Analysis of the process OF non-stationary heat transfer through the wall of a cylindrical container

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Abstract. The process of constructing a mathematical model of the heat exchange process between two media through the wall of a cylindrical container is described. In engineering, there are many problems of calculating the heat exchange process through the wall of a cylindrical tank (heat exchangers, heat supply systems, water tanks, etc.). There are usually no difficulties with the stationary process, while a unified calculation method has not yet been developed for the problems of non-stationary thermal conductivity due to the complexity of their mathematical description. Most of the problems encountered when using heat and mass transfer processes are reduced to solving partial differential equations. Differential equations describing real physical processes are usually very complex, and it is possible to obtain their solution in the form of a finite formula only in the simplest cases. With a strict formulation of the problem, assuming a minimum number of assumptions, these studies are comparable to the best physical experiments. In addition, numerical experiments have a number of advantages in comparison with physical ones: they allow relatively simple and wide-range variation of the problem parameters and boundary conditions, as well as obtaining comprehensive information about the process under study. The paper describes one of the possible ways to construct a method for calculating non-stationary heat transfer through the wall of a cylindrical container. A mathematical model has been developed that allows calculating thermal processes in cylindrical tanks.

1 Introduction

The solution of the problem of non-stationary thermal conductivity is reduced to determining the dependence of temperature and the transferred amount of heat on time for any point of the body and, above all, to determining the temperature field in the body in question in time.

Solving problems of non-stationary thermal conductivity is usually more difficult than stationary problems, due to the introduction of an additional independent variable - time. The temperature is a function of the coordinates in the area under consideration, but, in addition, the temperature distribution changes with time.

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There are various methods for solving problems of non-stationary heat transfer:

1. Analytical methods: thermal analysis using differential equations of heat transfer. This may include solving the heat equation to determine the temperature field and heat fluxes under specified boundary conditions.
2. Numerical methods: these methods involve the use of finite differences, finite elements, or finite volumes to numerically solve heat transfer equations and obtain an approximate solution.
3. Experimental methods: conducting experiments to measure temperatures and heat fluxes in real systems. This can help in validating mathematical models and numerical methods.
4. Modeling methods
5. Analogy method: using analogies between heat transfer problems and other physical processes such as electromagnetism or hydrodynamics to simplify the solution.
6. Source method: dividing the task into several simpler ones that can be solved separately. The results are then combined to produce a common solution.

The difficulties of solving problems of non-stationary heat transfer are related to the fact that the temperature field and heat fluxes vary in time and space. This requires consideration of additional factors such as initial temperature, boundary conditions, and material properties. In addition, non-stationary processes can lead to complex thermal regimes, such as thermal shocks and heat waves, which complicate the analysis and modeling of heat transfer.

Various methods can be used to solve problems of non-stationary thermal conductivity. The most general, but very complex, even for bodies of simple shape, is the analytical method. In this case, the differential equation of thermal conductivity is solved together with boundary and time conditions, and the results of the solution are presented in the form of graphs convenient for use.

Kutateladze S.S. suggests using generalizations of experimental results when solving such problems, applying similarity criteria to obtain computational equations [1].

Aboltins A., Kangro I., Kalis H. [2, 3] consider the special conservative averaging method for solving the heat transfer boundary-value problem in the 3-D domain, and also consider some nonlinear heat transfer problems.

The described methods require knowledge of special sections of mathematics (differential equations, numerical methods). At the same time, the practice of engineering calculations usually involves the use of relatively simple equations, methods and algorithms to solve practical problems.

We offer a simpler way to solve the problem, using a simplified mathematical apparatus. At the same time, our proposed method gives good accuracy, which is confirmed by experiment. The paper describes one of the possible ways to construct a method for calculating non-stationary heat transfer through the wall of a cylindrical container.

2 Materials and methods

Consider, for example, a metal cylindrical tank of a water tower (Figure 1). The heat flow through the cylindrical wall is calculated by the following expression:

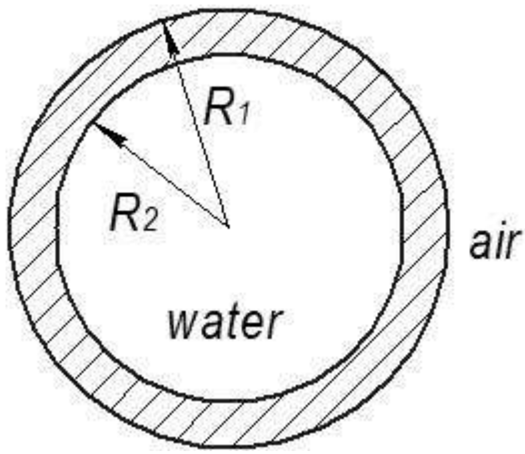

Fig. 1. The wall of the water tower.

$$Q = \frac{2 \cdot \pi \cdot \lambda_w \cdot l(T_2 - T_1)}{\ln \frac{R_1}{R_2}}, \tag{1}$$

where Q – heat flow, W;

l – height of the cylinder, m;

T_1 – temperature of the outer wall, K;

T_2 – temperature of the inner wall, K;

λ_w – the coefficient of thermal conductivity of the wall, $\frac{W}{m \cdot K}$;

R_1 – the radius of the outer surface, m;

R_2 – the radius of the inner surface, m.

The heat balance equation for the inner surface of the wall will be written in the following form:

$$-C \cdot l \cdot dT = (T_1 - T_a) \cdot \alpha_1 S_1 dt, \tag{2}$$

where C – heat capacity of the mass of water in a cylinder with a height of 1 m, $\frac{J}{m \cdot K}$;

dt – time interval, s;

dT – change in water temperature during dt, K;

T_1 – temperature of the outer wall, K;

T_a – temperature of air, K;

α_1 – heat transfer coefficient of the outer wall surface, $\frac{W}{m^2 \cdot K}$;

S_1 – area of the outer surface of the wall, m^2.

The minus sign indicates a decrease in water temperature. The heat balance equation for the wall is written as follows:

$$-C \cdot l \cdot dT = 2 \cdot \pi \cdot \lambda_w \cdot l \frac{T_2 - T_1}{\ln \frac{R_1}{R_2}} dt \tag{3}$$

The heat balance equation for the outer surface of the wall will be written in the following form:

$$-C \cdot l \cdot dT = (T_w - T_2) \cdot \alpha_2 S_2 dt, \tag{4}$$

where T_2 – temperature of the inner wall, K;

T_w – temperature of water, K;

α_2 – heat transfer coefficient of the inner wall surface, $\frac{W}{m^2 \cdot K}$;

S_2 – area of the inner surface of the wall, m^2.

Transform the equations:

$$-C \cdot l \cdot dT \frac{1}{\alpha_1 S_1} = (T_1 - T_a) dt \tag{5}$$

$$-C \cdot l \cdot dT \frac{\ln \frac{R_1}{R_2}}{2 \cdot \pi \cdot \lambda_w \cdot l} = (T_2 - T_1) dt \tag{6}$$

$$-C \cdot l \cdot dT \frac{1}{\alpha_2 S_2} = (T_w - T_2) dt \tag{7}$$

Adding these three equations together, we get:

$$-C \cdot l \cdot dT \left(\frac{1}{\alpha_1 S_1} + \frac{\ln \frac{R_1}{R_2}}{2 \cdot \pi \cdot l \cdot \lambda_w} + \frac{1}{\alpha_2 S_2}\right) = (T_w - T_a) dt \tag{8}$$

For a section element of a cylindrical vessel with a height of 1 meter , the equation will be written as follows:

$$-C \cdot dT \left(\frac{1}{\alpha_1 \cdot 2 \cdot \pi \cdot R_1} + \frac{\ln \frac{R_1}{R_2}}{2 \cdot \pi \cdot \lambda_w} + \frac{1}{\alpha_2 \cdot 2 \cdot \pi \cdot R_2}\right) = (T_w - T_a) dt \tag{9}$$

Next we have:

$$-\frac{c\cdot\rho\cdot R_2^2\cdot dT}{2}\left(\frac{1}{\alpha_1\cdot R_1}+\frac{\ln\frac{R_1}{R_2}}{\lambda_w}+\frac{1}{\alpha_2\cdot R_2}\right)=(T_w-T_a)dt, \tag{10}$$

where c – specific heat capacity of water, $\frac{J}{\kappa g\cdot K}$;

ρ – water density, $\frac{kg}{m^3}$.

Denote:

$$\frac{c\cdot\rho\cdot R_2^2}{2}\left(\frac{1}{\alpha_1\cdot R_1}+\frac{\ln\frac{R_1}{R_2}}{\lambda_w}+\frac{1}{\alpha_2\cdot R_2}\right)=K, \tag{11}$$

Then

$$-K\cdot dT=(T_w-T_a)dt. \tag{12}$$

After separating the variables , we get:

$$dt=\frac{-K\cdot dT}{(T_w-T_a)} \tag{13}$$

Integrate the resulting expression:

$$\int_0^t dt=\int_{T_b}^{T_e}\frac{-K\cdot d(T_w-T_a)}{(T_w-T_a)} \tag{14}$$

Finally we get:

$$t=-K\cdot\ln\frac{T_e-T_a}{T_b-T_a}=K\cdot\ln\frac{T_b-T_a}{T_e-T_a} \tag{15}$$

Formula 15 allows you to determine the cooling time of a substance in a cylindrical container from T_b до T_e.

3 Results and discussion

To confirm the theoretical positions stated earlier, we compared the calculated data obtained by equation (15) with the results of our experimental studies. Figure 2 shows the graphs obtained by us of the dependence of the water temperature in the experimental setup on time [4, 5, 6, 7]. The dots show experimental data, the line shows the theoretically obtained dependence [8, 9, 10].

Graph 2 shows that the theoretical and experimental data are approximately the same. The maximum discrepancy between the data was 10% [11. 12, 13, 14, 15]. Consequently, the mathematical model we have obtained adequately describes the dynamics of cooling of a substance and can be used to determine the time after which a particular temperature will be reached.

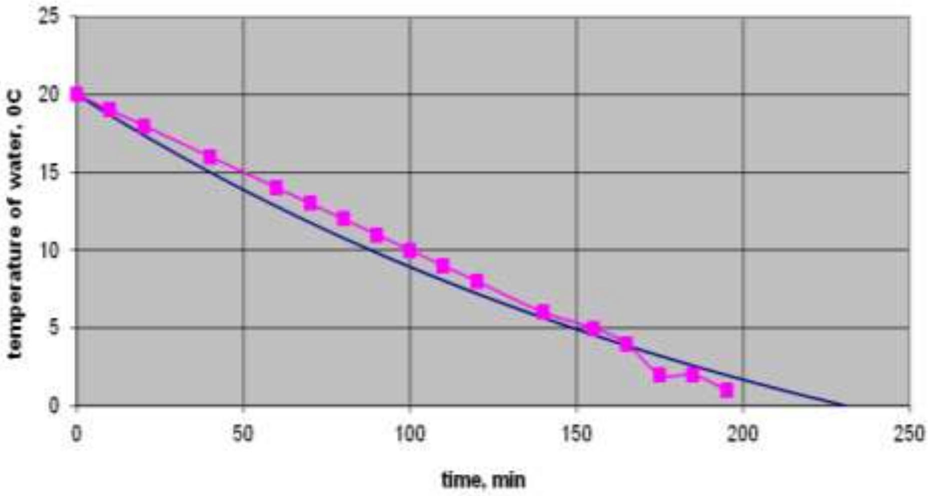

Fig. 2. The dependence of the water temperature in the experimental setup on time and the dependence obtained from the results of calculations.

4 Conclsions

1. A mathematical model has been developed that allows calculating thermal processes in cylindrical tanks.
2. As a result of the construction of this model, a fairly simple equation is obtained that can be used in the calculations of non-stationary processes.
3. This theoretical model simplifies obtaining the result. Nevertheless, the accuracy of this model is quite high compared to more complex methods.

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