Some practical applications of shear wave velocity measurements in dense sand

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ABSTRACT

Shear wave velocity (V_s) profiles were obtained using two techniques at a dense sand site in Blessington near Dublin, Ireland. It was shown that both the multichannel analysis of surface waves (MASW) and seismic CPT (SCPTU) techniques provided reliable data. Use was made of a novel approach involving a Monte Carlo inversion of the MASW output to assess the fit between the measured and theoretical surface wave data. Differences between the SCPTU and MASW data are likely to be due to a combination of natural anisotropy in the sand and to uncertainty in the two methods involved. Previous published correlations between CPTU data and V_s also work well for this site. The V_s data was also used to accurately predict (Class 3 prediction) measured settlement data from shallow footing tests at the site.

Keywords: shear wave velocity; dense sand; footings; settlement

1. Introduction

Shear wave velocity (V_s) is a fundamental measurement in all solids for example steel, concrete, wood, soil and rock (Mayne 2000). Because of this broad range of application, V_s values are an attractive means of characterising a range of natural geomaterials.

Over the last decades, V_s measurements have gained popularity in geotechnical engineering practice. Advances in cost effective and efficient methods of determination of V_s focused attention on this parameter, which originally was mainly used for seismic hazard assessment or dynamic analyses. However, its use has been extended to general site characterisation studies, ground movement analyses for tunnels and excavation problems, determination of strength and compressibility parameters by empirical correlation, prediction of the behaviour of deep and shallow foundations, assessment of sample disturbance effects and in the quality control of ground improvement schemes among other applications.

This paper reports on the results of trials of two techniques used to generate in-situ V_s profiles at a well characterised dense sand test site in Blessington, Ireland. These were the seismic CPT approach (SCPTU) and multichannel analysis of surface waves (MASW). The latter technique suffers from a possible non-uniqueness in the inverted V_s profile. Here a novel approach using a Monte Carlo inversion procedure is used to explore the uncertainty. Previously published correlations between V_s and other CPTU parameters are examined. Finally use is made of V_s to predict the settlement of shallow test footings at the site.

2. The site

2.1. General

The Redbog quarry at Blessington has been used as a test bed site by researchers at University College Dublin (UCD), Trinity College Dublin (TCD), Munster Technological University (MTU) and Delft University of Technology (TUD) since 2001 (Igoe et al. 2010, Igoe et al. 2011, Gavin et al. 2013). This is a sand quarry site on the outskirts of Blessington village, 25 km southwest of Dublin City, see Fig. 1. The main focus of the early research was on various aspects of foundation behaviour. The geotechnical characteristics of the site have previously been summarised by the above researchers (Igoe and Gavin 2019).

More recently the site has been used to explore bio-mediated and bio-inspired soil improvement schemes such as Microbial Induced Calcite Precipitation (MICP), and Enzyme Induced Calcite Precipitation (EICP) respectively (Judge et al. 2022). These are innovative soil remediation techniques that fuse soil particles together by growth of calcite.

Due to the ongoing quarrying operations several different areas in the quarry have been used for the research, see Fig. 1. The properties of the sand are broadly the same throughout the quarry. Following on the previously used terminology (Judge 2022) the original area used for the foundation studies is denoted Location 1 and the recent soil improvement test area is known as Location 2.

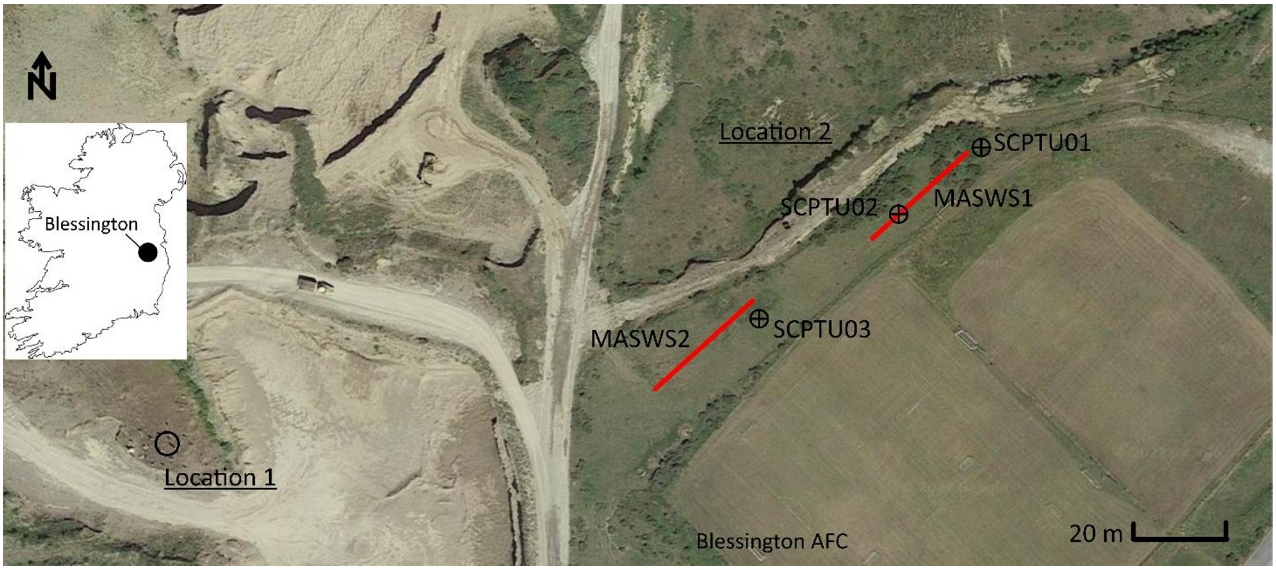


Figure 1. Test locations at Redbog Quarry, Blessington, Co. Wicklow, Ireland. The remains of the various pile tests can be seen at Location 1. Base map from Google Maps.

2.2. Geotechnical characteristics

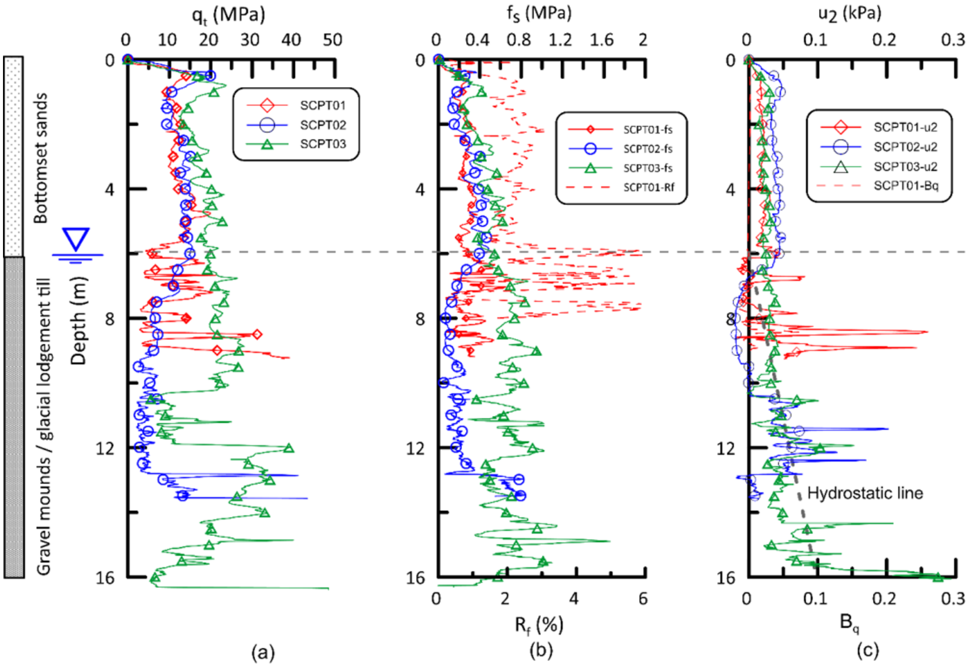
The sand that is the focus of this study is a Quaternary deposited sub-lacustrine deltaic bottomset deposit Glacial action and the recent removal by quarrying of the upper overburden material has resulted in the sand being in a heavily overconsolidated state, with an in-situ density of some 1.9 Mg/m^3 and a relative density close to 100%. The in-situ water content averages at 15.4%. Particle size distribution analyses indicate a fines content of 35%, a D_{60} = 0.11 mm and effective size D_{10} of 0.02 mm. Constant and falling head permeability measurements suggested hydraulic conductivity ranging from 9.4×10^{-4} to 8.8×10^{-6} m/s. The water table is generally deep due to quarry activities (Judge 2022).

The chaotic soil beneath the bottomset sands is likely to be material referred to geomorphologically by Philcox (2019) as gravel mounds. The gravel mounds are part of the pre-main delta deposits.

This material was deposited in a very different environment to the deltaic deposits and is more closely allied to that of a lodgement till (Judge 2022).

2.3. CPTU data – Location 2

Some CPTU data for Location 2 is shown on Fig. 2. The plots show corrected cone resistance (q_t), sleeve friction (f_s) and pore water pressure (u_2) as well as the derived parameters friction ratio (R_f) and pore water pressure parameter (B_q). The traces confirm that about 6 m of the bottomset sand deposits overlie gravel mounds and glacial lodgement tills at this location. The sands have q_t of about 15 MPa, f_s of some 0.3 MPa and show no excess pore water pressure during the CPTU push. These data fall in Zone 9 of the classical (Robertson et al. 1986) soil behaviour type chart confirming their classification as “sand”. The Location 1 data show very similar profiles (Igoe and Gavin 2019).



(a) (b) (c)

Figure 2. CPTU data from Location 2 (a) q_t, (b) f_s and R_f and (c) u_2 and B_q.

3. Techniques for determining shear wave velocity

Here the two techniques used in this study, i.e. MASW and SCPTU will be described. Trafford et al. (2022) also detail the use of distributed acoustic sensing (DAS) for V_s profiling at another location in the quarry.

3.1. Multichannel analysis of surface waves (MASW)

MASW is a non-invasive technique which allows estimation of seismic shear wave velocity (V_s). This technique was introduced in the late 1990s by the Kansas Geological Survey (Park et al. 1999). The method utilises the dispersion property of surface waves for the purpose of V_s profiling in 1D (depth) or 2D (depth and surface location) format. The entire procedure for MASW usually consists of four steps, see Fig. 3a.

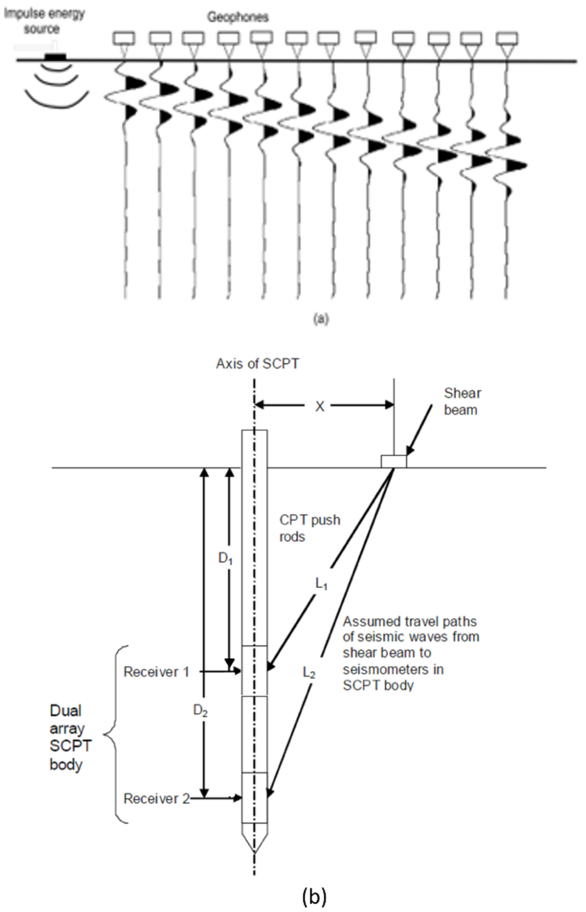


Figure 3. (a) MASW (Donohue 2005); (b) SCPTU (Butcher and Powell 1995)

(i) Acquire field records by using a multichannel recording system and a receiver array, like those used in conventional seismic reflection surveys. Typically, in geotechnical work the test configuration comprises twenty-four geophones spaced at 3 m centres over the survey length. An impulsive source (e.g. a sledgehammer) is used to generate the surface waves.
(ii) Use is then made of the dispersive properties of the soil, i.e. longer wavelength signals reflect the deeper soils and shorter wavelengths represent the shallower soils, to produce a phase velocity versus wavelength relationship from the measured data.
(iii) This phase velocity versus wavelength trace is converted to a dispersion curve (phase velocity versus frequency). Usually only fundamental mode dispersion is used.
(iv) The dispersion curve is inverted to obtain 1D (depth) V_s profiles (one profile from one curve). The inversion process involves defining a velocity model(s) whose calculated dispersion curve best matches that from the measured surface wave. The Monte Carlo forward modelling method was utilised to achieve this, using the "Dinver" module of the open-source software Geopsy; (Wathelet et al. 2004), (Wathelet 2008).

3.2. Seismic cone penetration testing (SCPTU)

SCPTU testing at Blessington was carried out by In Situ Site Investigations Ltd. using a system identical to that applied in the seismic dilatometer test (Marchetti 2015). A standard cone penetrometer is equipped with two horizontally aligned seismic sensors (Fig. 3b). Recordings are made during a pause in the cone penetration typically every 0.5 m. The seismic signals are generated by striking a horizontal beam which is coupled to the ground by the weight of the testing vehicle. The hammers mass was about 10 kg.

The beam was aligned parallel to the axis of the receivers. Assuming straight ray paths V_s is determined by the difference in the travel path to the two receivers divided by the difference in the shear wave travel time ($t_2 - t_1$) (Eq. 1).

$$V_s = \frac{L_2-L_1}{t_2-t_1} \tag{1}$$

4. Shear wave velocity measurements

4.1. Location 1

The deposits at Location 1 comprise a thick sequence of bottomset sands. No gravel mounds or glacial lodgement till was encountered in the investigations. Four MASW profiles for this area are shown on Fig. 4 (All profiles are at Location 1 but exact the positions are not known). All the data was inverted using the software Surfseis (KGS 2014) to provide a single set of V_s values with depth. It can be seen the measurements are very similar in all 4 tests with V_s increasing from some 180 m/s near the surface to 360 m/s at 14 m. These values correspond to a "dense" deposit as expected (EN 2004).

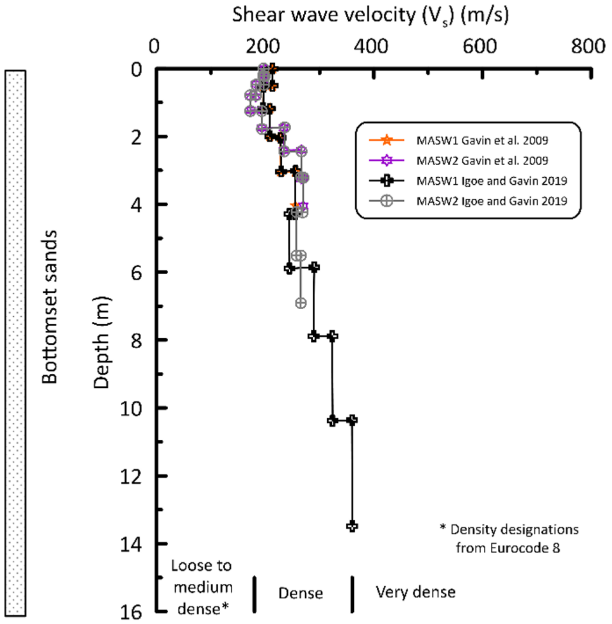


Figure 4. MASW Vs profiles for Location 1 (Igoe and Gavin 2019) and (Gavin et al. 2009) with stiffness designations (EN 2004). Note the x-axis range of 0-800 m/s will be used in all Vs plots.

4.2. MASW Location 2

A Monte Carlo inversion of the Blessington surface wave data (MASW S1 and S2 from Fig. 1) was carried out by assessing the fit between the measured and theoretical data using a quantitative misfit value, where the model with the lowest misfit was used as a new candidate model for the next iteration. Ten (10) runs with 10,000 iterations were carried out, each using 50 random starting models. This resulted in the generation of approximately 100,000 potential velocity models residing within the defined parameter space.

Fig. 5 shows the potential velocity models with a misfit of less than 0.05 (the modelled data is within ±5% of the field data). The parameterisation was defined using 11 layers, each assigned a range of values for thickness (H), V_s, compression wave velocity (V_P) and density (ρ). The average of the 10 best fit models (BFM) were used to generate the 1D V_s profiles presented in Fig. 6.

4.3. MASW v SCPTU Location 2

A comparison between the SCPTU data and the BFM MASW data from Location 2 are shown on Fig. 6. The actual V_s values for the bottomset sands are very similar to those from Location 1, i.e. corresponding to a dense material. For both profiles the MASW V_s values are less than those from SCPTU in the bottomset sand deposit. The two sets of values are closer to one another in the glacial till.

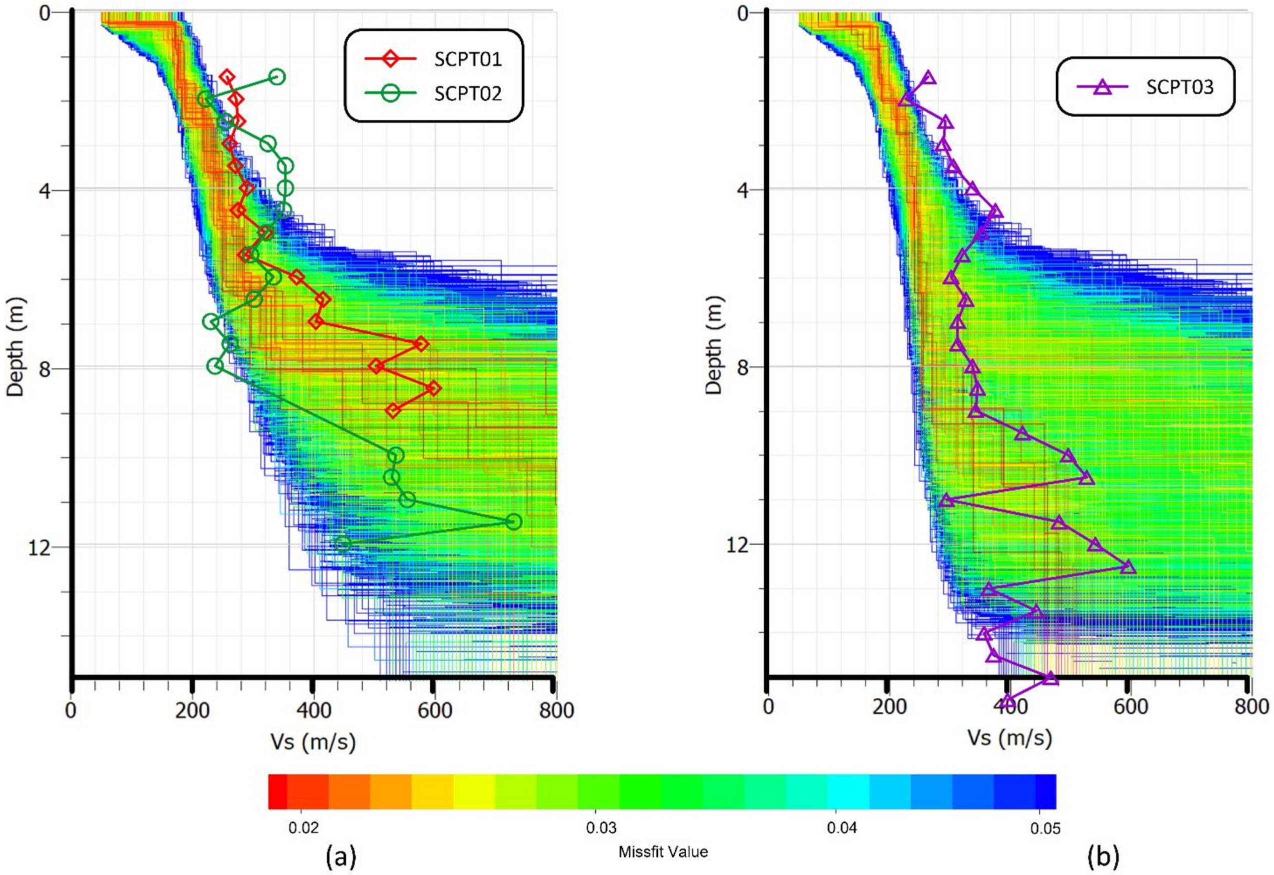


Figure 5. MASW Location 2 showing potential velocity models with a misfit of less than 5% compared to SCPTU results (a) MASWS1 and (b) MASWS2.

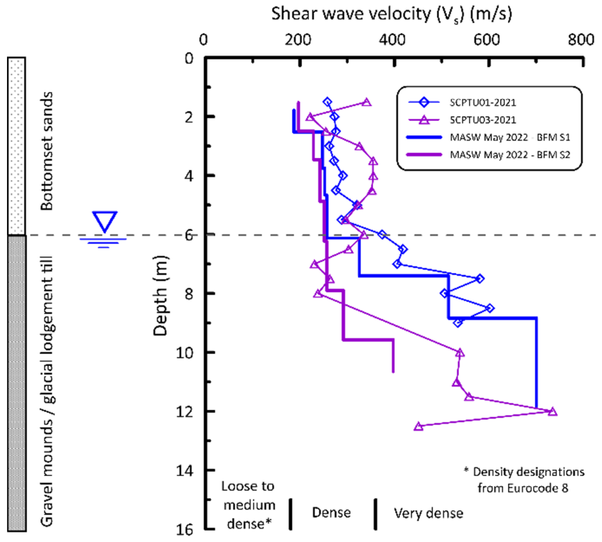


Figure 6. Comparison of BFM MASW profiles and SCPTU data for Location 2.

It is likely that the difference between the MASW and SCPTU profiles is due to a combination of uncertainty in the two methods and stiffness anisotropy. Anisotropy of shear wave velocity / stiffness may be significant in many soils. This is particularly the case for overconsolidated materials. Various authors (Butcher and Powell 1995) have suggested that to distinguish between shear wave velocities with different propagation and polarisation directions, subscripts can be used to donate these. For example $V_{s\text{-}vh}$ denotes a vertically propagating, horizontally polarised shear wave velocity.

$V_{s\text{-}vh}$ is measured in a downhole or SCPTU test. Similarly $V_{s\text{-}hv}$ or $V_{s\text{-}hh}$ would be measured in cross-hole testing. It is not clear which propagation and polarisation directions are represented by MASW. The Raleigh waves acquired in MASW surface wave testing are generated by the interaction of P and vertically polarised S waves and are measured by vertical geophones planted on the surface. These waves are horizontally propagating and predominantly vertically polarised.

The technique will provide a bulk measure of velocity underneath the full spread length, including all layers encountered within a wavelength of about 30 m (in the case of the Blessington data). It is therefore not entirely obvious how it would be affected by anisotropy and fabric. In a layered anisotropic soil it is possible that the interpreted V_s from MASW would possibly be a mix of $V_{s\text{-}hv}$ and $V_{s\text{-}vh}$.

A detailed discussion on the uncertainties in the methods is outside the scope of this paper. These have been dealt with comprehensively by others (Garofalo et al. 2016). Further research involving the use of SCPTU / Cross Hole Seismic / MASW would be required to more fully understand whether the observed differences between the shear wave velocity from the SCPT and MASW are due to anisotropy of the ground or uncertainty in the methodology.

5. V_s from CPTU data

Numerous correlations between V_s and CPTU data have been developed for sands. Some good reviews of this topic have been have previously been published (Kim et al. 2017) and (Long et al. 2020). These correlations include relationships which were derived using data from sands worldwide (Sykora and Stokoe 1983), (Andrus et al. 2001) and (Robertson 2009) and relationships which were derived for sands in a particular geographical location.

As no local correlation for Irish sands has been developed, the general expressions have been used here (Sykora and Stokoe 1983), (Andrus et al. 2001). The objective of using these correlations here is to check that the SCPTU and MASW V_s values are of the correct order and not to produce a definitive relationship between V_s and CPTU. The formulae used are as follows (Eq. 2 to 4) (Sykora and Stokoe 1983, Andrus et al. 2001):

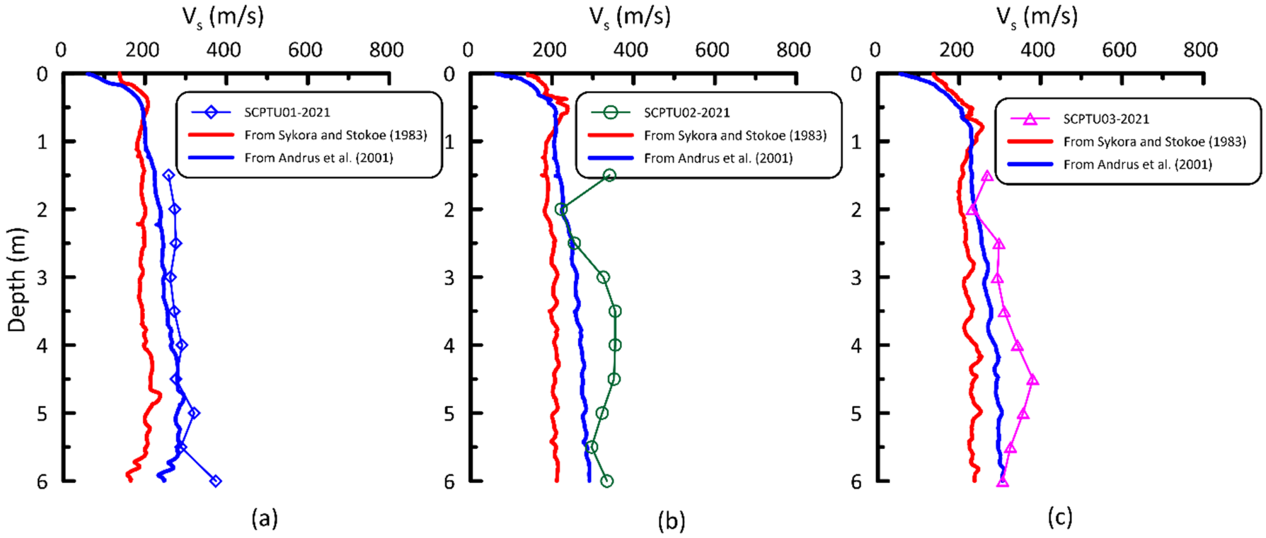


Figure 7. V_s profiles predicted from CPTU data (a) SCPT01, (b) SCPT02 and (c) SCPT03

$$V_s = 134 + 0.0052 q_c \tag{2}$$

where: q_c = measured (uncorrected) cone resistance in kPa

$$V_s = 77.4 (q_{c1})^{0.178} ASF \left(\frac{P_a}{\sigma_v}\right)^{-0.25} \tag{3}$$

$$q_{c1} = \left(\frac{P_a}{\sigma_v'}\right)\frac{q_c}{P_a} \qquad (4)$$

where: q_{c1} = normalized cone tip resistance in kPa, P_a = atmospheric pressure = 100 kPa, σ_v' = vertical effective stress and ASF = factor depending on the soil type typically 1.4 to 1.6.

Comparisons between measured and predicted V_s values for the three SCPTU tests at Location 2 on the Blessington site are shown on Fig. 7. The CPT based methods agree reasonably well with the SCPTU data, in particular that of Eq. 3 (Andrus et al. 2001). The CPT methods underestimate the measured values to some degree perhaps because they were originally intended for use in sands which were not as heavily overconsolidated as those at Blessington. The CPT based V_s profiles are arguably closer to the MASW data.

Although correlations such as these need to be treated with caution, the results presented here show that the correlations that have worked well for other sand sites also work well here, this giving some confidence in the correlations.

6. Prediction of settlement of shallow footings from V_s

Data from in-situ field loading trials of two small footings at Location 1 on the Blessington site have previously been presented (Gavin et al. 2009). The data for the 250 mm square footing is presented here. Here the measured response of the footings will be compared to the values that would be predicted directly from the V_s measurement.

For a full description of how to predict settlement from V_s for a variety of soils the reader is referred to Chapter 3 of the textbook on Engineering Geophysics (Klinkby and Bondo Medhus 2022), or to other similar work (Mayne 2000) or (Poulos 2021). A brief summary is given as follows.

According to elastic theory, the small strain shear modulus (G_{max}) may be calculated from V_s using (Eq. 5):

$$G_{max} = \rho V_s^2 \qquad (5)$$

where: G_{max} is the shear modulus (in Pa), V_s is in m/s, and ρ is the total mass density (in kg/m^3).

Unfortunately, stiffness is highly non-linear and will decrease with increasing strain. A stiffness value must be chosen consistent with the strain which occurs in the soil around the structure under consideration. Various techniques exist to allow G_{max}, as determined directly from V_s, to be reduced depending on the likely strains. Perhaps the most common technique is the use of the hyperbolic stiffness degradation formula suggested previously (Fahey and Carter 1993) (Eq. 6):

$$\frac{G}{G_{max}} = 1 - f\left(\frac{q}{q_{ult}}\right)^g \qquad (6)$$

Where: q and q_{ult} are the applied bearing pressure and ultimate bearing pressure respectively and f and g are fitting parameters. The ratio q/q_{ult} is analogous to the inverse of the factor of safety. Values of $f = 1$ and $g = 0.3$ appear to give reasonable estimates for unstructured and uncemented geomaterials and provide a general fit to experimental data (Mayne et al. 2009).

Following the previously used terminology (Mayne 2000) (Eq. 7):

$$\delta = \frac{QI}{BE_{max}\left[1-\left(\frac{Q}{Q_u}\right)^{0.3}\right]} \qquad (7)$$

where:
δ = predicted settlement
Q = applied load
Q_u = ultimate load capacity of footing
I = displacement influence factor considering finite depth to rock, footing shape and depth of burial
E_{max} = Young's modulus at small strain
$E_{max} = G_{max}(1 + 2\nu)$
ν = Poisson's ratio
B = footing width

Here Q_u was calculated according to the formula given in Eurocode 7 (CEN 2004) using an effective friction angle (ϕ') of between 45° and 50°, ν was assumed = 0.2 and I was determined to equal 0.67 (Christian and Carrier 1978). G_{max} was calculated to be equal to 80000 kPa (assuming V_s = 200 m/s from Fig. 3 and ρ = 2000 kg/m^3)

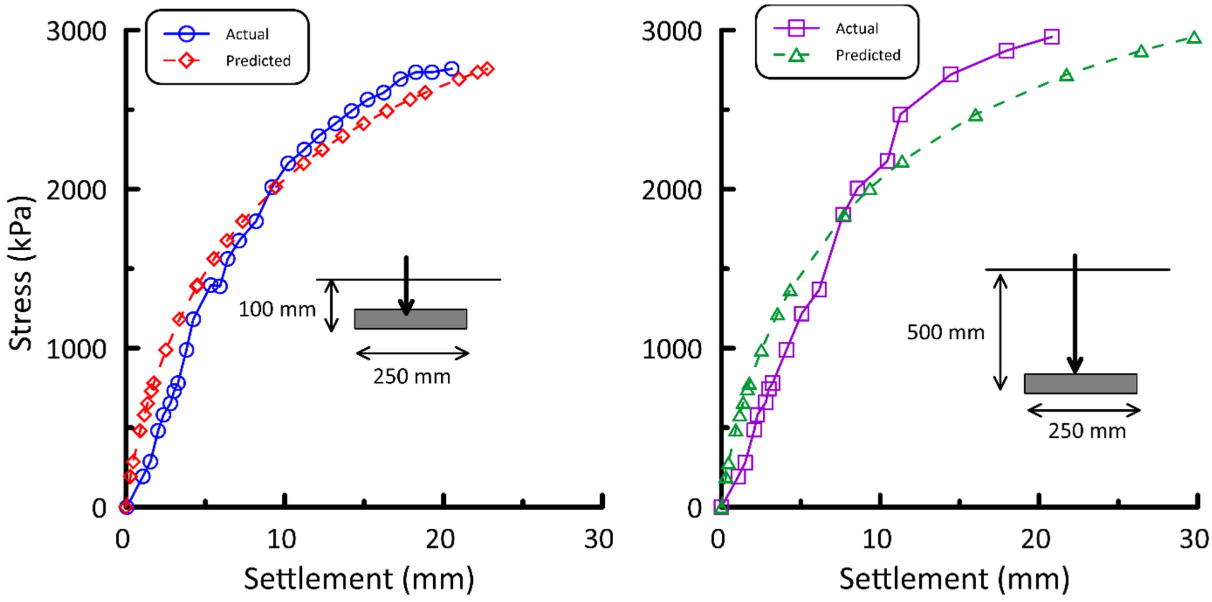
The resulting measured versus predicted settlements for the 250 mm square footing with depth of burial (D) = 100 mm (D/B =0.4) and 500 mm (D/B = 2) respectively are compared on Fig. 8.

The predicted and actual settlements match very well confirming the usefulness of the technique and consistent with the findings of others as outlined above.

It should be noted that this was a Class 3 prediction, i.e. carried out after the measurements.

7. Conclusions

Shear wave velocity values are being widely used in geotechnical engineering well beyond their original application in seismic hazard and dynamic analyses. Therefore the specific objective of this work was to determine the in-situ shear wave velocity profile for overconsolidated dense sand at the Blessington quarry site and to explore practical uses of V_s.



(a) (b)

Figure 8. Figure 1 Predicted versus measure settlement for 250 mm square footing for (a) D/B = 0.4 and (b) D/B = 2.

Some findings of the work were:

- V_s profiles can be determined reliably by several different methods including MASW and SCPTU.
- A novel technique was used to generate the MASW V_s profile using a Monte-Carlo inversion to assess the best fit between the measured and theoretical data.
- The MASW V_s values are somewhat greater than those generated by SCPTU. The differences are likely to be due to a combination of anisotropy in the material and in uncertainty in the two methods. Further work is required to assess these differences.
- The V_s values for Blessington sands are consistent with published values for other similar materials.
- The results also confirm that reliable V_s profiles can be estimated from correlations with CPTU data in the sandy materials encountered here.
- V_s values were used to make accurate predictions of the settlement of shallow test footings at the site.

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