

# Comparative Study of the Rosin-Rammler and Gates-Gaudin-Schuhmann Particle Size Distribution Models for Microwave-Assisted Crushing Analysis

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**Abstract.** Microwaves selectively heat dielectric minerals within the ore particles during a relatively brief exposure period, inducing thermal stresses that generate cracks, thereby diminishing the structural integrity of the ore particles. The immediate effect of employing this technique is an alteration in the particle size distribution (PSD) of the comminution product. In this research, two types of diamond-bearing kimberlite ore underwent microwave irradiation before the crushing process. The microwave experimentation was carried out utilizing a multi-variable power system with a maximum capacity of 15 kW, within a single-mode cavity structure. The exposure time was adjustable to vary the input microwave energy, which ranged from 6 kJ to 270 kJ. Three distinct narrow feed size classes were examined: -31.5 +16 mm, -16 +6.7 mm, and -31.5 +6.7 mm. Each sample weighed five hundred grams. Both the microwave-treated samples and the untreated samples (as-received) were subjected to crushing using a single-roll crusher. The PSD of the crushed product was determined through a standard sieving test and subsequently analyzed using empirical PSD models. These models are advantageous for enhancing the accuracy of PSD analysis by linearizing datasets that often exhibit skewness towards the distribution's tail end. Two widely recognized empirical PSD models, the Rosin-Rammler (RR) and Gates-Gaudin-Schuhmann (GGS) models, were employed to assess how microwave pre-treatment influences the PSD of the crushed product. Various comparative parameters derived from these models, such as the  $P_{50}$ , distribution uniformity (DP), and critical fine sizes, were evaluated for both untreated crushed samples and the samples treated with microwaves. The findings indicated that crushed products treated with microwaves generally displayed improved alignment with both PSD models in comparison to the untreated samples. Notably, the GGS model exhibited marginally higher overall r-squared values compared to the RR model. Moreover, the microwave-treated samples showcased a slightly coarser and more uniform PSD when contrasted with the untreated counterparts. Among the various microwave energy inputs tested, the lowest input at the highest power level yielded the most substantial increments in  $P_{50}$  and DP. Simultaneously, it led to the most significant reduction in fractions corresponding to critical fine sizes.

## 1 Introduction

Over the past few decades, ore pre-treatment has garnered significant attention within the field of mineral processing as a leading-edge innovation aimed at achieving more sustainable and efficient processes, particularly for challenging applications involving hard ores. It has been well established that most brittle rocks undergo physical weakening mechanisms following heat treatment [1]. However, traditional conductive and convective heating methods are ineffective and inefficient. This is due to the sluggish kinetics of heat transfer in rocks associated with these mechanisms, resulting in substantial heat loss to the environment. The gradual heating rate imposed on rocks restricts the development of thermal stress, thereby leading to minimal generation of cracks responsible for weakening the ore [2]. In pursuit of an ideal ore pre-treatment

technique, selective heating emerges as a more effective and efficient approach. Among the limited number of techniques employing selective heating principles, microwave pre-heating stands out as a promising avenue.

Microwaves selectively heat minerals within the ore according to their capacity to absorb and transform electromagnetic wave energy into heat energy—a characteristic referred to as dielectric property [3]. Minerals that possess a high capacity for absorbing microwaves are classified as dielectric minerals, whereas those with limited microwave absorption capabilities are termed transparent minerals [4]. Certain metal oxides and sulfides minerals, including magnetite ( $\text{Fe}_3\text{O}_4$ ), hematite ( $\text{Fe}_2\text{O}_3$ ), ilmenite ( $\text{FeTiO}_3$ ), pyrite ( $\text{FeS}_2$ ), bornite ( $\text{Cu}_5\text{FeS}_4$ ), chalcopyrite ( $\text{CuFeS}_2$ ), and galena (PbS) have demonstrated significant rise in temperature within relatively short duration [5]–[8].

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Conversely, common gangue mineral classes such as silicates and carbonates exhibit minimal microwave absorption. When subjected to microwave irradiation, particularly under conditions of high-power density and brief exposure duration [9]–[11], the dielectric minerals adeptly absorb and store the oscillating electromagnetic waves, resulting in rapid heat generation along with physical expansion. Stress arises between the expanding dielectric minerals and the transparent minerals. When this stress surpasses the bond strength, it initiates intergranular fracturing [1], [5], [12]. From the perspective of mineral liberation, these intergranular fractures play a pivotal role. They contribute to enhancing the recovery of valuable minerals in subsequent downstream processes, as reported by some scholars [13]–[20]. However, there is another facet to the enhanced liberation that has received less attention in literature. The improved liberation also signifies the potential to recover valuable minerals at coarser sizes, a concept explored in a limited number of sources. For instance, when copper carbonatite ore is subjected to microwave treatment, the recoverable magnetite at the coarsest size experienced a 9% increase by mass. Additionally, the liberation of carbonate gangue minerals occurred at larger sizes [21]. Noteworthy enhancements in gravity gold recovery, specifically at coarse sizes, have been documented following microwave treatment of free-milling gold ore [22]. A reported 30% increase in grind size has also been observed in the case of microwave-treated copper sulfide ore [23]. However, it is important to note that all these findings stem from grinding mill tests, as the liberation sizes of these ores are typically below 300 microns.

In kimberlite processing, the objective product size for comminution is primarily dictated by the marketable diamond dimensions, which typically range from 25 to 1 mm. This target size is notably coarse in comparison to the standards of mineral processing. Consequently, conventional diamond processing plants eschew the use of grinding mills and opt for high-pressure grinding rolls as their ultimate stage of comminution [24], [25]. Conversely, since the -1 mm product size is classified as waste, it becomes crucial to liberate diamonds at the coarsest feasible size. Over-crushing the bulk materials, which constitute approximately 99.99% of the material (considering the average diamond content in ore is 0.2 ppm), should be avoided. The fraction -0.5 mm necessitates processing in a thickener before being directed to fine tailing ponds. This process consumes additional water and incurs related expenses. Less than 0.1 mm poses even more challenges, particularly due to presence of clay group minerals commonly found in kimberlite ore. These clay minerals tend to render the fine tailings stream slimy, leading to a substantial consumption of flocculants [26], [27]. The application of microwave pre-treatment holds the potential to enhance the operational efficiency of kimberlite processing plants. This enhancement could be realized by increasing the likelihood of liberating diamonds at coarser sizes while concurrently reducing the size of waste products.

The investigation of product size resulting from comminution employs particle size distribution (PSD) analysis, recognized as the most prevalent method of size characterization within the mineral processing sector. PSD represents the frequency (often mass) of particle diameter at a specific size,  $x$ , assuming an equivalent sphere [28]. This data is collected within geometric intervals, adhering to established sieve series standards like the U.S. or Tyler standards, which ensure accuracy across multiple scales. To facilitate analysis, the data is often depicted in cumulative form. Additionally, it can be mathematically fitted into an algebraic format, yielding supplementary parameters that encapsulate the entire distribution. Within the field of mineral processing, two extensively employed PSD models are the Rosin-Rammler (RR) models and the Gates-Gaudin-Schuhmann (GS) models. The RR model involves a double-log integration of the probability functions of size distribution, expressed as [29]:

$$W_r = 100 \exp\left(\frac{-x}{a}\right)^n \quad (1)$$

$$\ln \left[ \ln \left( \frac{100}{W_r} \right) \right] = n \ln x - n \ln a \quad (2)$$

where  $n$  is the distribution uniformity parameter,  $a$  representing the characteristic particle size, and  $W_r$  denoting the cumulative weight percentage retained on sieve size  $x$ .

On the other hand, the GGS model employs a more straightforward approach, utilizing a single log integration in the following expression [29]:

$$W_p = 100 \left( \frac{x}{k} \right)^\alpha \quad (3)$$

$$\log \left( \frac{W_p}{100} \right) = \alpha \log x - \alpha \log k \quad (4)$$

In the GGS model, the equation features  $\alpha$  as the distribution coefficient (akin to  $n$  in the RR model),  $k$  as a theoretical top size (akin to  $a$ ), and  $W_p$  representing the cumulative weight percentage passing sieve size  $x$ . When plotting particle distribution data with the logarithm of size, the data exhibits relatively good fit with linear lines using these two models. Both models find widespread application in industry: RR is prominent in the coal sector, while GGS is commonly employed in metal ores grinding studies [30]. These models offer richer insight into the characteristics of PSD through multiple parameters, as opposed to relying on singular parameters like  $P_{50}$  or  $P_{80}$  (representing the size at which 50% or 80% cumulative weight passes).

Analysing PSD can serve as a preliminary study before embarking on liberation analysis, especially relevant for kimberlite ore investigations where diamonds might not even be present in the sample. Yet, applying these two PSD models in a comparative study of microwave-assisted comminution, specifically to examine multiple size parameters like  $P_{50}$ , DP, and specific size parameters (-1 mm and -0.1 mm), remains unexplored. Hence, the primary objective of this study is to assess the influence of microwave pre-treatment on the product size of crushed kimberlite ore. This will be achieved through the utilization of two PSD models.

## 2 Materials and experimental methods

### 2.1 Materials

A specific type of kimberlite rock known as hypabyssal kimberlite (HK) is the material tested in this study. The bulk HK material was procured from Northwest Territories province in Canada. X-ray diffraction analysis reveals that the sample is primarily comprised of minerals from the serpentine subgroup (~35%) and the hydromica group (~25%), both of which fall within the silicate mineral class. Calcite (~18%), representing carbonates, follows these in abundance. The HK material notably contains a significant proportion of magnetite (~9%), which is the sole dielectric mineral present. This composition is considered to closely approximate the ideal ore texture for microwave-assisted mineral processing [8].

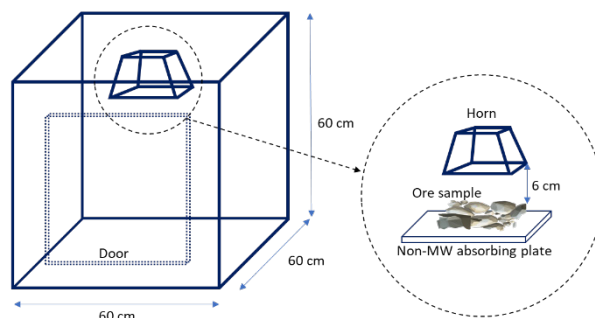
The entire material, amounting to approximately 120 kilograms, was pre-crushed into three distinct size categories:  $-31.5 +16$  mm,  $-16 +6.7$  mm, and  $-31.5 +6.7$  mm. Individual batch samples weighing 500 grams were meticulously prepared, ensuring consistent feed sizes (within a  $\pm 1\%$  margin of error). Some samples remained in their as-received state (untreated), while others were subjected to microwave treatment under various conditions (as detailed in Table 1). Due to limited availability of materials within the  $-31.5 +6.7$  mm size group, microwave treatment was exclusively conducted at low to moderate levels of microwave energy input.

### 2.2 Microwave setup

The microwave cavity takes on a metallic cubical form and features an antenna extending from its upper section (Fig. 1). This antenna is responsible for guiding the electromagnetic waves produced by a magnetron into the sample bed. The bed is positioned around 6 cm away from the antenna and is supported by microwave-transparent material. The microwave power is adjustable within a range of 5 to 15 kW, and the microwave input is modulated by altering the exposure time. (Microwave energy is computed as the product of microwave power and exposure time).

**Table 1.** Microwave conditions for all feed size group

Feed size group (mm)	Microwave Power (kW)	Net energy input (kJ)
$-31.5 +16$	5, 10, 15	15 – 260
$-16 +6.7$	5, 10, 15	10 – 260
$-31.5 +6.7$	10, 15	6 – 48



**Fig. 1.** Schematic illustration of 15-kW microwave single mode cavity at Geomechanics Laboratory, McGill University.

A laboratory-scale single-roll crusher (Goldbelt Global Olesi 4) is employed for the crushing of both untreated and treated samples in batch operations. The crusher features an opening (throat) measuring 6.25 x 10 cm, while the discharge gap is consistently set at 6 mm. The duration of a single crushing batch test involving a 500-gram sample ranges from 25 to 30 seconds, primarily contingent upon the feed size. The resultant crushed product is collected within an enclosed bucket and subsequently subjected to sieve tests utilizing the U.S. mesh standard sieve series (6.7, 4.75, 3.35, 2.38, 1.4, 1.0, 0.850, 0.600, 0.420, 0.300, 0.210, and 0.100 mm). The data pertaining to the distribution of product sizes are then processed using the GGS and RR mathematical formulations within Excel spreadsheets.

## 3 Results and discussion

### 3.1 GGS and RR models fittings

All microwave-treated data, irrespective of the microwave conditions, exhibit a tendency to display slightly superior fitting with both PSD models when compared to the untreated data, as demonstrated in Table 2. The quality of fitting is quantified by the coefficient of determination, denoted as r-squared, which ranges between 0 and 1, with 1 representing a perfect fit. The reported r-squared values represent averages, with no standard deviations exceeding 0.015. Both models exhibit a notably high degree of fitting, with r-squared values surpassing or equalling 0.94. In this study, however, the GGS model emerges as the superior predictor overall. The microwave-treated outcomes demonstrate a slight enhancement in fitting in comparison to the untreated data. This incremental improvement in fitting indicates an elevated level of precision in the distribution of crushed product sizes, particularly evident within the narrower feed size groups ( $-31.5 +16$  mm and  $-16 +6.7$  mm). In contrast, fitting accuracy of the crushed product PSD from broader feed size group ( $-31.5 +6.7$  mm) remains unaffected by microwave treatment.

**Table 2.** Fitness comparison between untreated and microwave-treated data with GGS and RR models

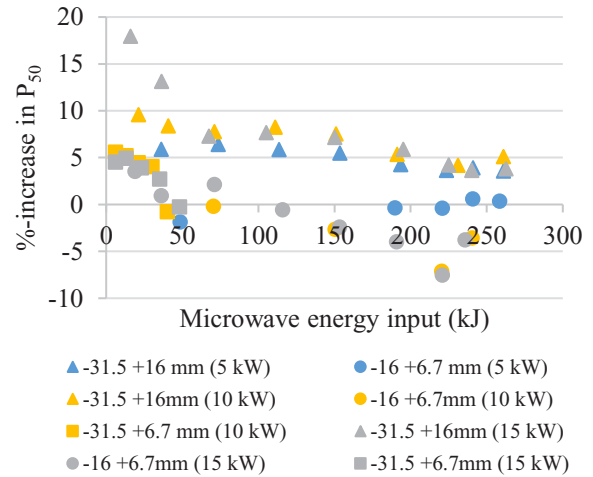
Feed size	Number of tests		RR r-squared		GGS r-squared	
	Untreated	Microwave-treated	Untreated	Microwave-treated	Untreated	Microwave-treated
-31.5 +16 mm	9	75	0.9421	0.9451	0.9912	0.9929
-16 +6.7 mm	7	62	0.9929	0.9448	0.9401	0.9919
-31.5 +6.7 mm	6	29	0.9676	0.9675	0.9855	0.9838

### 3.2 P<sub>50</sub>, P<sub>80</sub>, and distribution uniformity

Both PSD models can provide P<sub>50</sub> or P<sub>80</sub> values using Equations [2] and [4] (representing the size *x* at 50% or 80%, respectively). However, in terms of comparing these size parameters, including distribution uniformity, *k*, and critical sizes, the use of the GGS model would suffice due to its superior predictive capacity. Although P<sub>80</sub> is more commonly employed, possibly influenced by the prominence of Bond's equation, P<sub>50</sub> carries greater significance. P<sub>50</sub> not only denotes the median size but also holds practical relevance in kimberlite processing. It is particularly relevant as the subsequent downstream processes require division into a coarse stream (intended for coarse dense media separation) and a fine stream (geared toward fine dense media separation). P<sub>50</sub> can serve as a representative cut size. In terms of the results, the percentage increase of P<sub>50</sub> and P<sub>80</sub> relative to their respective reference samples exhibit notably similar patterns. However, the percentage increase of P<sub>50</sub> is 2 to 5 times greater. While only the P<sub>50</sub> results are presented in this manuscript, the discussion can equally apply to P<sub>80</sub>.

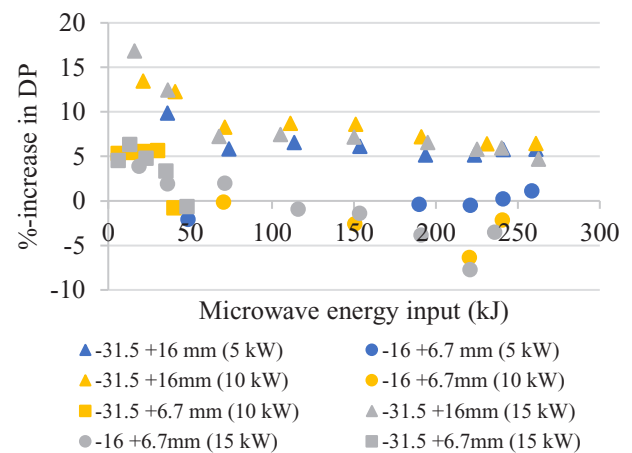
The impact of microwave treatment on P<sub>50</sub>, expressed as the percentage increase relative to the reference of untreated samples (y-axis), is plotted against the microwave energy input (x-axis), as shown in Fig. 2. The data points corresponding to microwave powers of 5 kW, 10 kW, and 15 kW are represented in blue, orange, and grey, respectively. Across most trends, the products' P<sub>50</sub> displays a higher percentage increase with lower microwave energy input, particularly notable for the coarse feed size of -31.5 +16 mm. An 18% increase in P<sub>50</sub> signifies that the median size of the microwave-treated crushed product is 18% larger than that of the untreated samples. This increase in P<sub>50</sub>, indicative of a coarser crushed product size, underscores the role of intergranular crack generation facilitated by microwaves.

Microwave treatment, specifically at low microwave energy input (equivalent to shorter exposure times), fosters the generation of intergranular cracks between mineral grains. However, these cracks are not extensive enough to propagate. Notably, the P<sub>50</sub> of the -16 +6.7 mm feed size crushing (represented by circular data points) does not experience a positive increase through microwave treatment, irrespective of microwave power level and energy input.



**Fig. 2.** Increase in P<sub>50</sub> (%) relative to untreated samples reference versus microwave energy input

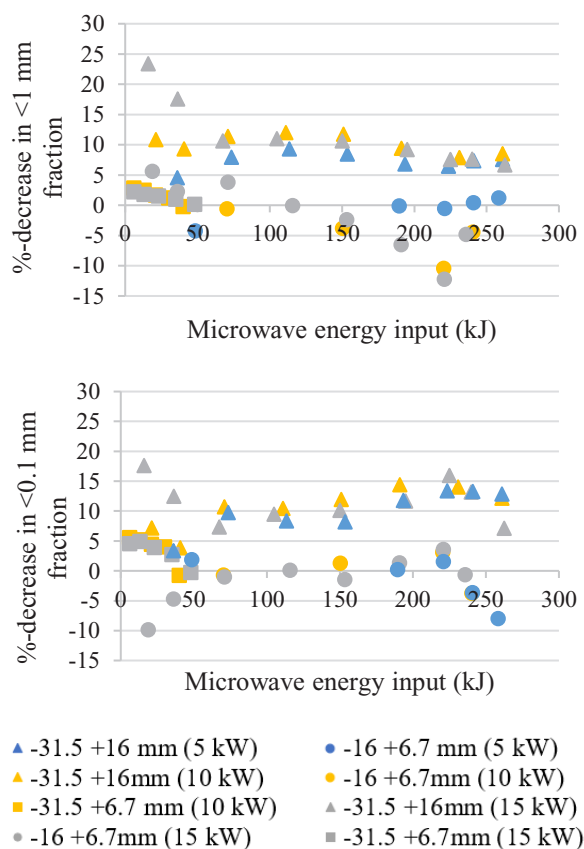
Distribution uniformity (DP) is calculated from equation (4), represented as *α* in the equation. Fig. 3 illustrates the relationship between the increase in DP caused by microwave treatment, relative to the untreated sample reference, and the microwave energy input. Notably, there are notable enhancements in DP observed in the microwave-treated samples, particularly when subjected to lower microwave energy inputs. A higher DP value corresponds to a narrower PSD or a steeper linear function within the models, a preference commonly sought in mineral processing. A narrower crushed product size holds significance as it not only implies a more pronounced shattering breakage mechanism in contrast to the prevailing cleavage and attrition mechanisms typical in roll-type crushers, but it also carries the potential to enhance screening and downstream process efficiency.



**Fig. 3.** Increase in distribution uniformity parameter (%) relative to untreated samples reference versus microwave energy input.

### 3.3 Critical fine sizes

This study considers two critical criteria for fine sizes in kimberlite processing. These are compared within this investigation: the fraction below 1 mm, representing the comprehensive tailings stream, and the fraction below 0.1 mm (100 microns), which substantially contributes to heightened tailing management costs. Both parameters can be derived from both PSD models. The optimal result of microwave treatment would involve a reduction in these fractions. The percentage decrease in the fractions below 1 mm and below 0.1 mm, relative to the untreated sample reference, is depicted for each microwave-treated sample in two separate graphs in Fig. 4. The trends in these decreases are related to the microwave energy input.



**Fig. 4.** Reduction in <1 mm and <0.1 mm fraction relative to untreated samples reference versus microwave energy input.

Once more, microwave treatment, particularly at low microwave energy input of 15 kW, leads to a reduction in both critical fine size fractions. Specifically, there is a 23% decrease for the fraction below 1 mm (Fig. 4 top) and a 17% decrease for the fraction below 0.1 mm (Fig. 4 bottom). The feed size emerges as the most influential factor influencing these critical fine size fractions. Remarkably, the coarse feed size group of -31.5 +6.7 mm consistently experiences favourable effects from microwave treatment, with reductions of no less than 5% observed across all microwave conditions. In contrast, different feed sizes produce divergent outcomes.

Significant reductions in critical fine size fractions have the potential to greatly save tailings management

costs. In a standard kimberlite processing plant, the rejection of the <1 mm stream involves pumping transportation as slurry, which demands substantial quantities of water, causes wear in pumping and piping systems, and consumes energy. The fraction below 0.1 mm is also crucial for determining the settling rate in the thickening process and impacts the consumption of flocculants and other reagents. This fraction becomes particularly problematic in the presence of swelling clay minerals within the kimberlite. Consequently, the findings from this study underscore the potential advantages of microwave pretreatment, extending beyond mere comminution optimization to encompass the cost-effective enhancement of tailings management.

### 4 Conclusions

The study has investigated the effectiveness of multivariable microwave heating in altering the crushed product size of kimberlite samples. This investigation was supported by the application of two PSD models: Gates-Gaudin-Schuhmann and Rosin-Rammler. Both models have demonstrated a high degree of fitting when applied to single-roll crushed kimberlite samples, encompassing both untreated and microwave-treated specimens. Microwave-treated samples exhibit even better and more consistent fittings, particularly evident within the GGS model. Microwave treatment selectively heats the magnetite within kimberlite samples, leading to the creation of intergranular cracks. These cracks subsequently influence the distribution of crushed product sizes. Comprehensive testing conducted across varying microwave power levels and energy inputs has revealed that the quality of parameters such as  $P_{50}$ , DP, and critical fine size parameters (<1 mm and <0.1 mm fractions) concurrently improves at the highest microwave power and the lowest energy input levels. Feed size is the most influential parameter in the study, with -31.5 +16 mm feed size consistently demonstrating improvement across different microwave treatment conditions, outperforming other feed sizes in this aspect.

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