x_i \right) \left( \sum_{i=1}^n x_i^\beta \right)^{-1} - n^{-1} \sum_{i=1}^n \ln x_i \right]^{-1},$ $\alpha = \left[ \frac{1}{n} \sum_{i=1}^n x_i^\beta \right]^{\frac{1}{\beta}}$

2.3. Root mean square error

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (y_i - x_i)^2}{N}} \tag{1}$$

2.4. Chi – square error

$$\chi^2 = \frac{\sum_{i=1}^N (y_i - x_i)^2}{N - n} \tag{2}$$

2.5. Variance

$$R^2 = \frac{\sum_{i=1}^N (y_i - z_i)^2 - \sum_{i=1}^N (y_i - x_i)^2}{\sum_{i=1}^N (y_i - z_i)^2} \tag{3}$$

Let N represent the total number of observations,  $y_i$  denote the frequency of each observation,  $x_i$  indicate the frequency of the Weibull distribution,  $z_i$  represent the average wind speed, and n stand for the number of constants employed.

Table 3. Weibull parameter estimation by numerical methods.

Method	Weibull Shape Parameter ( $k$ )	Weibull Scale Parameter (C m/s)
STDM	$k = \left(\frac{\sigma}{\bar{v}}\right)^{-1.086}$	$\frac{\bar{v}}{\Gamma\left(1 + \frac{1}{k}\right)}$
PDM	$k = 1 + \frac{3.69}{E_{pf}^2}$ $E_{pf} = \frac{\Gamma\left(1 + \frac{3}{k}\right)}{\Gamma\left(1 + \frac{1}{k}\right)^3}$	$\frac{\bar{v}}{\Gamma\left(1 + \frac{1}{k}\right)}$ $\Gamma(x)$ is gamma function.
MOM	$\sigma =$ $c \sqrt{\left[ \Gamma\left(1 + \frac{2}{k}\right) - \left\{ \left(1 + \frac{1}{k}\right)^2 \right\} \right]^{\frac{1}{2}}}$ $c, \sigma$ : standard deviation.	$\frac{\bar{v}}{\Gamma\left(1 + \frac{1}{k}\right)}$

MLM	$k = \frac{\left( \frac{\sum_{i=1}^n v_i^k \ln v_i}{\sum_{i=1}^n v_i^k} - \frac{\sum_{i=1}^n \ln v_i}{n} \right)^{-1}}$	$\left[ \frac{1}{n} \sum_{i=1}^n (v_i)^k \right]^{\frac{1}{k}}$
MMLM	$k = \frac{\left[ \frac{\sum_{i=1}^n v_i^k \ln(v_i) f(v_i)}{\sum_{i=1}^n v_i^k f(v_i)} - \frac{\sum_{i=1}^n \ln(v_i) f(v_i)}{f(v \geq 0)} \right]^{-1}}$	$\left[ \frac{1}{f(v \geq 0)} \sum_{i=1}^n (v_i)^k f(v_i) \right]$

2.6. probability density function of Weibull

The Weibull probability density function is defined by two parameters: a dimensionless shape parameter (k) and a scale parameter (c in m/s). These parameters are essential in calculating the wind speed necessary for the efficient functioning of a wind conversion system, as well as the speed range in which the device is projected to work. This mathematical relationship is represented in Equation.

$$f(v) = \frac{dF(v)}{dv} = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \times e^{-\left(\frac{v}{c}\right)^k} \tag{4}$$

Where *v*, *k* and *c* are wind speed (m/s), shape factor (dimensionless) and scale factor (m/s), respectively [1,2,3,4,6].

2.7. Cumulative distribution function of Weibull

The cumulative distribution function illustrates the integration of the Weibull density function, displaying the cumulative relative frequency for each speed range. The Weibull Function equation is defined as [1-6]:

$$F(v) = 1 - e^{-\left(\frac{v}{c}\right)^k} \tag{5}$$

All of these distributions are employed to calculate the likelihood of an event happening. The characteristics of the event impact the form of the probability curve, and when it comes to the wind pattern, the cumulative probability curve aligns predominantly with the Weibull Function.

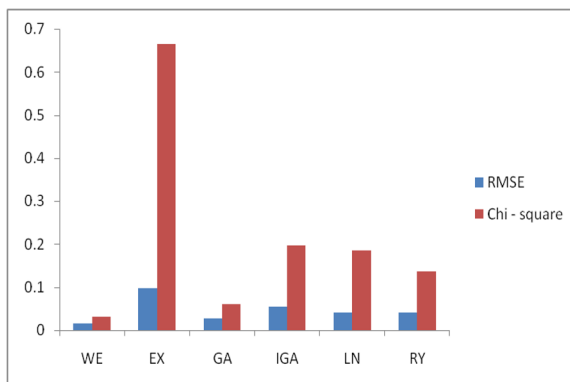


Fig. 1. Comparison of errors of wind speed probabilities.

2.8. Methods for estimating Weibull parameters

Five different numerical techniques, including standard deviation method (STDM), power density method (PDM), maximum likelihood method (MLM), method of moments (MOM), and modified maximum likelihood method (MMLM), are employed to estimate the Weibull parameters on a monthly basis at a height of 25m. Table 3 contains the equations for the shape parameter (k) and scale parameter (c m/s).

Table 1 comparison of errors of wind speed probability

	RMSE	Chi – Square error	Correlation coefficient
STDM	0.0154	0.0474	0.969
PDM	0.0164	0.041	0.9647
MOM	0.0154	0.0472	0.9689
MLM	0.0163	0.0544	0.957
MMLM	0.0154	0.0447	0.9661

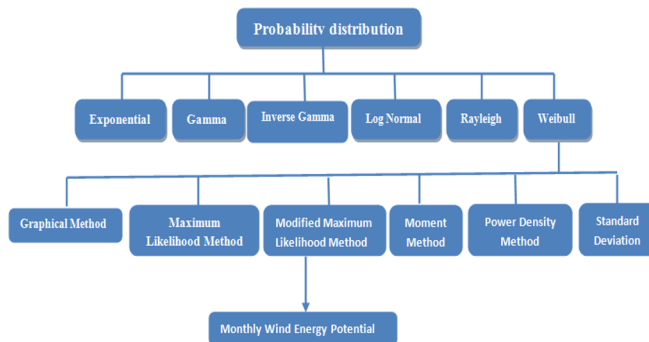


Fig. 2. Procedure flow chart.

2.9. Vertical profiles of steady wind

The wind speed at the surface of the Earth is stagnant, but it gradually increases as you ascend within the atmospheric boundary layer. The change in wind speed as altitude increases is known as the vertical wind speed profile or wind shear. The evaluation of wind energy resources and the design of wind turbines greatly rely on the significant importance of the relationship between wind speed and elevation. In wind energy, a power law is commonly employed to define the vertical wind profiles. It is essential to extrapolate the wind speed to the hub height if the measurements are not taken at the same height as the wind turbine hub. The following computational procedures may be employed to estimate the wind speeds at the turbine hub elevation. Let  $u_1$  denote the wind speed at the reference height  $z_1$ . The formulas for the scale and shape factors of the Weibull distributions express the height dependence. These are,

$$n = \frac{(0.37 - 0.0881 \ln c_1)}{[1 - 0.0881 \ln(z_1/10)]} \quad (6)$$

$$k_2 = \frac{k_1 [1 - 0.0881 \ln(z_1/10)]}{[1 - 0.0881 \ln(z_2/10)]} \quad (7)$$

$$c_2 = c_1 \left(\frac{z_2}{z_1}\right)^n \quad (8)$$

Where  $z_1, k_1, c_1$  are the reference height, shape and scale factors at reference heights. Where  $z_1, k_1, c_1$  are the reference height, shape and scale factors at reference heights respectively and  $z_2$  is the desired height.  $k_2, c_2$  are the shape and scale factors respectively at desired height [1,9]. Other parameters are considered from [10-14].

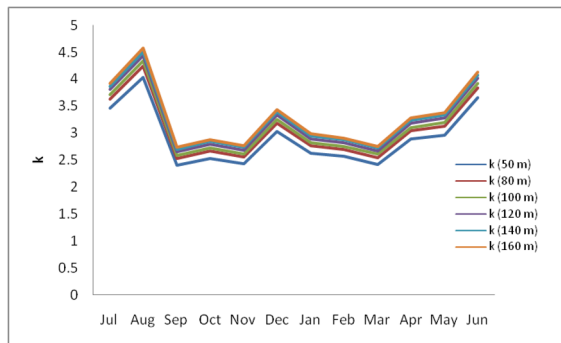


Fig. 3. Extrapolated values of  $k$  for higher heights

### 3. Results and discussions

Wind data collected during June 2004 to July 2005 was provided by the institute. The site specifications were previously discussed in the case description section. As efficient analysis of wind characteristics plays an important role in reducing the uncertainty in measuring wind energy potential for the site. In this paper six probability distribution functions are used to fit the data. Table 1 represents the overview of the distributions that are used. Maximum likelihood estimator is used in estimating the parameters as it will have less root mean square error. Table 2 shows the formulas to estimate the parameters. Figure 1 shows the histogram representations of root mean square error and chi-square error. It shows the Exponential distributions fits worst and Weibull distribution is superior over others. Therefore it can be concluded that Weibull distribution suits best for the data. Further parameters of Weibull distribution can be estimated using five different numerical methods. Table 3 shows the estimation of Weibull parameters by different numerical methods. Table 4 shows the comparison of RMSE, chi-square error, variance for estimating the wind speed probability by different numerical Weibull parameter estimation methods. Although all the numerical methods produce satisfied results it can be concluded that Maximum likelihood method performs relatively better in estimating the wind speed probability. These Weibull parameters estimated using Maximum likelihood method are extrapolated to higher heights using Justus and Mikhail method. Figure 4 shows the monthly Wind power potential at higher heights. From the graph monthly wind power potential is highest in the months June and June and very poor during October – December.

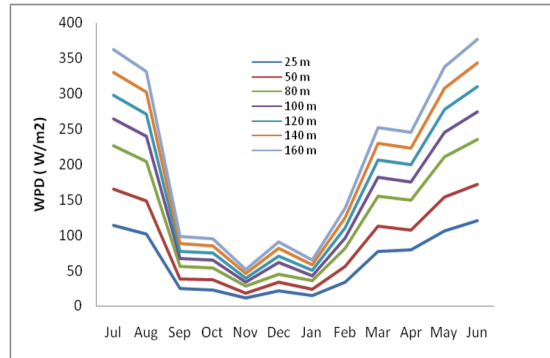


Fig. 4. Extrapolated values of WPD for higher heights

#### 4. Conclusion

This study examined the wind properties and applied five numerical techniques to analyze the Weibull wind speed distribution: monthly mean wind velocity and wind potential at higher heights of 50, 80, 100, 120, 140, 160 m are estimated using Justus and Mikhail method. The findings from this study can be summarized as follows:

1. Weibull distribution best suit the MANIT wind data.
2. Table.4 shows that maximum likelihood method is efficient to determine weibull parameters. Moment method and standard deviation methods occupied the succeeding positions.
3. The values of Weibull shape parameter are high during August.
4. The month of June exhibits the highest wind power density, whereas November experiences the lowest. This disparity can be attributed to the fluctuating weather conditions throughout the year. For the months of March – August the extraction of power from wind is economical at MANIT.

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