Constrained modulus of fine-grained soils from in situ-based correlations and comparison with laboratory tests

Luisa Dhimitri[1], John J. M. Powell[2#]
[1]In Situ Site Investigation, St. Leonards on Sea, UK
[2]Geolabs Limited, Watford, UK
[#]Corresponding author: jpowell@geolabs.co.uk

ABSTRACT

The Constrained Modulus of soil is an important parameter to quantify compressibility of soils and calculate consolidation settlements, especially for clays. It is often expressed in terms of 1D constrained modulus, M or its inverse coefficient of volume change, m_v. One method that has been suggested to assess M is from Cone Penetration Testing (CPTU) measurements through various correlations. Although, this is sometimes difficult, because of uncertainties regarding the appropriate stress range and variations in effective stress, σ'_{v0} which are highly affected by the groundwater and bulk density, γ of soils. The usual method to measure M (or m_v) is from oedometer tests in laboratory. Correlating in situ tests with laboratory tests results is crucial to increase the quality of soils strengths and stiffness information for design. However, most of the times it is a challenging process, which involves many uncertainties from different soils' tests.
This paper will look at comparison of CPTU derived M to oedometer tests taking into consideration various types of soils mostly from well-documented testbed sites where high quality in situ (CPTU for all sites and SDMT where available) and oedometer tests are carried out. The importance of CPTU measurements and site-specific correlations with laboratory tests to establish unique soil type coefficients for use in equations to derive CPTU-based M will be highlighted.

Keywords: constrained modulus; small strain shear modulus; cone penetration test; correlations.

1. Introduction

In geotechnical design it is important to quantify compressibility and stiffness of soils to assess displacements during the life of a structure and to control movements to satisfy ultimate and serviceability limit states requirements of design codes. This information is also crucial for geotechnical modelling. Demand for realistic parameters is increasing. Geotechnical software used to perform FE analysis require an extensive list of input parameters for each soil constitutive model, including (but not being limited to):

- constrained modulus at critical state M or coefficient of volume change, m_v
- small strains shear stiffness, G_0
- modulus degradation curve, γ
- pre-consolidation stress, σ_p (expressed in terms of over consolidation ratio, OCR)
- stress dependent stiffness parameter, E_{eod}
- unloading reloading stiffness, E_{ur}
- creep, compression, swelling index, μ, λ, κ
- Poisson's ratio, ν

The practical requirement is to perform advanced laboratory tests and in situ tests, Piezocone with seismic measurements (SCPTU), Pressuremeters (PMTs), Dilatometer Marchetti with seismic measurements (SDMT), etc. Due to its repeatability, reliability, and cost effectiveness many engineers rely just on SCPTU to assist geotechnical calculations and soil modelling. Using software packages with embedded correlations, measured SCPTU data is converted into design geotechnical parameters within seconds.

Although these packages perform the mathematical calculations correctly, they are often used with too little input, experience, and knowledge (Powell and Dhimitri 2022). The lack of laboratory tests results to cross check these derive parameters increase uncertainties.

Following the publication from Powell and Dhimitri, 2022, this paper will look at constrained modulus, M.

For this purpose, 9 well documented test bed sites and 5 commercial sites comprising various soil types varying from very soft/ quick to OC stiff clays, silts, silts mixtures and sands, where good quality in situ and laboratory tests were performed, have been studied.

2. Theoretical Background

2.1. Constrained Modulus, M

2.1.1 M from CPTU

Constrained Modulus, M or its inverse, coefficient of volume change, m_v are widely used to calculate settlements.

The most used CPTU-based correlation is from Senneset et al. 1982, presented in Eq. (1), proposed for overconsolidated clays, based on linear interpretation model in preconsolidation range. (Lunne et al. 1997).

$$M = \alpha_M(q_t - \sigma_{v0}) \quad (1)$$

The origin of Eq. (1) is from Eq. (2) (Sanglerat 1972). It was suggested that α_M varies with soil type, plasticity, and natural water content for a wide range of soils.

$$M = \alpha_M q_c \quad (2)$$

Following the work done by Sanglerat 1972 and Senneset et al. 1982, Mayne 1990 suggested a general value of $\alpha_M = 8.25$. Further research highlighted that although this value works in some cases, the prediction of consolidation deformation may be in error as much as ±100% and a warning is warranted (Lunne et al., 1997).

In 2006 Mayne compiled an extensive database with a general trendline of $\alpha_M = 5$, which falls in the middle of the dataset. It is suggested that $\alpha_M = 5$ is more suitable for 'vanilla' soils. Mayne's database plot is than modified to include the trendlines for $\alpha_M = 1, 8.25\ and\ 20$ to encompass most of the data (Fig. 1). It is evident from this modification, that as Sanglerat suggested, α_M could vary. For most sites data seems to follow the general slope/trend of the trendlines. The need to vary of α_M with soil type is also supported by many authors working on this topic. Table 1 presents a summary of suggested α_M values. However, these values may be influenced by the quality of laboratory samples they were calibrated to.

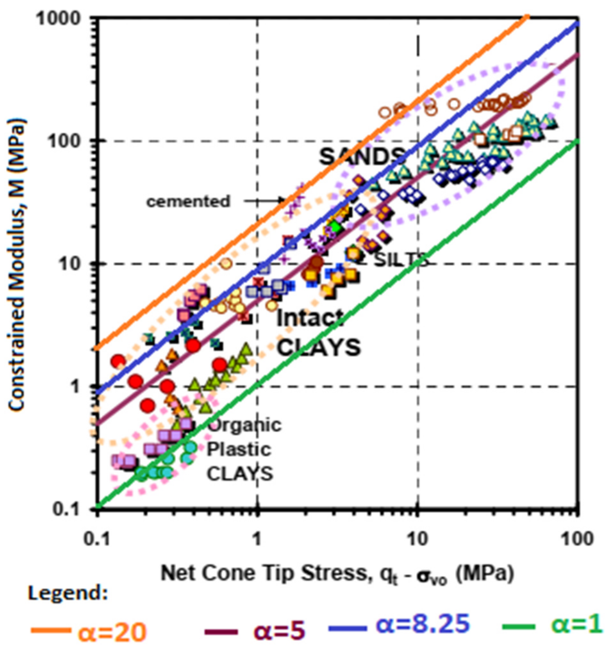

Figure 1. Modified Mayne 2006 database, relationships between M and q_t-σ_{v0}, for $\alpha_M = 1 - 20$ trendlines.

Another method to estimate CPTU-based M is by using Burns and Mayne 2002 correlation in Eq. (3).

$$M = \alpha_{MG0} G_0 \quad (3)$$

To distinguish from α_M used in Eq. (1), this coefficient is modified to α_{MG0}. Suggestions from various authors regarding α_{MG0} values are presented in Table 2.

Table 1. α_M from various authors

Reference	α_M
Sanglerat 1972	1 - 8
Senneset et al. 1982 & 1989	5-15 (most clays) 4-8 (NC clays)
Kulhawy and Mayne 1990	3 – 8 (NC sands) 7 – 25 (OC sands) 8.25 (recommended value)
Mayne 2006	5 (soft to firm clay & NC sands) 1 – 2 (organics) 10 – 20 (cemented clay) 5 (recommended value)
Robertson 2009	I_c >2.2 (fine grained) Q_t when Q_t < 14 14 when Q_t > 14 I_c < 2.2 (coarse grained) $\alpha_m[10^{0.55I_c+1.68}]$ where: $\alpha_m = 0.03$ (Robertson, 2009) $\alpha_m = 0.0188$ from experience, which better matches with Mayne, 2001 that M~G_0 in sands I_c soil behavior type index Q_t normalised cone resistance
Tschuschke et al. 2015	8.19 (clay) 9.57 (silty clay) 10.57 (sandy clay)
Młynarek et al. 2016	13.13 (NC tills)
Blaker et al. 2019	10 (clayey silt)
Di Buò et al. 2019	7.25 (soft sensitive clay)
Tonni & Gottardi 2011 **Tonni & Gottardi 2019***	$1.35I_c$ (silt) $1.22I_{cn}$ (silt)* where: I_{cn} modified soil behaviour type index as per Tonni & Gottardi, 2019*

Table 2. α_{MG0} from various authors

Reference	α_{MG0}
Burns & Mayne, 2002	0.02– 2 0.02 (organic plastic) 0.1 (clay) 0.2 (silt) 0.7-2 (sand)
Tschuschke et al., 2015	0.4 (Peat) – 14 (cemented clay) 0.12 (clay) 0.34 (silty clay) 0.45 (sandy clay)

Fig. 2 shows Mayne 2006 $M - G_0$ database, which clearly implies that α_{MG0} varies with soil type. We have also calibrated α_{MG0} for the soil types considered in this paper and added our trendlines to Mayne 2006 database in Fig. 2. For silts and stiff OC clays our values agree. For other types of soil α_{MG0} from the findings of this paper is summarised in Table 5 and the values are not similar to Mayne 2006.

G_0 is an important parameter that contributes to the success of Eq. (3), also important in many geotechnical design issues. For this reason, in Section 2.2 is explained how to estimate G_0 and correctly use in correlations.

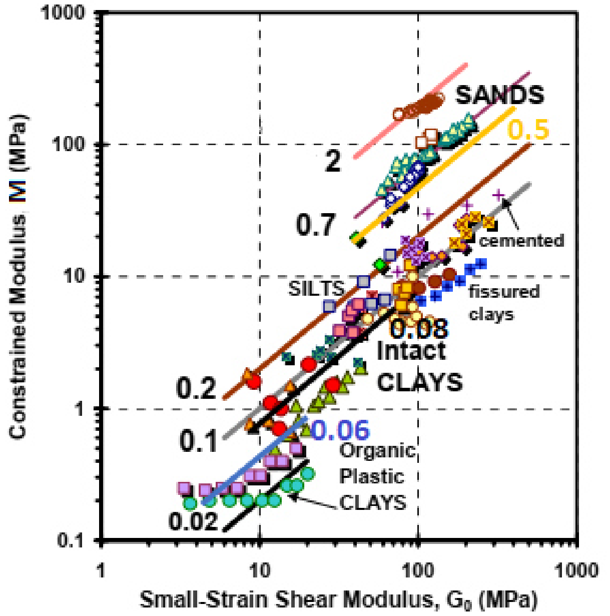

Figure 2. Modified Mayne 2006 soil SCPTU database relationships between M and G_0 for α_{MG0} 0.06-0.5 trendlines.

2.1.2 M from DMT

M can also be obtained from DMT through a correction factor, R_M, taken from Table 3 and applied to Dilatometer modulus, E_D using Eq. (4) (Marchetti 1980).

$$M = R_M E_D \tag{4}$$

where: $E_D = 34.7(p_1 - p_0)$
R_M calculation is given in Table 3.

Table 3. R_M values for various types of soils

Reference	R_M
Marchetti 1980	$I_D \leq 0.6$ (clay) $R_M = 0.14 + 2.36\log K_D$ $I_D \geq 3$ (clean sand) $R_M = 0.5 + 2\log K_D$ $0.6 < I_D < 3$ (silt and silty sand) $R_M = R_{M,0} + (2.5 - R_{M,0})\log K_D$ with $R_{M,0} = 0.14 + 0.15(I_D - 0.6)$ if $K_D > 10$ $R_M = 0.32 + 2.18\log K_D$ if $R_M < 0.85$ set $R_M = 0.85$ where: I_D dilatometer material index K_D horizontal stress index

2.1.3 M from Oedometer

Oedometer with Constant Rate of Strain (CRS), which allows for drained stress-strain behaviour of soils is the most common test to determine M. It should ideally be performed on high quality soil samples. Poor sample quality will result in misleading laboratory values. During the test, incremented vertical static loads are applied until full consolidation. After, the sample is unloaded. The representative value of the modulus is the tangent to the compressibility curve of effective overburden stress, σ'_v versus vertical strain, ε (Fig.3).

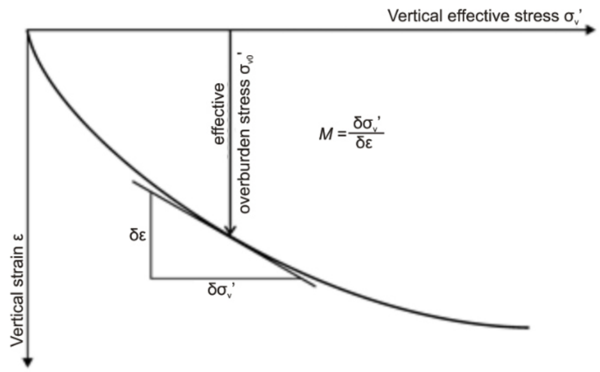

Figure 3. Constrained Modulus, M from oedometric test.

2.2. Small Strain Shear Stiffness, G_0

Small strain shear stiffness, G_0 can be easily estimated from the elastic theory using Eq. (5), if shear wave velocity, V_s and bulk density, γ_{bulk} are measured.

$$G_0 = \frac{\gamma_{bulk}}{g} V_s^2 \tag{5}$$

G_0 from Eq. (5) is considered a measured parameter. From SCPTU (or SDMT) it is estimated using measurements of vertically propagated horizontally polarised waves, V_{vh} (and can be better denoted as G_{vh}) This is the parameter that better relates to M to quantify the vertical compression. By way of note, it is a practice that when G_0 is the parameter of interest, seismic measurements are not always required by engineers, because they rely on CPTU derived V_s to assess G_0, rather than the measured V_s. There are 5 correlations commonly used to derive V_s from either corrected cone resistance, q_t, or friction sleeve, f_s, using also vertical stress σ_{v0} or soil behaviour type index, I_c (Eq. (6) – Eq. (10))

$$V_s = 118.8\log(f_s) + 18.5 \text{ (Mayne 2006)} \tag{6}$$

$$V_s = (10.1\log(q_t) - 11.4)^{1.67}\left(\frac{f_s}{q_t} * 100\right)^{0.3}$$

(Hegazy and Mayne 1995) (7)

$$V_s = 1.75(q_t)^{0.627} \text{ (Mayne and Rix 1995)} \tag{8}$$
$$V_s = 277(q_t)^{0.13} * \sigma_{v0}'^{\,0.27} \text{ (Baldi et al 1989)} \tag{9}$$

$$V_s = \frac{\alpha_{Vs} * \left(\frac{q_t}{\sigma_{v0}}\right)^{0.5}}{p_a} \quad \text{where } \alpha_{Vs} = 10^{(0.55 I_c + 1.68)} \text{ (Robertson 2009)} \tag{10}$$

Equation (8) was derived for clays, Eq. (9) for sands, and Eq. 10 is suggested for uncemented Holocene and Pleistocene soils. Users must be mindful about limitations of correlations and their relation to soil types.

Powell et al. 2016 showed that in fact, q_t correlates far better with G_{hh} then G_{vh}. For this reason, G_0 derived from V_s calculated using Eq. (6) – Eq. (10) is closer to G_{hh} than G_{vh}. The difference in G_0 due to anisotropy can cause significant discrepancy between M derived from Eq. (3) with G_0 from using V_s calculated as shown in Eq. (6) – (10), and M derived from Eq. (1) or measured in CRS oedometer laboratory tests. Furthermore, Powell

2017 and Powell and Dhimitri 2022 showed how erroneous derived G_0 can be without any site-specific calibration and anisotropy considerations.

G_0 can be measured in resonant column or by adding bender elements to triaxial tests. However, these measurements for anisotropic soils are G_{hh} and should not be used in Eq. (3). To add more uncertainties, laboratory results are affected by sample disturbance.

3. In situ derived Constrained Modulus, *M* and comparison with laboratory tests

Results presented in this section are based on soil types varying from very soft/ quick clays, alluvial clays (with microstructures of thin laminations and some cementation, a few with presence of organic material) to stiff heavily OC clays, comprising stiff, fissured, heavily OC aged clays or largely unstructured clay sized rock flour matrix with rock fragments. A summary of the typical background data compiled from laboratory reports from well documented sites is given in Table 4.

Table 4. Summary of soil types and properties from lab.

Soil type	Sites	OCR	s_u (kPa)	M (MPa)
organic clay	2	1.0-4.2	10-20	0.5-1.0
soft clay	3	1.1-1.3	20-50	0.6-2.0
stiff clay	3	2.9-103	60-270	5.0-12.0
silty clay	2	1.0-4.0	12-40	3.8-9.0
silts	2	1.6-3.4	20-50	19.0-23.4
sand mix.	1	23-50	-	14.7-58.8

CPTU - M is derived from Eq. (1) using global $\alpha_M = 8.25$ and $\alpha_M = 5$ regardless soil type, as per Kulhawy and Mayne 1990 and Mayne 2006, respectively and α_M based on I_c, as per Robertson 2009.

Another method used is through Eq. (3), estimating M from G_0, which requires the "constant" noted as α_{MG0}.

In Figs. 4-8 are shown CPTU - M results from Eq. (1) using 4 α_M values, CPTU - M results from Eq. (3) using best estimate α_{MG0} and DMT - M results, when available. CRS oedometer tests are plotted in dotted line.

Figure 4 shows M for a soft clay site in the UK. It is obvious that M is significantly overestimated but showing the right trend of increasing with depth. When using $\alpha_M = 2.5$ and $\alpha_{MG0} = 0.08$ an excellent agreement with oedometer results is achieved.

In Figs. 5-6 are shown results for glacial clay tills and heavily OC clays. $\alpha_M = 5$ used in Eq. (1) and $\alpha_{MG0} = 0.08 - 0.15$ in Eq. (3) give better estimation of M.

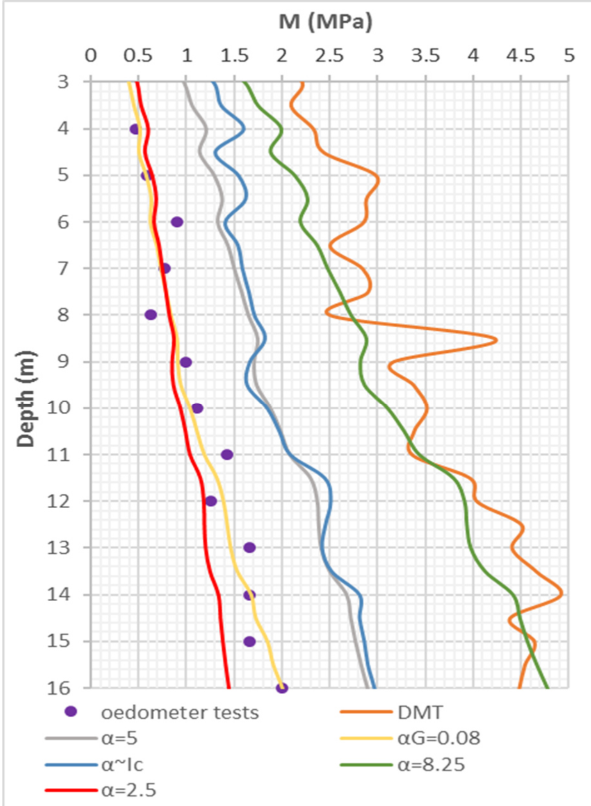

Figure 4. Constrained Modulus, M for Bothkennar.

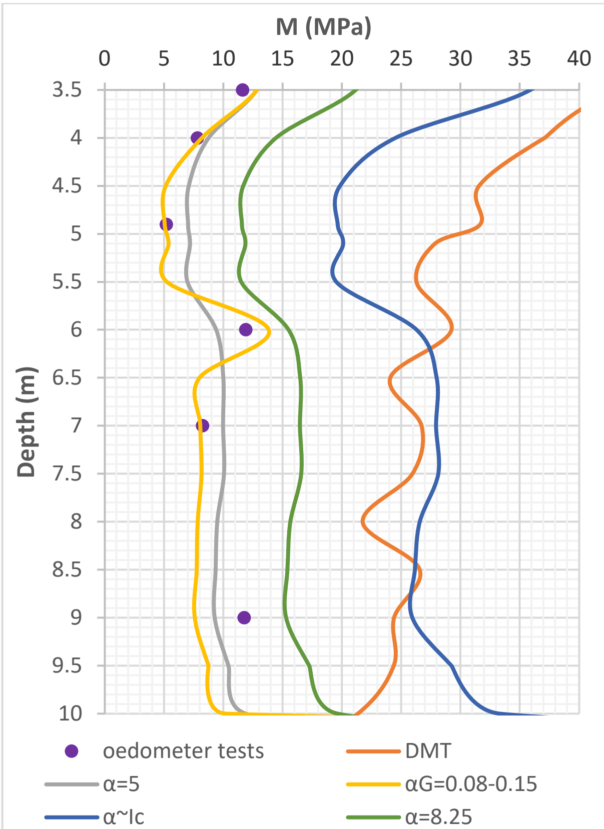

Figure 5. Constrained Modulus, M for Cowden.

For Halden, a post glacial clay/ silt testbed site (Fig. 7), derived M values are much higher. When modifying $\alpha_M = 12$ and $\alpha_{MG0} = 0.25$ excellent agreement with laboratory is observed. Also, good agreement is observed after modifying α_M=10 and α_{MG0}=0.2 for Tiller-Flotten, a quick clay site (Fig.8).

However, due to sample quality deterioration lower oedometer M values are observed at deeper depths (see L'Heurex 2019), affecting the good agreement observed in the upper meters. Looking at the results from I_C based α_M method, the peculiar shape of the graph shows that I_C might not always the right indication for α_M selection. α_M and α_{MG0} values used in Figs. 4-8 are given in Table 5. Overall ranges are similar to those in Tables 1 and 2.

Table 5. Summary of α_M and a_{G0}

The constant	Dhimitri & Powell, 2023
a_M	1.8-2.5 (organic clay) 2.0-3.5 (soft clay) 3.0-8.5 (stiff clay) 5.5-17.5 (soft-stiff silty clay) 12.0-20.0 (silt) 20.0-20.5 (sand mixtures)
a_{MG0}	0.06 (organic clay) 0.08 (soft clay) 0.1 (stiff clay) (also by Mayne, 2001) 0.2-0.25 (soft-stiff silty clay) 0.3 (silt) 0.5 (sand mixtures)

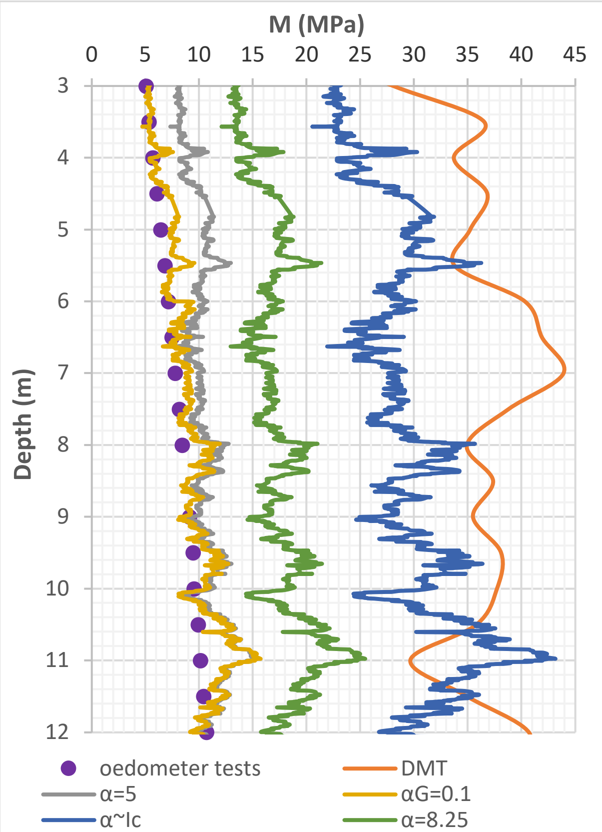

Figure 6. Constrained Modulus, M for Brent Cross.

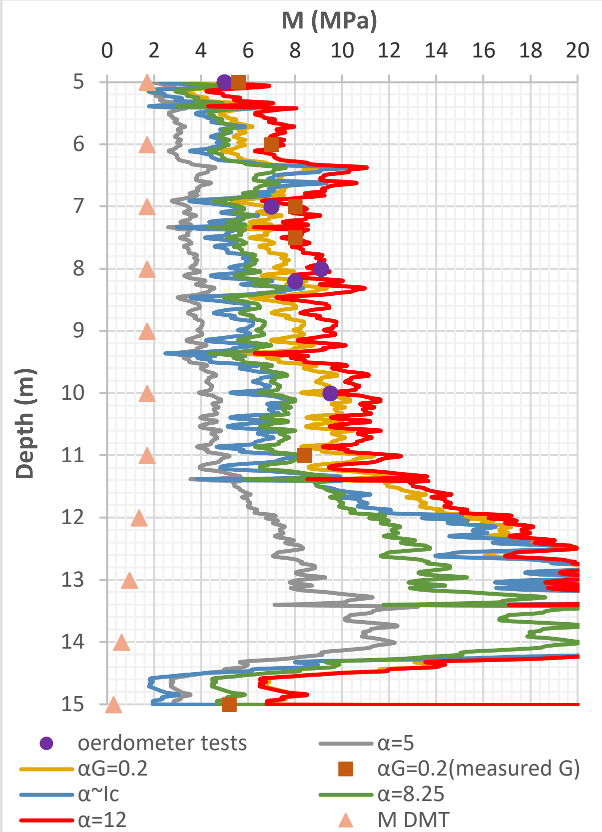

Figure 7. Constrained Modulus, M for Halden.

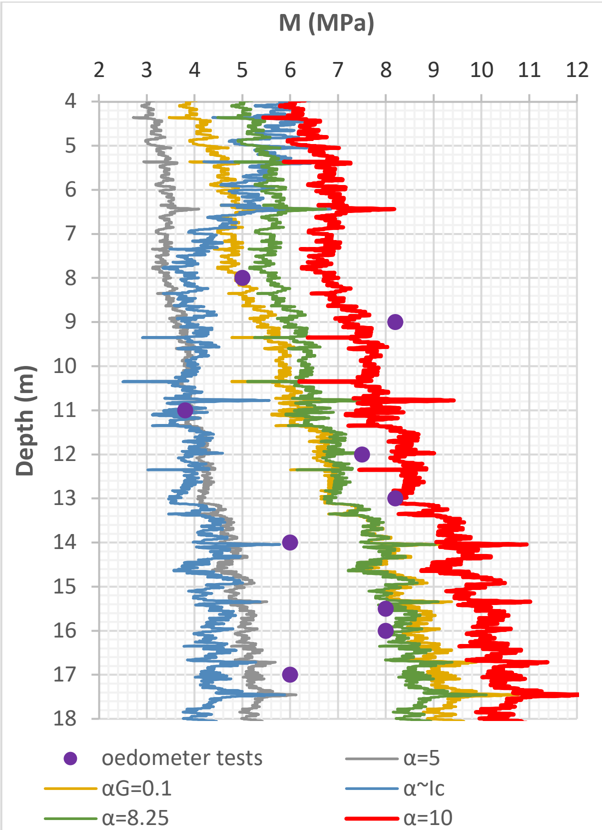

Figure 8. Constrained Modulus, M for Tiller-Flotten.

Some CPTU processing tools assign a default unit weight, γ to soil types. In Fig. 9 (Halden and Tiller-Flotten), from software generated γ used initially M was overestimated. Using the correct γ decreases M. Further M reduction is seen when ground water profile is corrected. This highlights the need of correct input for the tools, mostly relevant for methods using σ_{v0} and σ'_{v0}. This is not a criticism of the methods; it highlights the importance of using the correct basic input parameters.

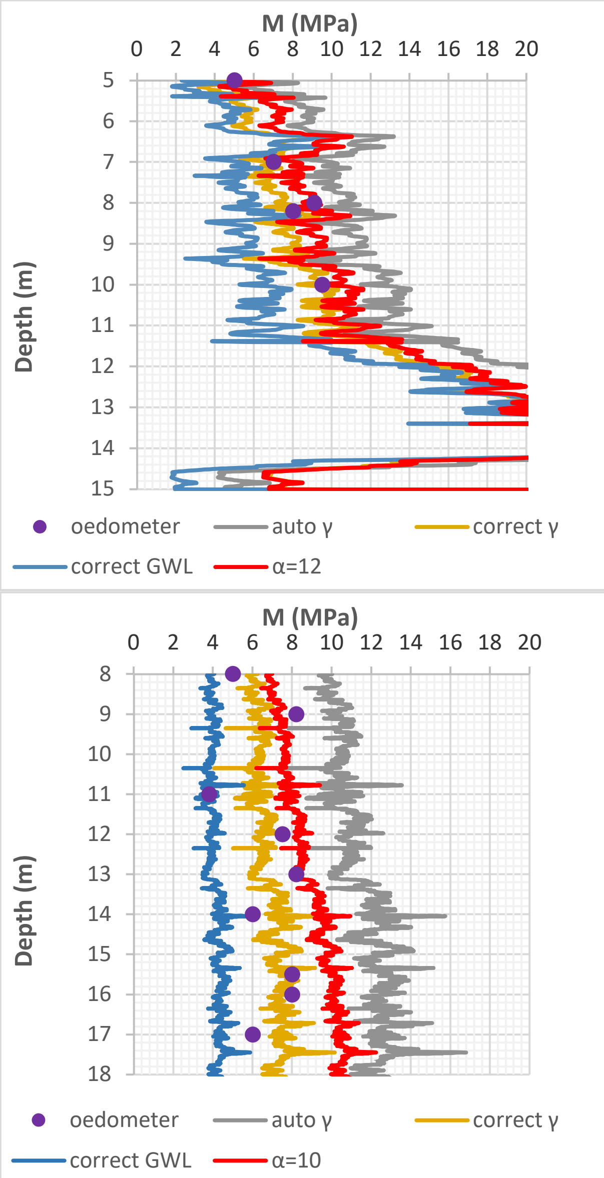

Figure 9. M with incorrect basic input data for 2 sites.

4. Conclusions

It has been shown in the foregoing that there is real potential for CPTU and SCPTU data be used to reliably profile constrained modulus, M with depth.

However, quite erroneous results can be obtained when using general software with inbuilt global correlations, typically overestimating M. These correlations need to be understood and to yield realistic values of M and site-specific calibrations are needed. The constants required for Eq. (1) and Eq. (3) need to be established by calibration to laboratory data from tests performed on high quality samples. Once established for a given soil type it may be possible to transfer to different sites on that same type of soil.

If realistic γ and ground water profiles are not included in analyses, then erroneous results are obtained.

On the sites where DMT data was available, is observed that it consistently gives erroneous results for M. Using DMT tests results for settlement calculations requires great care to accurately interpret M.

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