Influence of time on the small strain shear modulus of an allophanic volcanic ash

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ABSTRACT

The small strain shear modulus is an important parameter in the assessment of soil dynamics problems. Studies on the small strain shear stiffness of volcanic ash remain rare probably because globally they cover just under 1% of the land surface. However, on a regional scale, this figure may be consequential as in the case of Japan, where about one third of its total land surface is covered by andosols. In this research, we aimed at understanding the influence of confinement time, a non-negligible parameter, contingent on the soil type, which needs to be accounted for when assessing the shear modulus. Bender element tests were conducted on allophanic volcanic ash, kuroboku soil sampled from the southern island of Kyushu in Japan. The allophanic ashes present all the characteristics of a non-textbook soil, notably high natural water content, high liquid and plastic limits and high void ratios. From the micrographic images, it was observed that the soil structure consisted of different types of porous particles (allophane, imogolite, volcanic glass and so on) at different internal spatial scales. Strong electrostatic bonding between the allophane particles means that in normal conditions the soil material exist as aggregates. The consequence is that the end of the consolidation stage is reached within a few minutes. Thus, the threshold demarcating the onset of the creep stage is different compared to sedimentary materials or other clayey soils. Based on the test results, empirical equations for predicting the time-dependent behaviour of the shear modulus were proposed.

Keywords: Allophanic volcanic ash; bender element; confinement time; shear modulus.

1. Introduction

The small strain shear modulus, G_{max} is a fundamental parameter in the assessment and evaluation of soil dynamics problems. As a testimony of the importance of this topic, it has remained a common theme of the various symposia of the Technical Committee (TC) 101 (formerly TC-29) of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE) beginning from the 'International Symposium on the Pre-failure Deformation Characteristics of Geomaterials' in 1994 held at Sapporo, Hokkaido, Japan. While much progress has been made towards understanding the G_{max} of sedimentary and clayey materials, corresponding research on soils of volcanic origin has lagged behind (Senetakis *et al.*, 2012; Liu *et al.*, 2016). In part this may be explained from the fact that volcanic soils are not widespread. For instance, globally volcanic ash cover only about 1% of the Earth's land surface. Nonetheless, in the case of a seismically active country like Japan, where about a third of its total land surface is covered by andosols (National Agriculture and Food Research Organization, 2022), studies on the dynamic properties of the volcanic ash gain relevance. Furthermore, the presence of clay minerals like allophane in the volcanic ash confers uncharacteristic properties to the soils which do not invariably fit with sedimentary materials (Wesley, 2010).

G_{max} is known to be influenced primarily by the void ratio and effective confining stress as well as by gradation, overconsolidation ratio, particle shape, fines content and so on. In the 1970s, early works highlighted that the influence of time on the shear modulus is mostly neglected (Marcuson & Wahls, 1972; Afifi & Richart Jr, 1973; Anderson and Woods, 1976; Anderson & Stokoe, 1978). Using mostly resonant columns on diverse materials like sands, natural and artificial clays, the main conclusions from these studies were that all soils displayed an increase in stiffness with time. Changes in void ratio during secondary compression stage were not enough to explain the increase in stiffness. Rather, it was speculated that the increasing stiffness was a reflection of the modifications in the soil structure with time. It was also noted that under constant confinement, the stiffness increased almost linearly with the logarithm of time at the end of consolidation (Anderson & Stokoe, 1978). Schmertmann (1991) then emphasized that it was not certain whether the above observations would equally apply to the time dependent behaviour of G_{max} of residual soils, which had not been much studied.

The main objective of this research was thus to improve the understanding with respect to the long term variation of the small strain shear modulus of a residual soil material. In this paper, bender element (BE) tests were carried on allophanic volcanic ash, kuroboku soil sampled from two sites in the southern island of Kyushu in Japan to assess the influence of time on the small strain shear modulus, G_{max}. Based on the results obtained, empirical relations for estimating the long term variation in G_{max} were proposed. Micrographic images of the soil samples are also presented together with a brief description of the soil components, contact relation and pore space.

2. Test Material

The present study was performed using remoulded allophanic volcanic ash, kuroboku obtained at a depth of about 1.5m from two sites in central Kyushu in the south of Japan. Soil 1 was gathered from Naoiri (Site 1) in Oita prefecture and Soil 2 was sampled at Tateno, close to the Aso Bridge landslide spot (Site 2) in Kumamoto prefecture. The sampling locations are shown in Fig. 1. Soil 1 and Soil 2 are shown in Fig. 2(a) and Fig. 2(b) respectively.

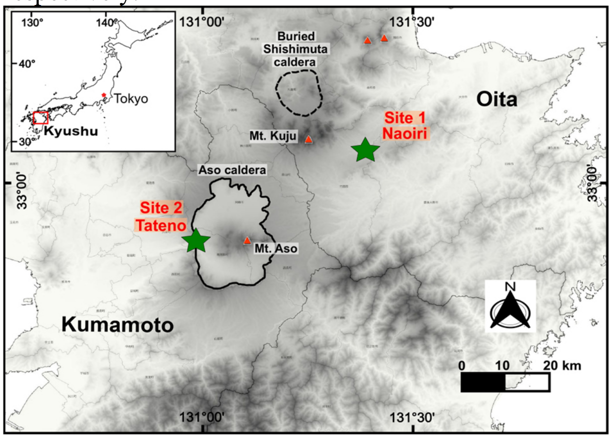

Figure 1. Sampling locations (Source map from the Geospatial Information Authority of Japan, 2022).

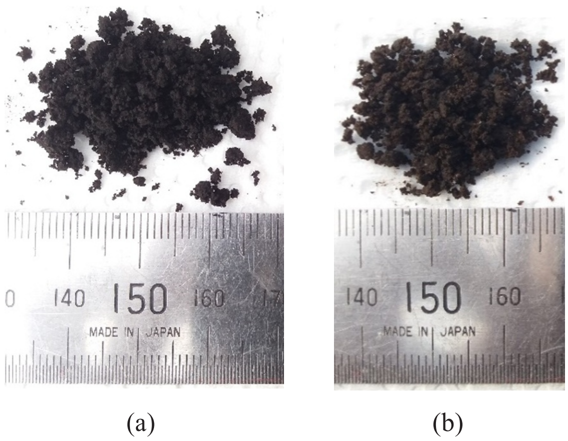

Figure 2. (a) Soil 1 (b) Soil 2.

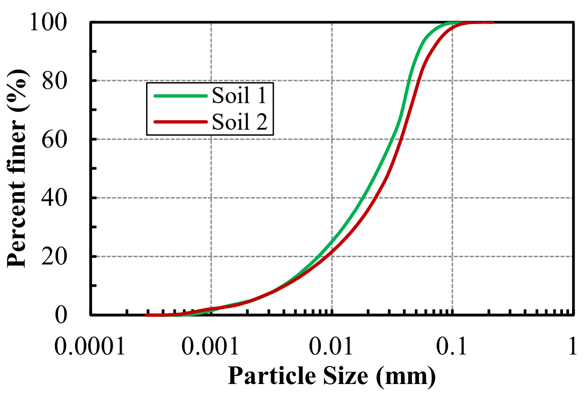

Figure 3. Grain size distribution.

The grain size distribution of the soils which was determined using a laser diffraction particle size analyser (SALD-3103, Shimadzu) is shown in Fig. 3. As can be observed, the tested volcanic ashes have a very high fines content in excess of 90%.

The Atterberg limits were determined as per Japanese Geotechnical Society Standards, JGS 0141. In accordance with the classification system of JGS 0051 – 2009, the allophanic volcanic ashes used in this study fall in the category of volcanic cohesive soil (type II) - VH2. The drying temperature of 110°C may be too high for some soils which owing to their mineral compositions have both “free water” and “water of crystallization”. Following Fourie *et al.* (2012), the water content of the allophanic ashes tested in this research was thus determined at a temperature of 50°C. For a more detailed explanation of this aspect, readers of this paper may refer to Gobin *et al.* (2023). The basic properties of the two soils are given in Table 1. It can be observed that the allophanic volcanic ashes are characterized by high natural water contents, high liquid and plastic limits (>130%).

Table 1. Soil Properties

Soil	G_s	Coefficient of Uniformity, U_c	Water content, w (%)	Liquid limit, LL (%)	Plastic limit, PL (%)
Soil 1	2.172	8.034	160	170	144
Soil 2	2.390	9.216	157	183	135

3. Methodology

3.1. Sample preparation and triaxial test procedures

Cylindrical triaxial specimens 50 mm in diameter and 100 mm in height were prepared in five layers using the moist tamping method. The soil at each layer was compacted by gentle tamping until the required thickness of each layer was achieved. The differential pressure was set to 15kPa. The specimens were saturated in three stages. At first carbon dioxide (CO_2) was circulated through the soil specimens for about 45 minutes. De-aired water was then percolated through the samples. A backpressure of 200 kPa was subsequently applied to saturate the soil samples. All samples were considered to be fully saturated when the Skempton pore pressure ratio was at least 0.95. The samples were consolidated at the target effective confining stresses of 50, 100 and 150 kPa.

3.2. Bender element

The bender element (BE), which can be incorporated in the triaxial equipment was used to estimate the shear velocity, V_s from which G_{max} was then evaluated. BE is suitable for studying time effects since the strain produced by the piezo-electrical transducer is of the order of 10^{-6}, which can be considered as non-destructive (Cai *et al.*, 2015). BE tests were performed at various time intervals during both the consolidation and creep stages.

 https://doi.org/10.1051/e3sconf/202454401005

The height and volume changes for the saturated specimens were captured by the transducers. Fig. 4 shows the schematic diagram of the testing equipment.

The shear velocity could be estimated by the following equation:

Shear wave velocity, $V_s = \frac{L}{\Delta t}$ (1)

where

L: length between tip to tip of bender elements (mm) and

Δt: shear wave travel time (ms)

The shear stiffness can be estimated using the following relation:

Shear stiffness, $G = \rho V_s^{\,2}$ (2)

where

V_s : shear wave velocity and

ρ: bulk density

The start-start method was chosen to determine the travel time for the allophanic volcanic ashes, similar to Nishimura (2006) for London Clay. Sine waves with a wide range of excitation frequencies (2–20 kHz) were initially used as input signals for the BE tests. Input frequencies between 6 and 8 kHz were deemed most appropriate to determine V_s, since it was found that the near field effects were negligible above this threshold.

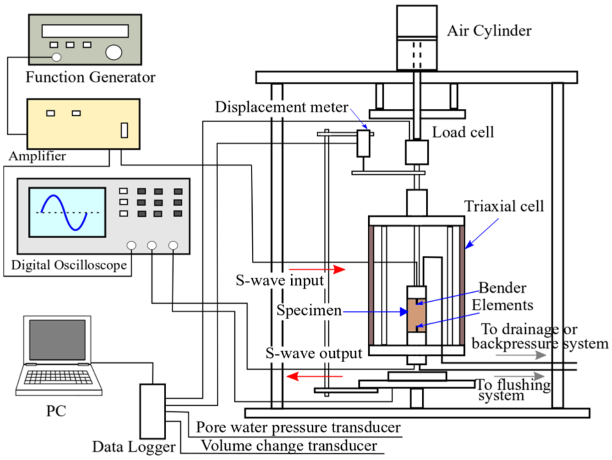

Figure 4. Schematic diagram of testing equipment

3.3. Microscopy analysis

Scanning Electron Microscope (SEM), Hitachi SU3500 was used to view the silt and sand sized components of the soil. Field Emission Scanning Electron Microscope (FE-SEM), Hitachi SU-8000 was used to observe the clay sized minerals.

4. Results and discussion

4.1. Microscopy analysis

The micrographs in Fig. 5(a) to (c) indicate that clay minerals like allophane and imogolite and volcanic glass are the main components of the tested soils. According to Wada (1978), allophanes are hollow spherules with a diameter of about 3.5 to 5nm. These individual hollow spherules can aggregate together to form domains (Rao, 1995). Imogolites exist as filiforms with individual fibres of about 2 nm. The length of imogolites can reach several micro metres in length (Wada, 1978).

From the SEM images in Fig. 5(a) to (b), it could be observed that the soil elements exist as aggregated particles, mostly of fine and medium sand sizes. In the FE-SEM image in Fig. 5(c), allophane domains with sizes varying from 25 nm to 60 nm can be seen. Yellow arrows in Fig. 5(c) indicate smaller allophane domains clustering into larger domains. Imogolite fibres enmeshing around the allophane particles could also be observed.

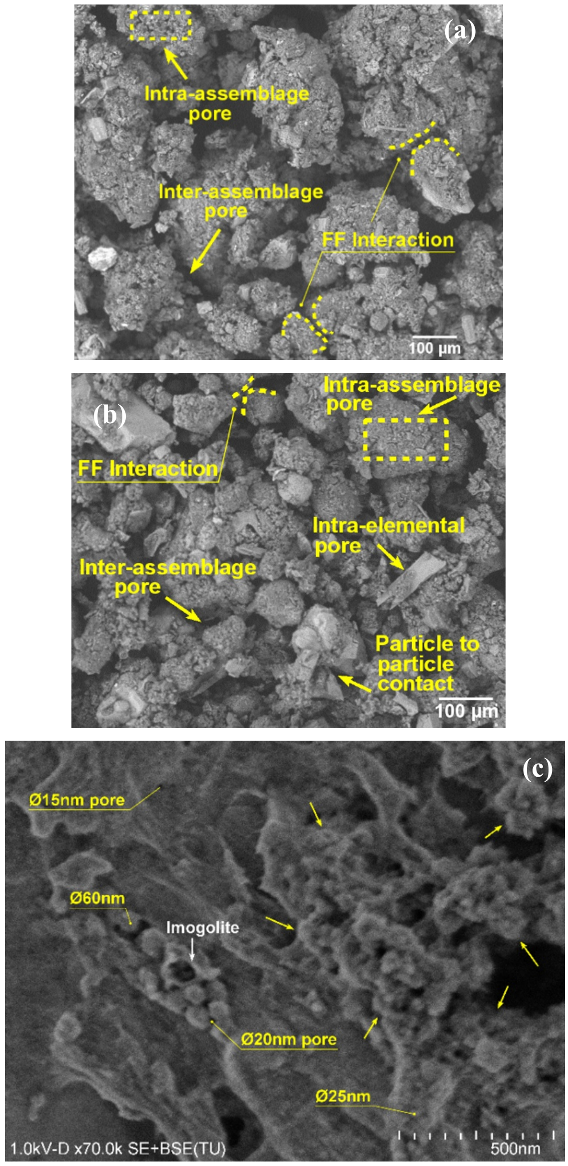

Figure 5. (a) SEM image of Soil 1 showing inter and intra assemblage pores and face to face (FF) contact. **(b)** SEM image of Soil 2 showing different types pores, face to face (FF) and particle to particle interaction. **(c)** FE-SEM image of Soil 2 showing presence of 25nm and 60nm allophane domains, imogolite fibres and presence of mesopores. Arrows indicate aggregation of allophane domains.

The main mode of interaction between the regular aggregates are most likely to be face to face (FF) contact. Particle to particle contact can occur occasionally between solid particles like volcanic glass (Fig. 5(b)).

The International Union of Pure and Applied Chemistry (IUPAC) has classified pores into three types based on their sizes, namely, micropores (<2 nm), mesopores (2-50 nm) and macropores (>50 nm). Macropores can exist as inter and intra-assemblage pores as well as intra-elemental pores in the vesicular volcanic glass particles (Fig. 5(a) to (b)). Mesopores of 15 and 20nm could be observed between allophane domains. Micropores, although not clearly visible from the FE-SEM image in Fig. 5(c) are found within and between allophane spherules and imogolite fibres (Wada, 1978).

4.2. Influence of time on shear modulus, G_{max}

The evolution of shear modulus, G_{max} and void ratio with time (on a semi-logarithmic scale), at effective confining stresses of 50, 100 and 150 kPa are depicted in Fig. 6(a) to (c) and Fig. 6(d) to (f) for Soil 1 and for Soil 2 respectively. It should be noted that the G_{max} results shown herein correspond to the vertically propagating, horizontally polarised shear wave (vh).

It can be observed that G_{max} increases significantly during the consolidation stage. The end of primary consolidation (EoP) for the allophanic volcanic ashes occurs quickly even if the latter are largely made up of clay sized particles. The EoP (estimated according to Anderson & Stokoe (1978)) was reached within 10 and 2 minutes for Soil 1 and Soil 2 respectively, with a slightly smaller time at higher effective confining stresses. As a comparison, for ball kaolinite the threshold demarcating the onset of the creep stage was about 1000 minutes (Anderson & Stokoe, 1978). Rao (1995) mentioned that due to the strong electrostatic bonding between the allophanic particles, the latter flocculate and exist as a pseudo-granular material. This explains why the consolidation stage proceeds at a fast rate for the allophanic volcanic ashes. In the pioneering works of Marcuson and Wahls (1972), Afifi and Richart Jr (1973) and Anderson and Stokoe (1978), it was assumed that the long term increase of shear modulus follows a linear trend with time on a logarithmic scale. At first glance, for the allophanic volcanic ashes, a linear trend with time during the creep stage also looks applicable. On closer scrutiny we can however observe that the G_{max} for the tested soils grows in phases, almost in step with the three stages of creep (as defined by Augustesen *et al.* (2004)). G_{max} increases at a decreasing rate during the primary (PC) and early parts of the secondary (stationary) creep stages. But for most of the secondary creep stage (SC), G_{max} increases at a near constant rate. At the cusp of the tertiary (acceleration) creep stage, a minor drop from the long-term growth in G_{max} was noted for all the soils at all effective confining stresses. This would appear to hint at a readjustment of the soil particles from a previously stable phase. During the tertiary creep stage (TC), the G_{max} of the allophanic volcanic ashes seems to grow at a marginally higher rate compared to the secondary stage at effective confining stresses of 100 and 150 kPa. Conversely, at an effective confining pressure of 50 kPa, G_{max} appears to start levelling off. The behaviour at a low effective confining stresses of 50kPa may indicate reorganization of soil particles by sliding and rolling mainly. The behaviour at higher confining stresses would suggest enhanced contact relations and entanglement of imogolite fibres or possibly deformation of solid particles like volcanic glass amongst others. Further tests are currently undergoing to confirm these observations.

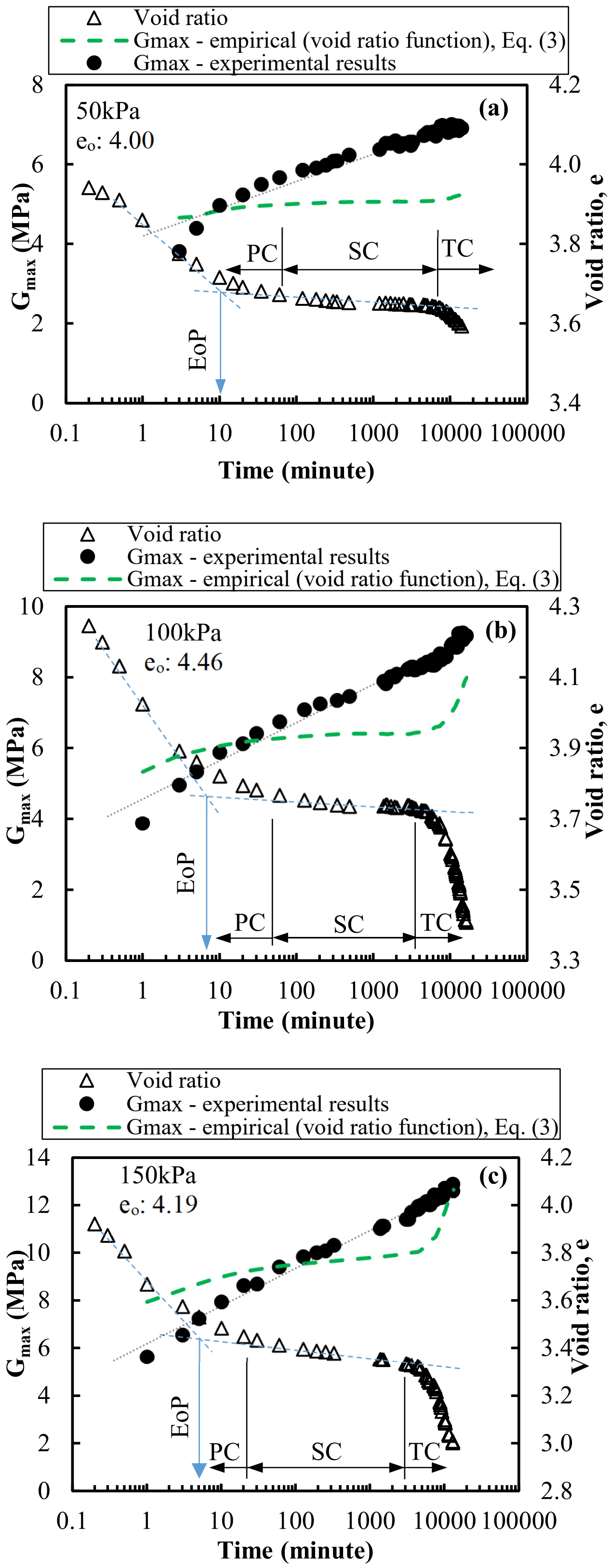

Figure 6. Variation of G_{max} with time **(a)** Soil 1 – 50 kPa. **(b)** Soil 1 – 100 kPa. **(c)** Soil 1 – 150 kPa.

Empirical formulae with void ratio and effective confining pressure as independent variables are commonly used to estimate G_{max} (Iwasaki & Tatsuoka, 1977; Yang & Gu, 2013). To assess the applicability over time, of empirical relationships between G_{max}, effective confining stress, σ' and void ratio, e, equations derived from data (50-150kPa) of Soil 1 and Soil 2 (at the end of primary consolidation) from Gobin *et al.* (2023) have been herein reproduced.

Soil 1

$$G_{\max} = 2842.7\frac{(e)^2}{(1+e)^{5.61}}\left[\frac{\sigma'}{P_a}\right]^{0.42} \quad (3)$$

Soil 2

$$G_{\max} = 2580.3\frac{(e)^2}{(1+e)^{5.68}}\left[\frac{\sigma'}{P_a}\right]^{0.54} \quad (4)$$

where, P_a is the atmospheric pressure which was taken as 100kPa for normalization purposes. The G_{max} obtained from equations (3) and (4) is in MPa.

The above two expressions have been represented as dotted green lines in Fig. 6 (a) to (f) to indicate the increase of G_{max} due to a decrease in void ratio. It can be observed that the increase in density of the soil as captured by the decreasing void ratio, cannot fully explain the increase in shear modulus.

The difference between the measured points and the data predicted by the empirical equations was greatest during the secondary creep stage for effective confining stresses of 100 and 150 kPa. At 50 kPa equations (3) and (4) very slightly undererestimate the measured values during the primary creep stage.

With the rapid decrease in void ratio during the tertiary creep stage, the difference in G_{max} estimated from void ratio functions largely narrows down compared to the actual values. Hence, it is fair to say that overall, in the case of the allophanic volcanic ashes, the variation between the measured points and empirical void ratio relationships are not so significant contrary to other soil materials like kaolinite (Afifi & Richart Jr, 1973; Anderson & Stokoe, 1978). But for a more realistic estimation of G_{max}, incorporating the time factor may be advisable.

To quantify the increase in stiffness with time, Anderson and Stokoe (1978) proposed the following expressions:

$$I_G = \frac{\Delta G}{\log_{10}\frac{t}{t_{\mathrm{ref}}}} \quad (5)$$

$$N_G = \frac{I_G}{G_{\mathrm{ref}}} \quad (6)$$

I_G is the rate of increase in shear modulus with time during the creep stage, at a constant effective confining pressure. I_G is commonly normalized with the value of shear modulus, G_{ref} at a reference time, t_{ref}, which is taken at the start of the creep stage. The normalized rate of shear modulus increase is denoted by N_G. t is a generic time and ΔG is the change in shear modulus from t_{ref} to t.

N_G varies with soil type. For sands, N_G values of 1 to 5% have been reported (Anderson & Stokoe, 1978; Santagata & Kang, 2007). In the case of overconsolidated clays values of 3 to 10% have been mentioned while for normally consolidated clays, N_G mostly varies from 5 to 20%. High N_G values in the range of 40 have been reported by Anderson and Woods (1976) for Leda Clay I from Canada and Gulf of Mexico clay.

As mentioned previously, the growth in G_{max} is not strictly linear and so is the case with N_G. But for easy application, a linear trend can be assumed provided that the experimental data extend through the secondary stage of creep.

In this study, the t_{ref} was taken as 30 and 10 minute respectively for Soil 1 and Soil 2 . N_G was found to be

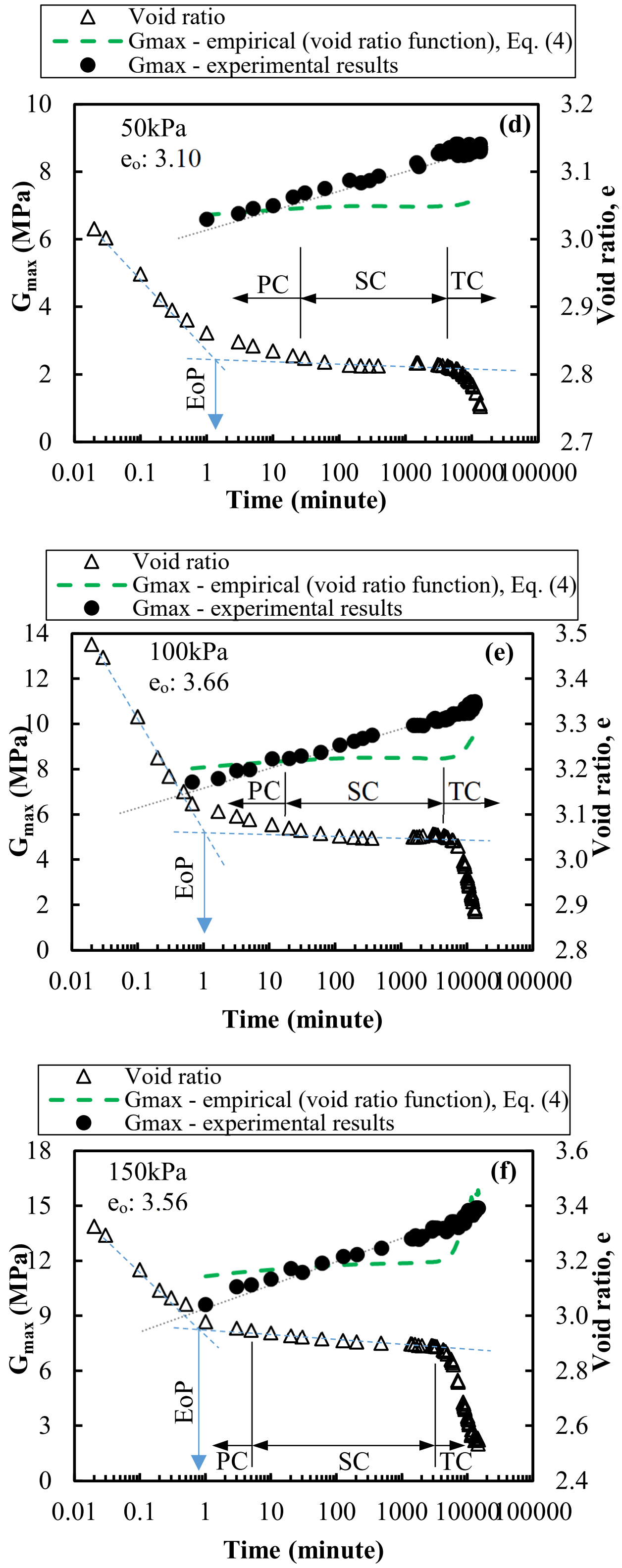

Figure 6. Variation of G_{max} with time **(d)** Soil 2 – 50 kPa. **(e)** Soil 2 – 100 kPa. **(f)** Soil 2 – 150 kPa.

a little more pronounced at higher effective confining stresses. At effective confining stresses of 100 and 150 kPa, N_G was found to be around 16 to 17.5% and 9.5 to 10.7% for Soil 1 and Soil 2 respectively. On the other hand, at an effective stress of 50 kPa, N_G was found to be slightly smaller with a value of about 11% and 8% respectively for Soil 1 and Soil 2. These results suggest that the normalized rate of increase in shear modulus for the allophanic volcanic ashes is comparable to clayey soils in Michigan like Chevy clay (10-16%) or Detroit clay (10-20%) (Anderson & Woods, 1976; Santagata & Kang, 2007), even if the consolidation characteristics are similar to granular materials. N_G was slightly higher for Soil 1 in comparison with Soil 2. This follows the findings of Afifi and Richart Jr (1973) and Anderson and Stokoe (1978) who observed that soils with a lower D_{50} generally display a higher N_G value.

For the purpose of estimating the long term increase in stiffness of the allophanic volcanic ashes, the following time-based equations are proposed:

Soil 1

$$G_{max} = 2842.7 \frac{(e)^2}{(1+e)^{5.61}} \left[\frac{\sigma'}{P_a}\right]^{0.42} \left[1 + N_G \log_{10}\frac{t}{30}\right] \quad (7)$$

N_G values:
50kPa: 0.11
100 - 150kPa: 0.16-0.175

Soil 2

$$G_{max} = 2580.3 \frac{(e)^2}{(1+e)^{5.68}} \left[\frac{\sigma'}{P_a}\right]^{0.54} \left[1 + N_G \log_{10}\frac{t}{10}\right] \quad (8)$$

N_G values:
50kPa: 0.08
100 - 150kPa: 0.095-0.107

where t is in minutes.

It should be added that from previous studies on the long term gain in shear stiffness, Ishihara (1996) reported that there is not much difference from using either remoulded or undisturbed samples. Hence, the long term shear modulus for the remoulded allophanic volcanic ashes may be expected to converge to a similar value as undisturbed samples.

5. Conclusions

With the aim of assessing the influence of time on the small strain shear modulus, G_{max} of non-textbook soils, bender element tests were carried out on remoulded allophanic volcanic ash, kuroboku collected from two sites in the south of Japan.

The main conclusions from this study are as follows:

1. Clay minerals allophane and imogolite together with volcanic glass are the main constituents of the allophanic volcanic ash, kuroboku. The pore sizes varied across the whole spectrum of macropores, mesopores and micropores. Particles interact mostly through face to face contact, with direct contact between volcanic glass less frequent.
2. With strong electrostatic bonding between the allophane particles, the soil exist as a pseudo-granular material. Consequently, the end of consolidation occurs within a few minutes.
3. It was observed that in general, the shear modulus, G_{max} increases with time in phases, almost reflecting the three creep stages. But for easy application, the growth in G_{max} with time may be approximated using a linear relationship on a semi logarithmic scale.
4. The normalized rate of shear modulus increase, N_G for the allophanic volcanic ashes was about 8 to 17.5%, which is akin to some normally consolidated clays.
5. Incorporating the time factor, empirical equations were proposed to estimate the long term growth in shear modulus for the allophanic volcanic ashes. The time based relationships give a more realistic estimation of G_{max} in comparison with empirical equations, solely based on effective confining pressure and void ratio.

Acknowledgements

The first author is thankful to the Japanese Government (Monbukagakusho: MEXT) for sponsoring his studies. The support of Kyushu University Library for procuring research materials and the help of Mr. Michio Nakashima, technical staff are also acknowledged.

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