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Algebraic quasi-fractal dynamics: modeling of the risk system of a complex energy-saving system

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Abstract. Algebraic quasi-fractal is a convenient form of recording a process. Currently, digital transformation's process permeates almost all areas of human activity. In this regard, digital twins of risk management methods become necessary. In this article, we approach the study of the risk system of a complex system from the following position: a risk event in the system occurs in the event of an intermission of the system's connection. The main idea to describe the risk system of a complex system is the description as model with intermission connections. The main tool for it the algebraic quasi – fractal systems. So, it will be used: modeling of the risk system of a complex system in accordance with a model in the form of a group of factors determining the complex system, with a help of a group with an intermission a definite connection of a system; modeling of the risk system of a complex system in accordance with a quasi – fractal group of factors determining the complex system, with a help of a quasi – fractal group with an intermission a definite quasi – fractal connection of a complex system. Such approach allows one to connect properties of a complex system with properties of its risk system.

1 Introduction

The monographs [1], [2, Chapter 9] present the direction of studying the sustainability of complex systems using models in the form of algebraic and algebraic quasi–fractal systems. The following results were obtained:
- Methods to increase system reliability [3,4];
- The algorithm of tensor estimation of system sustainability [2];
- Sustainability of a system modeled by a quasi-fractal algebraic system [2];
- System sustainability violation: violation of the system's closeness property. Malfunction of one of the system's factors;
- Violation of feedback in the system. Factor-flexible quasi-fractal system;
- Substitution of system's functions. System's compensational possibilities.

Using modeling of the risk system of a complex system in accordance with a model in the form of a group of factors determining the complex system, with a help of a group with

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an intermission a definite connection of a system and modeling of the risk system of a complex system in accordance with a quasi – fractal group of factors determining the complex system, with a help of a quasi – fractal group with an intermission a definite quasi – fractal connection of a complex system, one can transfer results mentioned previously onto the risk system of a complex system.

2 Research methodology

The research methodology is based on the Theory of algebraic systems, introduced by A. I. Malt'sev [5], and methodology of the Theory of fractals, introduced by Benoit Mandelbrot [6], and on the following models:

- Modeling of the risk system $\boldsymbol{R_S}$ of a complex system S in accordance with a model $\boldsymbol{G_S}$, of factors determining the system S , with a help of a group $\boldsymbol{G_{Sj}}$ with an intermission connection j of a group $\boldsymbol{G_S}$;
- Modeling of the risk system $\boldsymbol{R_S}$ of a complex system S in accordance with a quasi – fractal group $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{G}^{\alpha}_k = (\langle G^{\alpha}_k; \Omega_k = \langle \cdot, \ ^{-1}, e\rangle \rangle, \alpha \in \Lambda_k))$, of factors determining the system S , with a help of a quasi – fractal group $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{G_{J}}^{\alpha}_{k} = (\langle G_{J}{}^{\alpha}_{k}; \Omega_k = \langle \cdot, \ ^{-1}, e\rangle \rangle, \alpha \in \Lambda_k))$, here ω_γ is an ordinal, with an intermission connection j of a quasi – fractal group $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{G}^{\alpha}_k = (\langle G^{\alpha}_k; \Omega_k = \langle \cdot, \ ^{-1}, e\rangle \rangle, \alpha \in \Lambda_k))$

We shall also use the concept of system state, see, for example, [7]**:**

Definition 1

The state of a system $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{A}^{\alpha}_k = (\langle A^{\alpha}_k; \Omega_k \rangle, \alpha \in \Lambda_k))$ is a quasi – fractal function $f = QF^{\alpha}_{k<\omega_\gamma}(f^{\propto}_k) = QF^{\alpha}_{k<\omega_\gamma}(f\colon \boldsymbol{A}^{\alpha}_k = \langle A^{\alpha}_k; \Omega_k \rangle \to M), \alpha \in \Lambda_k$

where M is $\widehat{Z_{2^\infty}}$, $f^{\propto}_k\colon \boldsymbol{A}^{\alpha}_k = (\langle A^{\alpha}_k; \Omega_k \rangle \to M$ is a digitalization function for $\boldsymbol{A}^{\alpha}_k = \langle A^{\alpha}_k; \Omega_k \rangle$ at each level $k < \omega_\gamma$.

For more details see also [8].

Let's remind, that the definition of a quasi – fractal function runs as follows:

Definition 2 (the concept of a quasi - fractal function)

Let $\boldsymbol{A}^{\alpha}_k = (\langle A^{\alpha}_k; \Omega_k \rangle, \alpha \in \Lambda_k, k < \omega_\gamma$, ω_γ – an ordinal, be a quasi – fractal algebraic system with ω_γ levels,

M be arbitrary non-empty set. Let $f^{\propto}_k\colon \boldsymbol{A}^{\alpha}_k = (\langle A^{\alpha}_k; \Omega_k \rangle \to M$ for every $\alpha \in \Lambda_k, k < \omega_\gamma$**.** Then $f = QF^{\alpha}_{k<\omega_\gamma}(f^{\propto}_k) = QF^{\alpha}_{k<\omega_\gamma}(f\colon \boldsymbol{A}^{\alpha}_k = \langle A^{\alpha}_k; \Omega_k \rangle \to M), \alpha \in \Lambda_k$ is called a quasi – fractal function from $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{A}^{\alpha}_k = (\langle A^{\alpha}_k; \Omega_k \rangle, \alpha \in \Lambda_k))$ into M

3 Research results

Problem Statement.

- Studying the chaotic risks presystems of a system S modelled by a group of factors $\boldsymbol{G_S}$**.**

The investigation of the internal risks of these systems. As a rule, the risk of functioning of a complex system is manifested in the rupture of the connections of this system. Examples of that conclusion can be found in training systems, financial and economic systems, and technical systems.

The following results are presented:

- Modeling of the risk system $\boldsymbol{R_S}$ of a complex system S in accordance with a model in the form of a group of factors determining the system S , with a help of a group with an intermission connection j of a group $\boldsymbol{G_S}$;
- Modeling of the risk system $\boldsymbol{R_S}$ of a complex system S in accordance with a quasi

– fractal group of factors determining the system S , with a help of a quasi – fractal group with an intermission connection j of a quasi – fractal group of factors determining the system S.

The existence a Brownian motion in the risk system of a complex closed associative system S .

The transition the following results:

- the algorithm of tensor estimation of system sustainability [2];
- sustainability of a system modeled by a quasi-fractal algebraic system [2, Chapter 9];
- system sustainability violation: violation of the system's closeness property;
- malfunction of one of the system's factors; - violation of feedback in the system;
- factor-flexible quasi-fractal system; - substitution of system's functions;
- system's compensational possibilities, onto the risk system of a complex system.

4 Discussion of the results

4.1 Main definitions and theorems

An important role in risk management of complex systems belongs to the investigation of the internal risks of these systems. As a rule, the risk of functioning of a complex system is manifested in the rupture of the connections of this system. Examples of that conclusion can be found in training systems, financial and economic systems, and technical systems. In this regard, we propose the following definition

Definition 3. Risk systems of the system S in accordance with model G

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Then the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \backslash J\rangle, J \subseteqq I$ is modeling subsystem risk of the system's model of group $\boldsymbol{G}$ for every $J \subseteqq I$, by a model $\boldsymbol{G}$**.**

In accordance with this definition one can construct the definition of a quasi – fractal risk system's S model

Definition 4. Let the system S be modeled by a quasi – fractal group of factors

$QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{G}^{\alpha}_k = (\langle G^{\alpha}_k; \Omega_k = \langle \cdot, \quad ^{-1}, e\rangle \rangle, \alpha \in \Lambda_k))$,

$G^{\alpha}_k = \langle G^{\alpha}_k = \{a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k} | i \in I\} | \{w_{i}{}^{\alpha}_{k}(a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k}) = 1, i \in I^{\alpha}_k\rangle$ with generators $\{a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k} | i \in I\}$ and defining relations $\{w_{i}{}^{\alpha}_{k}(a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k}) = 1, i \in I^{\alpha}_k;$ here ω_γ is an ordinal.

Then the quasi – fractal system with intermittent connections simulated by the quasi – fractal groups

$QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{G_J}{}^{\alpha}_{k} = (\langle G_J{}^{\alpha}_{k}; \Omega_k = \langle \cdot, \quad ^{-1}, e\rangle \rangle, \alpha \in \Lambda_k))$,

$G_J{}^{\alpha}_{k} = \langle G_J{}^{\alpha}_{k} = \{a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k} | i \in I\} | \{w_{i}{}^{\alpha}_{k}(a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k}) = 1, i \in I^{\alpha}_k \setminus J^{\alpha}_k\rangle$ with generators $\{a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k} | i \in I\}$ and defining relations

$\{w_{i}{}^{\alpha}_{k}(a_{1}{}^{\alpha}_{k}, \dots, a_{n}{}^{\alpha}_{k}) = 1, i \in I^{\alpha}_k \setminus J^{\alpha}_k$ is modeling subsystem risk of the system's model of group $\boldsymbol{G}$ for every $J \subseteqq I$, here ω_γ is an ordinal. The advantages of the latter definition run as follows. Using quasi – fractal technic one can construct algorithms one can improve a risk analysis algorithm for smart planning systems and a risk analysis algorithm for smart control systems, [9, 10].

In [2] approaches and the concept of system stability are described in the language of quasi- fractal algebraic systems. These results, using definitions of risk systems as systems with intermittent connections, are easily transferred to the case of risk management systems of a complex system. This result and the Algorithm of tensor estimation of system sustainability allow one to manage risks systems of the complex system, [2, Chapter 9, paragraph 9.3, section 9.3.1] in specified limits.

Besides it:

Theorem 5

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$ by a model $\boldsymbol{G}$**.**

There exists a Brownian motion in the risk system $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ of a system S modelled by a group of factors $\boldsymbol{G_S}$, if $I = J$

Proof One can get the statement of the theorem in accordance with [7], applied the results to $\boldsymbol{G_J}$**.**

Theorem 6

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$ by a model $\boldsymbol{G}$**.**

There exists not only Brownian motion in the risk system $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ of a system S modelled by a group of factors $\boldsymbol{G_S}$, if $I = J$.

Proof One can get the statement of the theorem in accordance with [7], applied the results to $\boldsymbol{G_J}$**.**

Here we continued to study the risk system of a complex system and compare properties of a complex system and its risk system using modeling by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$.

- The state of system S and the state of the risk system $\boldsymbol{G_J}$ of this system according to model of factors $\boldsymbol{G}$**,** defining the system S / using homological methods

Corollary 7

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$**.** The description of the risk system system's $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ of a system S modelled by a group of factors $\boldsymbol{G_S}$ states through a model A_S , which is an abelian group ,

$$Hom(\boldsymbol{G_J}, A_S)$$

coincides with the description of the system's $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ states through an abelianization $\boldsymbol{G_J}/[\boldsymbol{G_J}, \boldsymbol{G_J}]$ of the group $\boldsymbol{G_J}$ by the model A_S.

What is the connection between $Hom(\boldsymbol{G}, A_S)$ and $Hom(\boldsymbol{G_J}, A_S)$, here

$$\mathbf{1} \to \boldsymbol{ker\, \varphi_j} \xrightarrow{\boldsymbol{\varepsilon}} \boldsymbol{G_J} \xrightarrow{\boldsymbol{\pi}} \boldsymbol{G} \cong (\boldsymbol{G_J} \,/\, \boldsymbol{ker\, \varphi_j}) \to \mathbf{1}$$

Let's consider

$$\mathbf{1} \;\to\; \boldsymbol{ker\,\varphi_j} \overset{\varepsilon}{\to} \boldsymbol{G_J} \overset{\pi}{\to} \boldsymbol{G} \cong (\boldsymbol{G_J} \,/\, \boldsymbol{ker\,\varphi_j}) \to \mathbf{1}$$

(1.1)

$$\boldsymbol{A_S}$$

So, any homomorphism from $\boldsymbol{G}$ to $\boldsymbol{A_S}$ extends to a homomorphism from $\boldsymbol{G_J}$ to $\boldsymbol{A_S}$. Inverse proposition runs as follows: any homomorphism from $\boldsymbol{G_J}$ to $\boldsymbol{A_S}$ extends to a homomorphism from $\boldsymbol{G}$ to $\boldsymbol{A_S}$. If $\boldsymbol{A_S}$ is divisible, for example

$$\mathbf{1} \;\to\; \boldsymbol{ker\,\varphi_j} \overset{\varepsilon}{\to} \boldsymbol{G_J} \overset{\pi}{\to} \boldsymbol{G} \cong (\boldsymbol{G_J} \,/\, \boldsymbol{ker\,\varphi_j}) \to \mathbf{1}$$

(1.2)

$$\boldsymbol{A_S}$$

it is true

So, any homomorphism from $\boldsymbol{G}/[\boldsymbol{G},\boldsymbol{G}]$ to $\boldsymbol{A_S}$ extends to a homomorphism from $\boldsymbol{G_J}/[\boldsymbol{G_J},\boldsymbol{G_J}]$ to $\boldsymbol{A_S}$

Inverse proposition: any homomorphism from $\boldsymbol{G_J}/[\boldsymbol{G_J},\boldsymbol{G_J}]$ to $\boldsymbol{A_S}$ extends to a homomorphism from $\boldsymbol{G}/[\boldsymbol{G},\boldsymbol{G}]$ to $\boldsymbol{A_S}$ is true, for example, if $\boldsymbol{A_S}$ is a divisible group.

Corollary 8

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \backslash J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$ by a model $\boldsymbol{G}$.

Any state of system G coincides with one of the states of the risk system of system G. The opposite is not true.

There are states of the risk system $\boldsymbol{G_J}$ that cannot be described using system states $\boldsymbol{G}$ using algorithm of Corollary 7.

In accordance with [7], one can put the following question:

Question. Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \backslash J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$ by a model $\boldsymbol{G}$.

If P is a unary predicate given on the class of all groups, such that it is closed under taking subgroups and factor-groups, $P(A)$ is true. Find for every group G such its normal subgroup $B_P(\boldsymbol{G_J}) \trianglelefteq \boldsymbol{G_J}$ that

$$Hom(\boldsymbol{G_J}, A) \cong Hom(\boldsymbol{G_J}/B_P(\boldsymbol{G_J}), A) \tag{1.3}$$

$$1 \to B_P(\boldsymbol{G_J}) \overset{\varepsilon}{\to} \boldsymbol{G_J} \overset{\pi}{\to} G/B_P(\boldsymbol{G_J}) \to 1$$

$f \quad \varphi$

(1.4)

$$A$$

$$f = \varphi\pi.$$

4.2 Quasi - fractal risk -– envelope of the complex system S according to the model G of factors defining S

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$ by a model $\boldsymbol{G}$**.** So, $\boldsymbol{G/ker\varphi_J} \cong \boldsymbol{G_J}$;

$$1 \to \boldsymbol{ker\varphi_J} \overset{\varphi_J}{\to} \boldsymbol{G} \overset{\tau_J}{\to} \boldsymbol{G/ker\varphi_J} \cong \boldsymbol{G_J} \to 1$$

(1.5)

Let's use the construction of p – adic completion, p is a prime number, [Fucks, v 1] to embed the complex system S into inverse limit

$$G_R{}^* = \lim_{\leftarrow}\{G_J\,;\, \pi_J^K; J, K \subseteq; J \subseteq K\} \quad (1.6)$$

$$\pi_J^K : G_K \to G_J \quad (1.7)$$

$$\boldsymbol{G/ker\varphi_K} \overset{\pi_J^K}{\to} \boldsymbol{G/ker\varphi_J} \quad (1.8)$$

$$\boldsymbol{gker\varphi_K} \overset{\pi_J^K}{\to} \boldsymbol{gker\varphi_J};\ ker\varphi_K \subseteqq\ ker\varphi_J$$

(1.9)

If $J, K \subseteq I; J \subseteq K$ then π_J^K - epimorphism

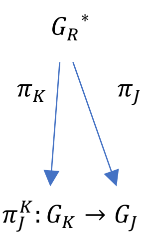
$$\lim_{\leftarrow}\{G_J\,;\, \pi_J^K; J, K \subseteq I; J \subseteq K\} \subseteq \prod_{J \subseteq I} \boldsymbol{G/ker\varphi_J}$$

(1.10)

$$\lim_{\leftarrow}\{G_J\,;\, \pi_J^K; J, K \subseteq I; J \subseteq K\} = (\dots, gker\varphi_K, \dots, gker\varphi_J, \dots);\ \pi_J^K(gker\varphi_J) = gker\varphi_K$$

(1.11)

(1.12)



Group G can be embedded into its completion written in the form of inverse limit of its factor groups:

$$G\ \ni g \mapsto (\dots, gker\varphi_K, \dots, gker\varphi_J, \dots,) \in G_R{}^*;\ J, K \subseteq I; J \subseteq K$$

(1.13)

In [2] we write down the inverse limit of algebraic systems in the form of a quasi – fractal algebraic system. So, $G_R{}^*$ is a quasi - fractal risk – envelope of the complex system S according to the model G of factors defining S

Example

1. Let system S be modeled by a group of factors

$$\boldsymbol{Z} = \langle Z, +, -, 0\rangle \cong \boldsymbol{Z} \oplus \boldsymbol{Z} \cong (\boldsymbol{F_2}/[\boldsymbol{F_2}, \boldsymbol{F_2}])\ /\boldsymbol{Z}$$

$G = (\boldsymbol{F_2}/[\boldsymbol{F_2}, \boldsymbol{F_2}])\ /\boldsymbol{Z} \cong \boldsymbol{Z},$ one can choose a prime number p and consider $\boldsymbol{G_R}^* \cong \boldsymbol{Z_{p^\infty}}$. So, $\boldsymbol{Z_{p^\infty}}$ is one of the risk -– envelope of the complex system S according to the model G.

4.3 The connections of elementary theory of a model G_S of a complex system S and system's S risk model G_J by a model G_S

One can use the connections between an elementary theory of a model $\boldsymbol{G_S}$ of a complex system S and an elementary theory of a system's S risk model $\boldsymbol{G_J}$ by a model $\boldsymbol{G_S}$ to investigate common properties of $\boldsymbol{G_S}$ and $\boldsymbol{G}$. This approach allows one to use the notion of decidability of elementary theories to manage properties of a complex system S. We propose the usage of the theorems on preserving formulas of NPC (Narrow Predicate Calculus), [5] by homorphisms of algebraic systems of the same signatures, and, in particular, preserving formulas of NPC by homorphisms of groups. So, from [5] we have the following

Theorem 9

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$ by a model $\boldsymbol{G}$.

Then positive formulas belonging to the elementary theory $Th(\boldsymbol{G})$ belongs to the elementary theory $Th(\boldsymbol{G_J})$

The negative formulas belonging to the elementary theory $Th(\boldsymbol{G_J})$ belongs to the elementary theory $Th(\boldsymbol{G})$.

Let's remind that a closed formula of NPC is called a positive one, if it is constructed from the simplest formulas of the form $P_m(x_1, \dots, x_n)$ or $f = g$, here P_m is a predicate symbol, $x_1, \dots, x_n$ are subject variables, f, g terms, using the operations of conjunction, disjunction and quantization.

A closed formula of NPC is called negative if it is formed using a conjunctions, disjunctions, and quantifiers from expressions of the form $f \neq g$; $\neg P_m(x_1, \dots, x_n)$, here $x_1, \dots, x_n$ are subject variables, f, g terms, P_m predicate symbols.

Let's note, that the use of elementary theories and algebraic quasi-fractal logic of the first order (NPC) instead of isomorphism allows the use of probabilistic methods in situations where it is impossible to do this directly (in the case of non-amenable algebraic systems, for example, free groups). In [10] we have used Erdös–Renyi second model to finding $\boldsymbol{P} -$ giant component in algebraic systems and in algebraic quasi – fractal systems, here $\boldsymbol{P}$ is a unary predicate, given on a class of algebras of the signature Ω , closed on taking subalgebras and factor – algebras.

5 Conclusions

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Let the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ be modeling subsystem risk of the system's model of group $\boldsymbol{G}$ by a model $\boldsymbol{G}$.

The description the risk system of a complex system as a model with intermission connections allows one to formulate the following conclusions.

Any state of system S coincides with one of the states of the risk system of system S by a model $\boldsymbol{G}$. The opposite is not true.

When managing complex system's risks, the connections of the complex system S change, that is, the system itself and the system of its risks $\boldsymbol{G_J}$ change according to the model $\boldsymbol{G_S}$ of the group of factors that determine the complex system $\boldsymbol{S}$

The complex system $\boldsymbol{S}$ operates in its quasi - fractal risk – envelope $\boldsymbol{G_R}^*$, according to the model $\boldsymbol{G}$ of factors defining the complex system $\boldsymbol{S}$**.** That is in accordance to proposed complex system's risk model, there are no risk-free complex systems. Let's note that when we consider financial market's model, for example, the risks - free securities risk – free investments are usually spoke about [11]. In the frame of our model of quasi - fractal risk – envelope $\boldsymbol{G_R}^*$ one can speak about only of a level of risk of securities: bonds and so on and of a level of risk of investments. According to our proposed model $\boldsymbol{G_R}^*$, it is possible to calculate the level of the quasi-fractal risk shell of the system, which determines this level of risk

In 2021 we published results of practical usage of smart systems' modelling with the help of quasi –fractal algebraic systems' [12]. Such technic is used since 2002 in practice. Now we continue to develop detailed courses "Algebraic Methods in Digitalization of Smart Systems" for students of economic areas with an emphasis on modeling in the field of the fiscal system, and for students of technical specialties with an emphasis on solving specific applied problems in the field of technology.

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