# Mathematical model and analysis of the dynamics of a system of the screw-cutting lathe

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**Abstract.** Analysis of machining processes on metal-cutting machines is an important area of research in the field of metal processing. The purpose of this research is to study machining processes on metal-cutting machines in order to improve the efficiency and quality of processing. In order to achieve this purpose, an analysis of cutting processes and machining processes is carried out. The influence of cutting conditions on the machining process, such as the position of the part and tool, feed and depth of cut, is considered. As the object of research of the operating modes of a metal-cutting machine, 16K20 screw-cutting lathe was chosen as the most widely used type of machine. The 16K20 screw-cutting lathe is a metal-cutting machine that is used for processing various metal parts. The article covers a mathematical model of the system “frequency-controlled electric drive – asynchronous engine – metal-cutting lathe” that allows evaluating and analyzing the dynamics of the system, predict its behavior under various conditions and optimize operating parameters in order to achieve optimal results.

## 1 Introduction

Modern manufacturing enterprises are faced with the need to improve equipment efficiency to ensure high levels of productivity and competitiveness in the market. Particular importance is attached to optimizing operating modes, as well as increasing energy efficiency based on the technological parameters of the cutting process on metal-cutting lathes in order to improve the quality and accuracy of metalworking [1,2]. Scientific research by scientists is aimed at creating schemes for connecting compensation devices to the electrical network to compensate for the reactive power of asynchronous engines of metal-cutting lathes, frequency regulation, as well as determining dynamic and energy indicators based on the requirements of technological processes. In this direction, priority is given to research on improving the energy efficiency of metal-cutting lathes with frequency-controlled asynchronous electric engines [3].

A mathematical model of the operating mode of a 16K20 screw-cutting lathe is given in the article and the factors influencing its efficiency are indicated.

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The 16K20 screw-cutting lathe is a high-precision equipment designed for processing parts of varying complexity and shape. It is equipped with an engine that drives a spindle and a tool that processes the surface of the part.

The mathematical model of the operating mode of the 16K20 screw-cutting lathe includes the following parameters:

- spindle rotation speed (n);
- tool feed speed (s);
- cutting depth (h);
- cutting forces (F);
- power consumption (P).

The operating mode of the 16K20 screw-cutting lathe largely depends on the relationship between these parameters [4]. The optimal ratio allows achieving maximum machine efficiency and high-quality parts processing.

The operating efficiency of a 16K20 screw-cutting lathe depends on many factors, such as:

- correct choice of operating mode;
- accuracy of equipment setup;
- quality of the cutting tools used;
- condition of cutting tools;
- quality of the material of the workpiece;
- operating conditions of the machine (temperature, humidity, dust, etc.).

## 2 Methodology

An important factor influencing the efficiency of a screw-cutting lathe is the correct choice of operating mode. When choosing an operating mode, it is necessary to consider many factors, such as the type of material being processed, the shape and size of the workpiece, the required accuracy and roughness of processing, cutting speed, type of cutting tool, etc.

The mathematical model of machine engine control can be represented as an equation:

$$J\frac{d\omega}{dt} + b\omega = u, \tag{1}$$

where: J is moment of inertia of the machine engine; ω is rotation speed of the machine engine; t is time; b is damping coefficient; u is control action (for example, voltage on the engine)

This equation describes the dynamics of the machine engine and allows determining the necessary control action to achieve a given rotation speed or acceleration.

Various algorithms and methods can be used in order to control the machine engine, such as proportional-integral-derivative controller (PID controller), adaptive controller, fuzzy controller, etc.

One of the most important tasks of engine control is maintaining a given rotation speed. To do this, it is necessary to calculate not only the required engine power, but also consider changes in the load on the machine, for example, when processing different materials or changing the depth of cut [5,6].

Let us assume that there is a model of the load on the machine depending on time:

$$F(t) = F_0 + A \cdot \sin(\omega t), \tag{2}$$

where: $F_0$ is the constant component of the load, $A$ is the oscillation amplitude, ω is the oscillation frequency.

Then the equation of motion of the engine can be written as:

$$J\frac{d\omega}{dt} = M_{eng} - M_L, \tag{3}$$

where $J$ is the moment of inertia of the rotating mass, $M_{eng}$ is the moment created by the engine, $M_L$ is the load moment on the machine.

Considering that the engine torque is related to its power $P_{eng}$ as follows:

$$M_{eng} = \omega\, P_{eng}, \tag{4}$$

and taking into account the load model, we obtain the following equation:

$$J\frac{d\omega}{dt} = \omega\, P_{eng} - F(t) \cdot r, \tag{5}$$

where $r$ is the radius of the cutting tool.

In order to control the engine speed, it is necessary to maintain the required power $P_{req}$, which can be calculated using the following formula:

$$P_{req} = k \cdot F(t) \cdot v_{cut}, \tag{6}$$

where: $k$ is a coefficient that takes into account the efficiency of the cutting process, $v_{cut}$ is the cutting speed.

Thereby, we can determine the law of the engine power control:

$$P_{eng} = K_p \cdot \left(P_{req} - P_{eng}\right) + K_i \int 0t \left(P_{req} - P_{eng}\right)dt, \tag{7}$$

where: $K_p$ and $K_i$ are the coefficients of the proportional and integral components of the PI controller.

Let u(t) be a control signal that differs from zero only in the interval [0, T], where T is the engine operating time. Then the engine dynamics are described by the equation:

$$J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + k\theta = Ku(t), \tag{8}$$

where:

J is the moment of inertia of the engine,
b is viscous friction coefficient,
k is stiffness coefficient,
θ is the angle of rotation of the engine shafts,
K is the engine amplification factor.

In order to control the engine, it is necessary to determine the optimal value of the control signal u(t). This can be done by minimizing the power consumption functionality:

$$J = \frac{1}{2}\int \left(b \cdot \left(\frac{d\theta}{dt}\right)^2 + k\theta^2 + Ru(t)^2\right) dt, \tag{9}$$

where R is the motor circuit resistance.

In order to assess the operating efficiency of a screw-cutting lathe, it is necessary to analyze energy consumption and compare it with the requirements and standards of energy consumption for a given type of machine [7,8]. To do this, the mathematical models that allow estimating energy consumption depending on the operating mode of the machine and other parameters can be used.

The energy consumed by the lathe is defined as the integral of power over time:

$$W = \int_{t_0}^{t_f} P(t)dt, \tag{10}$$

where P(t) is the power consumed by the machine at time t; $t_0$ and $t_f$ are the start and end times.

Power P(t) can be expressed through torque M(t) and rotation speed ω(t):

$$P(t) = M(t) \cdot \omega(t), \tag{11}$$

The average power consumption of a lathe can be expressed as:

$$\bar{P} = \frac{1}{t_f - t_0}\int_{t_0}^{t_f} P(t)\, d(t) = \frac{1}{t_f - t_0}\int_{t_0}^{t_f} M(t) \cdot \omega(t)d(t). \tag{12}$$

The energy consumed by the lathe during the time $t_f$ - $t_0$is equal to:

$$W = \int_{t_0}^{t_f} P(t)\, d(t) = \int_{t_0}^{t_f} M(t) \cdot \omega(t)d(t). \tag{13}$$

The efficiency coefficient of the machine can also be calculated:

$$\eta = \frac{P_0}{P}, \tag{14}$$

where $P_0$ is the useful power, P is the power consumption of the lathe.

Useful power is defined as the work performed by the lathe per unit of time:

$$P_0 = M(t) \cdot v(t), \tag{15}$$

where *v(t)* is the tool movement speed.

## 3 Results and discussion

Figure 1-4 show: the speed of the part, the angular speed of the shaft, the angle of rotation of the shaft, as well as the kinetic and potential energies of the metal-cutting lathe in a function of time, respectively.

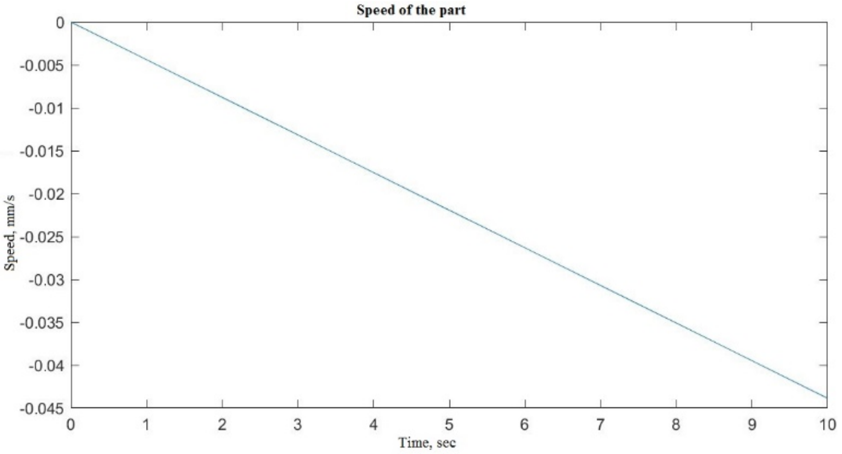


**Fig. 1.** Speed of the part depending to time.

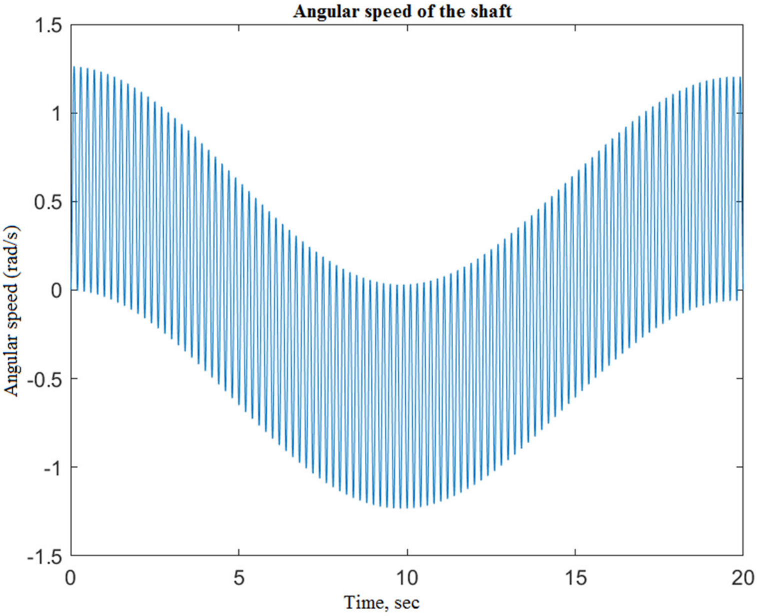


**Fig. 2.** Angular speed of the shaft depending to time.

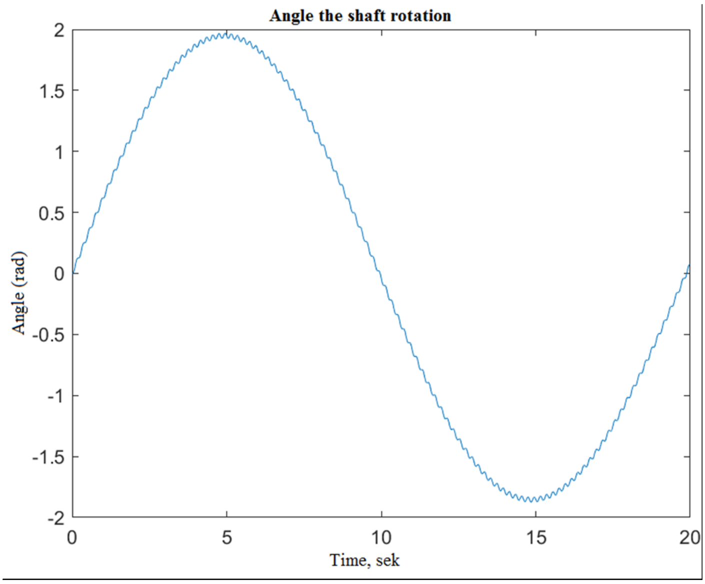


**Fig. 3.** Angle of the shaft rotation depending to time.

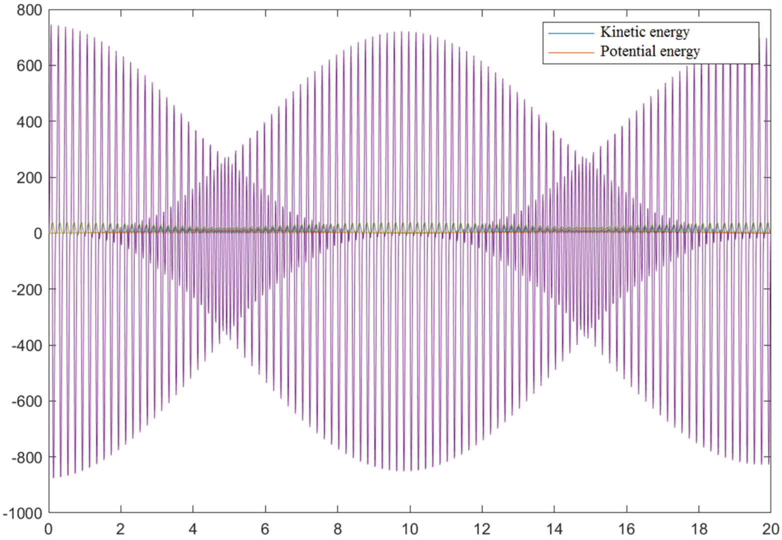
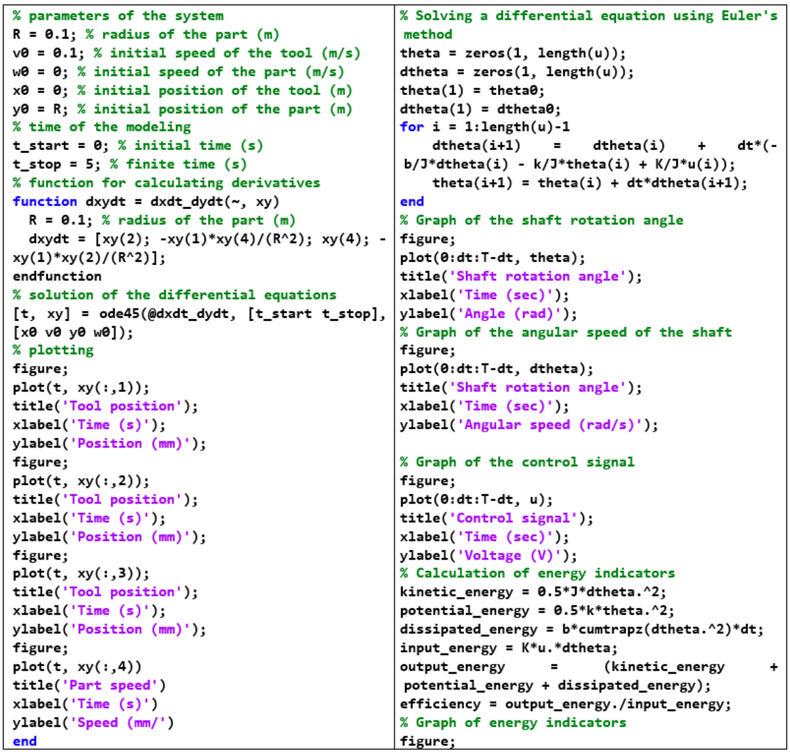


**Fig. 4.** Kinetic and potential energies of a metal-cutting lathe depending on time.

A cutoff of the listing of the mathematical model of the operating mode of the 16K20 screw-cutting lathe and its implementation in the MATLAB programming language is shown in Figure 5.



```
% parameters of the system
R = 0.1; % radius of the part (m)
v0 = 0.1; % initial speed of the tool (m/s)
w0 = 0; % initial speed of the part (m/s)
x0 = 0; % initial position of the tool (m)
y0 = R; % initial position of the part (m)
% time of the modeling
t_start = 0; % initial time (s)
t_stop = 5; % finite time (s)
% function for calculating derivatives
function dxydt = dxdt_dydt(~, xy)
  R = 0.1; % radius of the part (m)
  dxydt = [xy(2); -xy(1)*xy(4)/(R^2); xy(4); -
xy(1)*xy(2)/(R^2)];
endfunction
% solution of the differential equations
[t, xy] = ode45(@dxdt_dydt, [t_start t_stop],
[x0 v0 y0 w0]);
% plotting
figure;
plot(t, xy(:,1));
title('Tool position');
xlabel('Time (s)');
ylabel('Position (mm)');
figure;
plot(t, xy(:,2));
title('Tool position');
xlabel('Time (s)');
ylabel('Position (mm)');
figure;
plot(t, xy(:,3));
title('Tool position');
xlabel('Time (s)');
ylabel('Position (mm)');
figure;
plot(t, xy(:,4))
title('Part speed')
xlabel('Time (s)')
ylabel('Speed (mm/')
end
```

```
% Solving a differential equation using Euler's
 method
theta = zeros(1, length(u));
dtheta = zeros(1, length(u));
theta(1) = theta0;
dtheta(1) = dtheta0;
for i = 1:length(u)-1
    dtheta(i+1)    =    dtheta(i)    +    dt*(-
 b/J*dtheta(i) - k/J*theta(i) + K/J*u(i));
    theta(i+1) = theta(i) + dt*dtheta(i+1);
end
% Graph of the shaft rotation angle
figure;
plot(0:dt:T-dt, theta);
title('Shaft rotation angle');
xlabel('Time (sec)');
ylabel('Angle (rad)');
% Graph of the angular speed of the shaft
figure;
plot(0:dt:T-dt, dtheta);
title('Shaft rotation angle');
xlabel('Time (sec)');
ylabel('Angular speed (rad/s)');

% Graph of the control signal
figure;
plot(0:dt:T-dt, u);
title('Control signal');
xlabel('Time (sec)');
ylabel('Voltage (V)');
% Calculation of energy indicators
kinetic_energy = 0.5*J*dtheta.^2;
potential_energy = 0.5*k*theta.^2;
dissipated_energy = b*cumtrapz(dtheta.^2)*dt;
input_energy = K*u.*dtheta;
output_energy      =      (kinetic_energy      +
 potential_energy + dissipated_energy);
efficiency = output_energy./input_energy;
% Graph of energy indicators
figure;
```

**Fig. 5.** A cutoff of the listing of the mathematical model.

The developed mathematical model of the "frequency-controlled electric drive – asynchronous engine – metal-cutting lathe" allows evaluating and analyzing the dynamics of the system, predict its behavior under various conditions and optimizing operating parameters to achieve optimal results. In order to achieve this purpose, mathematical models and control methods were developed that allowed adjusting the trajectory of the cutting tool and cutting modes depending on the characteristics of the materials being processed and the requirements of the technological process. Modern technologies and control algorithms, such as feedback systems, frequency regulation and process automation were also used.

## 4 Conclusion

In order to achieve the purpose of research, presented in the article, an analysis of cutting processes, including an analysis of the cutting tool, material properties and processing was carried out. The influence of cutting conditions on the machining process, such as the position of the part and tool, feed and depth of cut, is also considered.

Having conducted a study of processing on metal-cutting machines in the metalworking process and studied the processes in order to increase the efficiency and quality of processing, we came to the following conclusion.

Thereby, the choice of operating mode affects not only the processing efficiency, but also the energy consumption of the machine. Therefore, energy consumption increases when the machine operates at high cutting and feed speeds. The condition of the cutting tool also affects energy consumption, since a dull tool requires more power to process the material.

Rotation speed control is the most effective both from the point of view of meeting the requirements of the technological process and from the point of view of energy efficiency.

At present, the best way to regulate the speed of asynchronous motors on metal-cutting machines is a variable frequency drive (VFD). A variable frequency drive allows changing the frequency and voltage of the electrical signal supplied to the motor, which leads to a change in the rotation speed of the motor shaft and, accordingly, the processing speed on the machine.

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