Rheological behavior of granular materials under different relative densities

Jithin S. Kumar[1#] and *Ramesh Kannan* Kandasami[1]

[1]*Department of Civil Engineering, Indian Institute of Technology Madras, India*
[#]*Corresponding author: ce19d012@smail.iitm.ac.in*

ABSTRACT

Granular materials typically exhibit complex behaviour especially during shearing under different states. The material behaviour transitions from fluid-like to unfluid-like primarily due to various fundamental mechanisms involved during the shearing process such as momentum transport, compaction, and jamming. Describing this transition is particularly relevant in a variety of natural phenomena like landslides, debris flow, mudflow, and liquefaction. The mechanism involved in the transitional response is predominantly governed by the initial packing of granular medium, shear rate and boundary stresses. In this study, the influence of relative densities (20% to 74%) on the rheological behaviour of sand is studied using a vane-shear apparatus. The variation of critical shear strength (during jamming), yield strength, apparent viscosity, and the maximum shear rate with relative density is determined from this experimental program. Further, the characterization of the flow behaviour of sand revealed the existence of pre-jamming and post-jamming regimes. The post-jamming response is evaluated by using slope parameters (m and ξ) which predicted an increasing trend with increase in relative density and overburden pressure.

Keywords: Flow regime; vane-shear; jamming; shear rate

1. Introduction

Gravity flow of dry granular materials through hopper, chute and silos in food, processing, and pharmaceutical industries while shear-induced flow of wet granular materials such as landslides, mudflow and liquefaction (Gnoli et al. 2018, Kou et al. 2017, Forterre and Pouliquen 2008, Kalyan and Kandasami 2023) are some of the commonly observed kinematic process where the material transitions to different state. The transition from fluid-like to unfluid-like or vice-versa of granular materials is primarily due to various fundamental mechanisms such as momentum transport, compaction, and jamming (Dent 1986). The initial packing of granular medium and the stress state predominantly governs the mechanisms involved in this complex response of dry granular materials. Under shear loading, the particles move along the direction of force and interact with each other to offer more shear resistance. The flow of granular materials is typically categorized into three regimes; (a) under very low shear rate - quasi-static regime (no flow), (b) as shear rate increases - viscous regime or flow regime (shear-rate-dependent stress), and (c) gaseous regime (under high velocity or low density) (Pouliquen and Forterre 2009). Many researchers have performed experimental and numerical studies on the rheological behaviour of granular materials such as glass beads (dry or fully saturated) of different shapes and sizes (Pellegrino and Schippa 2017, Higashi and Sumita 2009, Macaulay and Rognon 2021).

Granular viscosity for a dry state has been studied using scaling laws by comparison with Newtonian fluids. The apparent granular viscosity (η) (Eq. 1 - ratio of shear stress - τ and shear rate - $\dot{\gamma}$) and apparent friction (μ) (Eq. 2 ratio between shear stress - τ and overburden pressure – P) describes the dynamics of granular materials under fluid-like and solid-like conditions which makes the physical behaviour difficult to depict.

$$\eta = \frac{\tau}{\dot{\gamma}} \quad (1)$$

$$\mu = \frac{\tau}{P} \quad (2)$$

Explaining the granular flow is complex due to the uncertain nature of the constitutive parameters involved in defining the flow properties. The flow behaviour of the dry granular materials are controlled by the friction and collision of the particles (packing density and particle morphology) (Forterre and Pouliquen 2008, Gnoli et al. 2018). However, in cases of granular suspensions, the rheological behaviour are mainly governed by interstitial fluid properties in addition to the grain size distribution, and particle concentration (Pellegrino and Schippa 2017, Scotto di Santolo et al. 2012, Pellegrino et al. 2013). Recently, studies were carried out using Discrete Element Method (DEM) to understand the flow behaviour of granular materials under the microscopic scale (Macaulay and Rognon 2021, Artoni et al. 2015). Currently, most granular flow models (Ancey 2007, Scotto di Santolo et al. 2012) that have been developed to describe the motion did not consider the effect of relative density on the rheology. Thus in order to quantify the influence of relative density on the rheological behaviour of granular materials, sand specimens prepared at different densities are sheared using a vane shear apparatus which utilizes a rotating geometry as shown in Fig. 1. The transition of fluid-like to unfluid-like behaviour is carefully examined to quantify the influence of relative density on different flow regimes. During the experiments, as the particles remains in contact most of the time, collision plays a crucial role suggesting a slow flow rate (van Hecke 2015).

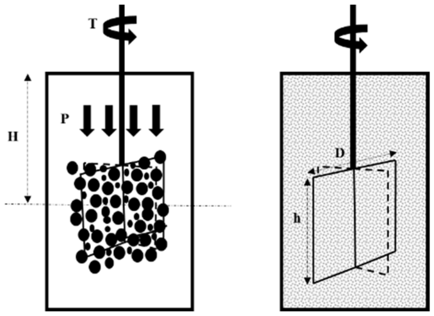

Figure 1. Vane-cup geometry used in this study.

2. Experimental program

Ennore sand of grade 3 (ES G3) is used as the granular material in this experimental program to understand the rheological properties. The uniformly graded particle size disribution of this Ennore sand is shown in Fig. 2. The sand is sub-rounded having a sphericity and roundness value of 0.6 and 0.45 respectively. The material properties of the sand are listed in Table 1. Different relative densities are achieved using the air pluviation technique (Hossain and Ansary 2018). With the height-of-fall method, a modified hopper setup having different nozzle sizes are used to achieve different densities. Further, a stress controlled vane shear apparatus with a vane-in-cup geometry (diameter 16 mm and height 52 mm) is used to study the flow properties of sand for different experimental conditions (such as overburden pressure, vane height and relative density).

The primary benefit of utilizing vane geometry lies in its capacity to diminish the sample's sliding effects during measurements. This is because the cross blades enlarge the contact area between the sample and the measurement geometry, resulting in a decrease in sliding effects during measurements (Beugre and Gnagne 2022). Vanes attached to the shaft are placed at an effective depth, H, at the center of the cup, to shear the uniformly packed granular material (as shown in Fig. 1). Utmost care has been taken to minimize the disturbance while placing the vanes inside the cup filled with sand. During shearing of the sand ensemble, due to the inter-particle friction, the vanes are initially jammed (static jamming) until the imposed stress exceeds the static friction because of yield strength, after which it rotates until the applied stress balances the dynamic friction (flow jamming). Hence, the spindle/vane rotates intermittently in a stick-slip manner. A suite of experiments is carried out for different vane heights (12.7 mm to 25.0 mm), placement of vane at different depths (to achieve different overburden pressure - 180 Pa to 850 Pa) and for different relative densities (20%, 64% and 74%) as mentioned in Table 2. Once the rotation of the vane stops, the corresponding failure strength is noted. The spring used in this vane-shear setup is calibrated and kept constant throughout the experiments for uniformity of angle of deformation (θ) under low strain. During experiments, the time is measured along with the deformation angle (θ). Later, the deformation angle is converted into shear stress using the equations (Eq. 3) derived from the first principles, where N is the calibration constant (Monney 1974). The shear rate ($\dot{\gamma}$) is calculated using the angle of rotation and the vane geometry (vane diameter, D and height, h) as given in Eq. 4. Moreover, the repeatability of these experiments are verified.

$$\tau = \frac{\theta N}{\pi D^2 \left(\frac{h}{2} + \frac{D}{6}\right)} \quad (3)$$

$$\dot{\gamma} = \frac{\theta D}{2 h t} \quad (4)$$

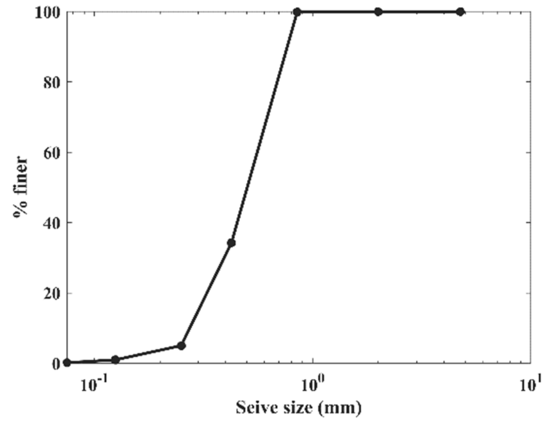

Figure 2. Particle size distribution of ES G3.

Table 1. Properties of ES G3

Properties	Value
Specific gravity, G	2.65
d_{10}	0.3 mm
d_{50}	0.6 mm
c_u	2.17
c_c	0.82
e_{max}	0.86
e_{min}	0.59

3. Methodology

The rheological behaviour of sub-rounded sand was exclusively understood by performing a series of controlled experiments. Later, these experimental results are analysed based on the methodology discussed below.

3.1. Visco-plastic rheology

Consider sand having an effective particle diameter d (d = d10) (Ismail 2008) and packed in a cylindrical cup for a particular packing density (ρ) under an overburden pressure of P. The overburden pressure, P is calculated based on the relative density and effective height (P = ρgH). A torsional force, T is probed through a rotating vane, which will shear the particles. For a large system of rigid particles (Young's modulus >> overburden/ confining pressure), the granular flow is controlled by a non-dimensional parameter, called inertial number (I)

(Pellegrino and Schippa 2017, Macaulay and Rognon 2021). Moreover, I should be less than one to maintain a slow flow (Artoni et al. 2015). The inertial number, I (Eq. 5) is defined as,

$$I = \frac{\dot{\gamma}d}{\sqrt{P/\rho}} \qquad (5)$$

In the slow flow, when $I < 10^{-3}$, the flow will be quasi-static or otherwise dense flow ($10^{-3} < I < 10^{-1}$). Under slow flow, the regimes of dry granular flow for a typical case are shown in Fig. 3 under low shear rates. The initial relative density provides a yield strength that resists the flow behaviour by inducing static friction. The regime under which the granular material provides static resistance is called a quasi-static regime. Once the yield strength due to static frictional resistance is reached, particle mobilization occurs, leading to particle rearrangement.

This particle rearrangement leads to shear thickening behavior (non-Newtonian response) under the viscous/ flow regime. Prior to the limiting inertial number, Im (Fig. 3), the continuous particle rearrangement during flow results in jamming (interlocked state). The shear rate and apparent friction corresponding to Im is called as maximum shear rate and jamming friction (μ_j) respectively. The apparent friction at the time of failure (sand packing fails to carry shear load - vane rotates rapidly for several revolutions) is called failure friction (μ_f), which is obtained from failure stress (τ_f). The increase in apparent friction even after jamming is due to the increase in contact stress due to lack of mobility of the particle. The slip behavior during post-jamming can be studied using a non-dimensional number m which is the slope of the line joining μ_j and μ_f. Analysis of the experimental results on apparent friction, inertial number, shear stress and shear rate provides more insight into the rheological behaviour of granular material under different relative densities in a vane-cup geometry.

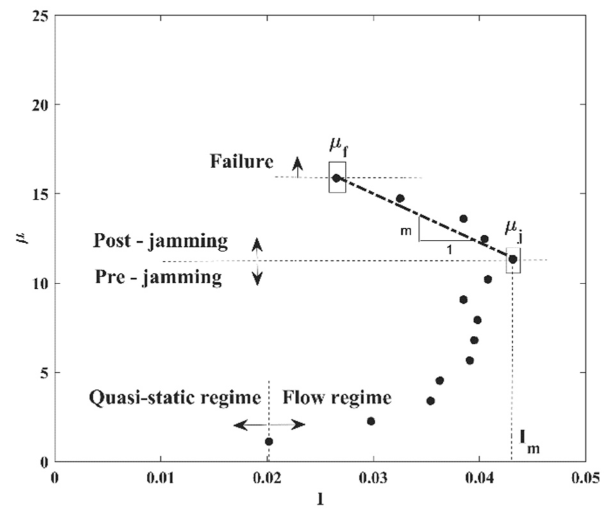

Figure 3. Regimes under dry granular flow (Case 3).

The analytical expression for the apparent friction (μ) and volume fraction or porosity (ϕ) as a function of the inertial number for a dry granular flow (Eq. 6), which is given as (Macaulay and Rognon 2021):

$$\phi(I) = \frac{\phi_m}{1+\sqrt{I}} \qquad (6)$$

where, ϕ_m is the maximum porosity. Using this model, porosity variation in each experimental condition are predicted for better understanding.

3.2. Herschel – Bulkley model

Continuum fluid mechanics models such as the Herschel–Bulkley and the power law models are of considerable interest in predicting the dynamics of natural and industrial granular flows (Huang and Garcia 1998). Under pre-jamming regime, with adequate experimental data, Herschel–Bulkley model can be used to capture the rheology of granular materials. In this model, the rheology is quantified using three parameters such as yield strength (τ_0), consistency index (k) and flow index (n), as shown in Fig. 3. This model is mathematically described as given in Eq. 7 (Konan et al. 2022). The sand when subjected to shear exhibits a shear-thickening behaviour i.e., increase in viscosity under high stresses which is modelled by increasing the power of strain rate term greater than 1 (n > 1).

$$\tau = \tau_0 + k\,\dot{\gamma}^n \qquad (7)$$

During pre-jamming or flow regime, the apparent viscosity, η_a (Eq. 8) is defined as (Lirer and Mele 2019)

$$\eta_a = \frac{\tau_{max} - \tau_{min}}{\dot{\gamma}_{max} - \dot{\gamma}_{min}} \qquad (8)$$

where τ_{max} and τ_{min} are the maximum and minimum values of the shear stress and $\dot{\gamma}_{max}$ and $\dot{\gamma}_{min}$ are the corresponding shear strain rates.

The above discussed methodology is used to understand the rheological behaviour of sand by quantifying the granular viscosity, frictional parameters and rheological parameters which are elucidated in detail in the following sections.

4. Results

In this experimental study, the effect of packing, overburden pressure and vane height are studied using a suite of five experiments. The bilinear variation of angular displacement (θ) with time for different test cases are shown in Fig. 4. Further, the failure stress (τ_f) and duration of the experiments/ failure time (t_f) for each experimental case are listed in Table 2. Fig. 4 shows an increase in angular displacement with an increase in relative density, overburden pressure and vane height (h). As the relative density increased by about 4 times, the angular displacement at failure increased by 5 times. Similarly, when overburden pressure is increased by about 5 times and vane height increased by 2 times, the angular displacement at failure increased by about 3 times and 2 times respectively. Thus, from the variables used in the experiments, it is observed that mobilization of granular materials during flow increases with an increase in these variables.

With the increase in overburden pressure, the time required to reach failure decreases, exhibiting a shorter flow regime as a consequence of an increase in contact stress between the particles which reduces the mobility. In case of increase in relative density, the failure time (t_f) is increasing which is due to the availability of more number of sand particles which improves the duration of quasi-static regime due to increase in static friction. An increase in vane height induces more mobilization of particles (larger influence zone) near the vane causing more resistance (frictional and colloidal force) which leads to jamming in a shorter period of time. Post-processing of these experimental data provides more insight into the granular flow in a vane-cup geometry (discussed in section 5).

Table 2. Failure parameters obtained from vane shear experiments

	τ_f (kPa)	t_f (s)
P = 180 Pa, RD = 20%, h = 12.7 mm (Case 1)	1.94	300
P = 850 Pa, RD = 20%, h = 12.7 mm (Case 2)	8.33	210
P = 850 Pa, RD = 64%, h = 12.7 mm (Case 3)	13.43	325
P = 850 Pa, RD = 64%, h = 25.0 mm (Case 4)	13.47	190
P = 850 Pa, RD = 74%, h = 12.7 mm (Case 5)	50.85	330

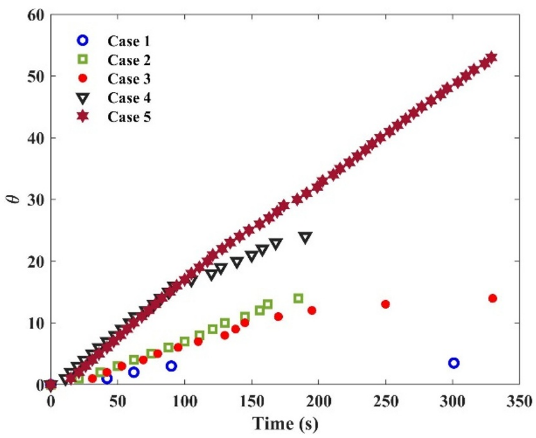

Figure 4. Variation of angle of deformation (θ) with time for different experimental conditions.

5. Discussion

The observations made from experiments are critically analysed using suitable non-dimensionalized parameters to understand various physics involved in determining the flow behaviour of dry granular material. From the analysis carried out on experimental observations as shown in Table 2, it can be inferred that, with increase in relative density from 20% to 74%, τ_f increases from 8.33 kPa to 50.85 kPa and t_f increases from 210 s to 330 s. This increase in τ_f and t_f is due to the reduction in effective mobility of the packing around the vane due to increase in flow parameters such as yield strength, and apparent viscosity. An obvious observation from Table 2 is that different vane heights under same overburden pressure and relative density does not influence the failure stress. However, there is an increase in failure stress with an increase in overburden pressure. Further, in this study, it is also observed that, with the increase in relative density, the bilinear response of the deformation angle with time vanishes which is trivial. This is due to more solid-like behaviour after jamming with an increase in relative density.

For better understanding, these experimental results are transfigured to non-dimensional parameters such as apparent friction (μ) and inertial number (I) (discussed in section 3). The variation of the apparent friction (μ) with the inertial number (I) is shown in Fig. 5. The shear response from the experiments clearly show the quasi-static and slow flow regimes ($I < 1$). Moreover, a critical post-jamming behaviour (slip flow) is observed beyond jamming friction. This is due to the localised particle rearrangement around the vane after jamming. The limit of the shear rate at which jamming occurs is mentioned as the jamming shear rate ($\dot{\gamma}_m$). The shear stress corresponding to the jamming shear rate is called jamming shear stress (τ_j). The variation of jamming friction (μ_j), failure friction (μ_f), m and I_m are listed in Table 3, where I_m is related to the jamming shear rate which together with jamming friction (μ_j) defines the flow regime. Observations from experiments predicted that all the frictional parameters (μ_j *and* μ_f) are increasing with an increase in relative density. With an increase in relative density, the domain of the flow regime is increasing due to increase in inter-particle interaction during this dynamic process (increase in I_m). The results show that the initial relative density plays a crucial role in the flow behaviour.

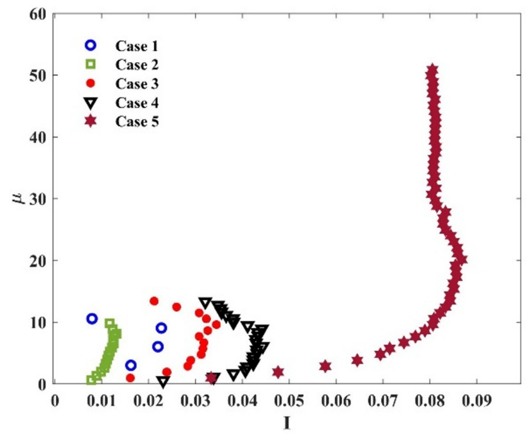

Figure 5. Variation of apparent friction with the inertial number for different experimental conditions.

Table 3. Frictional parameters

	μ_j	μ_f	I_m	m	ξ (1/s²)
Case 1	10.5	9.1	0.02	102.1	2.6*10⁵
Case 2	11.0	8.2	0.01	950.3	6.3*10⁵
Case 3	14.8	10.6	0.04	1302.5	5.2*10⁵
Case 4	14.7	9.8	0.05	1301.9	5.3*10⁵
Case 5	55.6	42.0	0.11	1868.1	10.0*10⁵

A post-jamming slope, ξ (similar to m) which is defined as the ratio between strain rate and time is observed in Fig. 6 where ξ shows a decreasing trend with an increase in relative density and overburden pressure. But with an increase in vane height, ξ remains constant (Table 3). Both post-jamming slope parameters such as ξ and m predict a similar sort of trend with respect to increase in relative density, overburden pressure, and vane height. These two parameters depict the failure plane during the post-jamming phase.

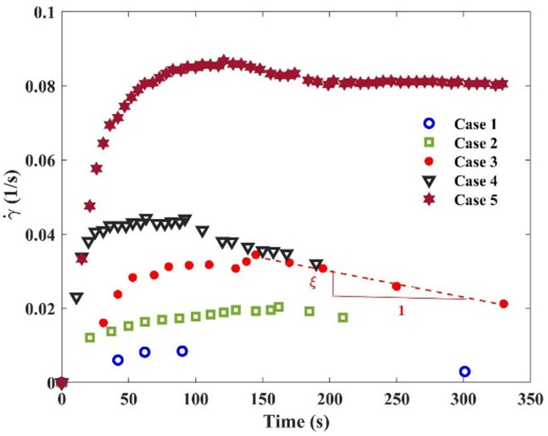

Figure 6. Variation of shear rate with respect to time for different experimental conditions.

For each case, the variation of granular viscosity (η) with apparent friction (μ) is quantified and plotted in Fig. 7. The variation of η with μ predicts a bilinear variation (due to prominent jamming). However, with the increase in relative density, this bilinear trend vanishes and shows a linear relation between η and μ. A similar kind of behaviour is also observed in the variation of deformation angle with time (Fig. 4). It is observed that the relative density enhances both granular viscosity and apparent friction which are functions of inertial number, I.

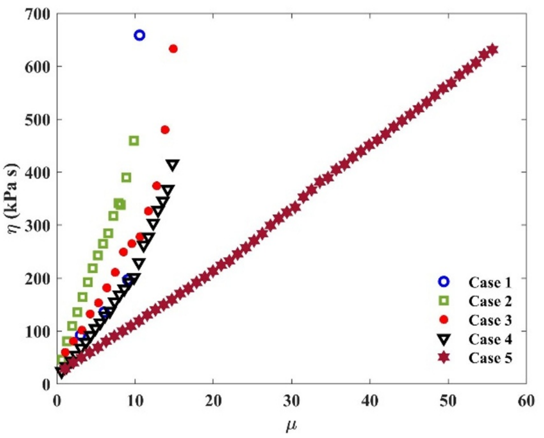

Figure 7. Relation between granular viscosity and apparent friction for different experimental conditions.

The characteristics of the flow of granular material under different conditions (Fig. 8) are studied by fitting it to a Herschel - Bulkley model (section 3.2) with the experimental results. The fitting parameters such as yield strength, consistency index, and flow index are listed in Table 4. This prediction shows that the relative density improves the yield strength and the flow index of the packing along with overburden pressure. The increase in flow index indicates the increase in the degree of shear thickening. Whereas the apparent viscosity shows a decreasing trend due to the increase in maximum inertial number, I_m. The increase in vane height from 12.7 mm to 25.0 mm significantly improved the consistency index and flow index, indicating an enhancement in shear thickening behaviour.

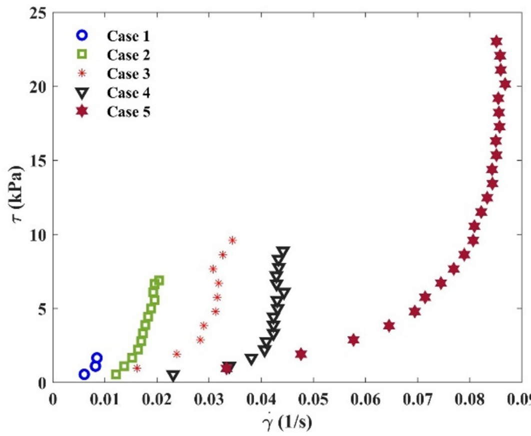

Figure 8. Pre-jamming rheological behavior of dry granular material for different experimental conditions.

Table 4. Rheological properties

	τ_0 (kPa)	k (kPa s^{-n})	n	η_a (kPa s)
Case 1	0.35	$1.15*10^{11}$	6.4	459.5
Case 2	0.55	$4.45*10^{11}$	7.1	465.5
Case 3	0.88	$6.22*10^{10}$	7.5	470.5
Case 4	0.91	$3.66*10^{16}$	11.3	395.4
Case 5	2.03	$4.56*10^{10}$	8.6	311.4

Using Eq. 6, the porosity variation is predicted for all the cases as shown in Fig. 9. With an increase in relative density, I_m increases (as shown in Table 3), which predicts a decrease in minimum porosity achieved prior to jamming. Similarly, the empirical model also predicted a decrease in minimum porosity with the increase in overburden pressure.

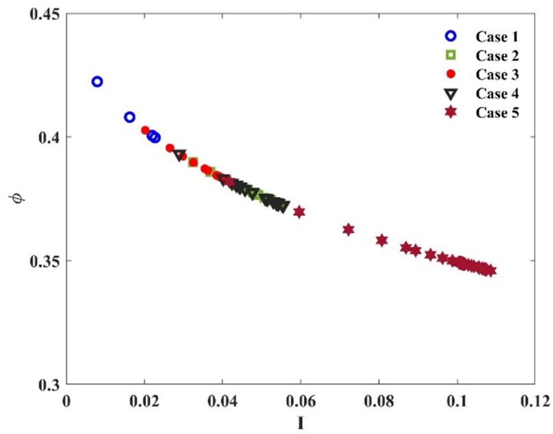

Figure 9. Prediction of porosity variation for different experimental conditions.

This study suggests that the relative density and overburden pressure improves the flow behavior of

Ennore sand (Grade 3). In addition, the above-discussed aspects of the rheology (on frictional and viscosity parameters) propose that vane cup geometry can be used to study the flow properties under slow flow conditions. Moreover, this study also suggests the occurrence of a post-jamming response (characterized by m and ξ) which is directly related to relative density.

6. Conclusions

The experiments are carried out to understand the rheological properties of granular material - ES G3 by varying the vane height (12.7 mm to 25.0 mm), overburden pressure (180 Pa to 850 Pa) and relative densities (20% to 74%) under slow flow regime. The observations made from the study are:

- The inertial number (I) controls the apparent friction under slow flow regime while the maximum inertial number, I_m increases with an increase in relative density
- An increase in relative density enhances the rheological parameters of the granular material, resulting in improved jamming stress and failure stress
- With an increase in relative density, and overburden pressure the yield strength also increases
- The post–jamming response about the failure plane (m and ξ) changes significantly with initial packing and overburden pressure
- The shear rate under which the granular material remains under the quasi-static regime and flow regime increases with an increase in initial relative density

The discussed results suggest that relative density is critical in describing the flow behaviour of granular material especially under slow flow regime.

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