

Development of a mathematical model for analyzing the perturbed state of a thermal power hydraulic system

Svetlana Sazonova^{1,2*}, *Irina Shcherbakova*³, *Nadezhda Akamsina*¹, *Nikolai Mozgovoy*¹, and *Olga Sokolova*¹

¹Voronezh State Technical University, 84 October 20th Anniversary Street, Voronezh, 394006, Russia

²Voronezh State Pedagogical University, 86 Lenin Street, Voronezh, 394043, Russia

³Voronezh State Forestry University named after G. F. Morozov, 8 Timiryazev Street, Voronezh, 394087, Russia

Abstract. The results of developing a model for analyzing the perturbed state of a thermal power hydraulic system are presented using the example of a heat supply system. The parameters of the operating mode of the thermal power hydraulic system were assessed after exposure to disturbing factors. It is noted that the choice of boundary conditions must be carried out taking into account the use of energy equivalence when analyzing the perturbed state of the investigated fragment of the hydraulic system. The developed model of steady flow distribution with non-isothermal flow of a viscous medium is presented. It is concluded that the developed mathematical model represents a qualitatively new approach to formalizing problems of flow distribution analysis in systems with adjustable parameters. The model can be considered as a generalized form of representing particular models of flow distribution when describing the object under study, which can be considered as a hydraulic circuit with adjustable parameters for non-isothermal flow of a viscous medium. It is shown that this model can be used to analyze and describe the flows and properties of a viscous medium in systems where it is possible to regulate parameters and non-isothermal flow is taken into account.

1 Introduction

According to [1], analysis of the flow distribution of a thermal power hydraulic system (TPHS), in particular a heat supply system (HSS) in a perturbed state, means that the operating mode parameters of the system are assessed after exposure to disturbing factors. Such factors can significantly affect the dynamics of the system or its elements.

In case of design or parameter changes, such as installing a new heat source or changing the hydraulic characteristics of the pump, this may lead to a loss of information certainty of the boundary conditions [2, 3].

* Corresponding author: ss-vrn@mail.ru

2 Materials and methods

The classification of stationary states of the studied fragment of the system (SFS) with a stable distribution of the target product flow (TP), proposed in [1], is focused on boundary conditions (BC), and not on missing initial conditions. In general, BC for the j -th energy unit (EU) can be expressed by the relationship:

$$H_j = \zeta_j(g_j); \quad (1)$$

where H_j, g_j - respectively, nodal potential and inflow through EU j ; ζ_j - relationship between potential and flow, described as a function.

Based on the function type ζ_j , BC can be divided into four types.

For boundary conditions of the first type, one of the parameters of equation (1) is set at the node, namely $H_j = \hat{H}_j$ or $g_j = \hat{g}_j$, and the second is obtained by analyzing the flow distribution. This type of BC is more often used for design tasks and for analyzing the current state of the system based on field measurements (for example, data obtained during a manometric survey).

If one of the node parameters is set (for example, coolant flow), another parameter is calculated (for example, node potential) depending on the flow distribution in the system. This can be useful, for example, when considering specific system requirements or when analyzing operational data to capture the current state of the system.

For the second type of BC, select the function ζ_j , for which may be unknown H_j, g_j , but if any of the parameters has been found, then using (1) the second of them is determined. The second type of BC is usually used to describe the pressure characteristics of injection equipment, such as pumps and compressors, and the pressure characteristics of head or pressure regulators.

Such boundary conditions make it possible to take into account the characteristics of devices and regulators and configure the system in accordance with their parameters and characteristics, which is important when designing and operating technical systems of the type in question.

The third type of BC assumes that the form (structure) of these conditions is known, but their content (meaning) is considered unknown. This means that only the type or form of the function is set ζ_j , defining a BC of the third type, but the specific value of the parameters of this function H_j, g_j remain uncertain.

The third type of BC permits the description of certain structures or relationships in a system, even if the exact values of the parameters are still unknown. It is advisable to apply these conditions when analyzing and modeling various processes and systems.

Boundary conditions of the fourth type do not have a predetermined form or content and are formed during the calculation process.

BC of the fourth type are conditions generated in an algorithm that do not depend on specific data, but define the methods, means and rules used to generate data or other conditions. In such conditions it is necessary to create structures or rules based on calculations.

Boundary conditions of types 1 and 2 are explicitly described using the function ζ_j with given values. Boundary conditions of types 3 and 4 are an implicit form of boundary conditions. To determine them, it is necessary to determine the algorithm for their formation. These conditions can be more complex and depend on various factors or conditions, in particular on energy equivalence [2, 3], and the main concept of this problem becomes the field of design zone (DZ) calculations, which makes them more flexible for various modeling problems.

The process of implicit BC formation may include the definition of restrictions and rules governing the interaction between the SFS and the DZ to ensure proper modeling and analysis of internal processes within a given DZ.

This method allows you to simulate internal processes for a more accurate and convenient study and take into account external influences only through explicit or implicit boundary conditions at certain points or boundaries of the DZ [4, 5, 6].

When forming a BC of the third type, we use energy equivalent. When analyzing the perturbed state of the TPHS using the energy equivalent, two main situations arise: firstly, in an open system (for example, in a hot water supply system), when the load of the system is influenced by external disturbing factors, then the amount of load on the system inevitably leads to uncertainty, which leads to significant changes in the operation of the system; secondly, if the system load is influenced by external factors, then an unaccounted amount of load on the system will inevitably lead to uncertainty, which, in turn, will lead to significant changes in the operation of the system [7, 8, 9]. Failure to include booster pumps in the model may not fully or accurately assess the system's performance and response to disturbances, and may result in the system being classified as open. All of the above can lead to significant changes in the operation of the system, and therefore energy equivalence has become an important tool for the analysis and modeling of such systems.

Energy equivalent is a useful tool for both open and closed systems where load uncertainty analysis is important. Although a closed system can be classified as open if booster pumps are included in it, which in turn can improve the quality of the simulation.

Let us consider the mechanism of formation of the third type of BC [2, 3] for HSS as for TPHS. We will equivalent the hot water supply subsystems of each subscriber, that is, we will replace such a subsystem with a subsystem corresponding to the conventional part, taking into account energy dissipation [10, 11, 12].

In the fictitious section, we express the energy losses using the Darcy Weisbach expression, and then the required transcendental equation takes the form of the Bernoulli equation:

$$\left[Z_j + H_j + \frac{w_j^2}{2g} \right] = \left[Z_{je} + H_{je} + \frac{w_{je}^2}{2g} \right] + \xi_c \frac{q_{je}^\alpha}{D_c^\beta} \rho L_e; \quad (2)$$

where the indices j and j' respectively refer to the boundary EU, and the index "e" entered for a fictitious section.

When solving equation (2), it is necessary to determine the missing parameters of power units (EU) in the HSS. These parameters depend on the selected BC of the first type and the analysis of the initial state of the system. In this case, the kinetic energy of the transported medium in the EU system is insignificant, so it can be assumed that the velocity pressure cancels out [13, 14, 15].

The addition of equivalents may be considered to establish a third type of BC in the HSS. This method allows you to determine the parameters of equivalent sections and describe the functions in the EU SFS. It is important to establish an explicit relationship between the system parameters and the parameters of equivalent sections. This can enable more efficient modeling and analysis of HSS taking into account the effects of equivalent sites [16, 17, 18].

The hydraulic equivalent (HE) of the hot water supply (HW) subsystem is designed taking into account the balance of energy dissipation in its real elements and depending on the situation in the studied location of the system. HE helps analyze and model the operation of the hot water system and simplifies its mathematical description and analysis. The mathematical apparatus used to design a HE can be very complex and depends on the specific HW, its components and operating conditions.

For the HW subsystem, we construct the HE based on the dependence relationship based on the equilibrium conditions of energy dissipation between the fictitious element and the real segment:

$$\sum_{j \in J_{\eta(p)}^z \cup J_{\eta(q)}^r} \sum_{i \in I_j^r} s_i Q_i^\alpha = \sum_{i \in I^{\text{ac}}} s_i Q_i^\alpha; \quad (3)$$

where I_j^{ar} - j segments of HW associated with EU.

Equivalence of elements in the HW system and in the HSS is used mainly in open systems, where all the necessary parameters of the Bernoulli equation are known, and the processes occurring in the elements are not of interest. Converting plant design diagrams can simplify system analysis and modeling, but also introduces additional errors. The balance between detail and simplification plays an important role in the accuracy of the calculations and the presentation of the system.

In the problem under consideration, the equivalent is used to analyze the perturbed state of the system. This makes it possible to simplify the calculation by transforming a real element into a boundary node of a selected fragment of the system, where it is possible to record the barometric pressure and solve the problem of analyzing the perturbed state, taking into account only the BC of first and second types. This is true if fictitious elements are used to equivalent subscriber subsystems (SS) and fixed air pressure [19, 20, 21].

The equilateral nature of the flow from the SFS to the metasystem [1] (regardless of the type of disturbance) ensures the stability of the equivalent circuit, especially with regard to the metric properties of the equivalent section. The guarantee of stability lies in the fact that the equivalence scheme may require changes only if the symbol of the material balance changes at the connection point between real and fictitious elements. The energy equivalent of the HSS corresponds to the formation of the hot water subsystem in the connecting node with the BC of the third type. Taking into account the above factors, from the point of view of mathematical modeling of flow distribution in the HSS, the equivalent mechanism is reasonable and effective.

The conditions set out in the study [1, 2, 3], according to which, in the absence of various types of sources and regulators, any infrastructure can be considered as a metasystem.

As an alternative, it is quite reasonable to preserve the individual properties of the active elements, especially when modeling, the DZ region is still an autonomous object, and not an SFS. This makes it possible to create an information-oriented problem that takes into account the various interactions between the DZ and the metasystem.

To achieve information certainty of the problem and take into account known information about the interaction between the DZ and the metasystem, it is important to preserve the properties of the active elements, since they play a key role in this interaction, and this method allows them to more accurately model and analyze the system.

The use of variational principles in binary calculation schemes (BCS) to form a mathematical model of flow distribution is similar to the use of these principles in a unary calculation scheme (UCS). The BCS can be expressed as an ordinary network, and well-known network laws and theorems of graph theory apply to it.

The principles of variation can be used to find optimal solutions, minimize energy costs, optimize resource allocation, or achieve other goals related to network operation. These principles allow us to formalize problems and find solutions that meet certain performance standards or better standards. The use of graph theory and network laws also provides capabilities for analyzing network topology, determining resource flow, identifying bottlenecks, analyzing stability, and other aspects of resource balancing system functions. The use of variational principles to formalize problems and the application of graph theory to resource balance systems are reasonable methods for modeling and analyzing such systems.

There is no geometric similarity between the model and the original in hydraulic equivalent, which is important for the analysis of unsteady hydraulic processes. This means that the equivalent model is not an exact copy of the original model from a geometric point

of view, so limitations may arise when considering non-stationary processes. This leads to the fact that analytical expressions need to be considered only in a stationary formulation.

The variational problem can be broken down into various groups of terms to take into account various aspects and parameters of non-stationary hydraulic processes. This may include processes related to fluid dynamics, temporal changes in parameters, and other factors that may be relevant in the analysis of transient processes. In some cases, transient processes can be important, and their analysis is necessary to understand the behavior of the system. The choice between steady-state analysis and non-stationary analysis depends on the specific problem and the requirements for modeling accuracy.

When modeling the flow distribution in a TPHP, the variation problem is described by the following sets of elements [1]. The change in source performance is represented by the first set of elements. The change in the power of wastewater through the EU relative to the air pressure is represented by the second set of elements. The third group of elements represents a change in the efficiency of existing and new regulated flows that connect consumers. The fourth group of elements represents a change in power dissipation to overcome the hydraulic resistance on the SFS element and on the fictitious element corresponding to the SS. The remaining sets of elements allow us to represent the equations of motion of the system in the form of a free variational problem.

3 Results and discussion

After eliminating the uncertain Lagrange multiplier, the desired current distribution model is obtained.

For DZ TPHP, the topology of the system is formed in the form of a BCS, which has only SS equivalents, and the model of the steady-state distribution of a non-isothermal flow of a viscous medium has the form

$$C_{p \times n1} \quad C_{p \times n2} \times \begin{Bmatrix} R_{n(d)} + R(Q_{n(d)}^u) \times Q_{n \times 1}^u \\ 0 \end{Bmatrix} \quad 0 \quad \times \begin{Bmatrix} Q_{n \times 1} \\ Q_{n \times 2} \end{Bmatrix} = M_{p \times e}^t \times \hat{H}_{exl} \pm \sum_i H(Q_i^u); \tag{4}$$

$$K_{r \times n1} \quad 0_{r \times n2} \times \begin{Bmatrix} R_{n(d)} + R(Q_{n(d)}^u) \times Q_{n \times 1}^u \\ 0 \end{Bmatrix} \quad 0 \quad \times \begin{Bmatrix} Q_{n \times 1} \\ Q_{n \times 2} \end{Bmatrix} = 0_{r \times 1} \pm \sum_i H(Q_i^u); \tag{5}$$

$$A_{m \times n1} \quad A_{m \times n2} \times \begin{Bmatrix} Q_{n \times 1} \\ Q_{n \times 2} \end{Bmatrix} = \hat{g}_{m \times 1} \tag{6}$$

$$E_{n(d)} \times (B_{n(d)} \times \Theta_{n \times 1} + T''_{n \times 1}) = -\bar{A}_{n \times m}^t \times T'_{m \times 1}; \tag{7}$$

$$\bar{A}_{m \times n} \times Q_{n(d)}^u \times T''_{n \times 1} - \bar{A}_{m \times n} \times Q_{n(d)}^u \times T'_{n \times 1} = \bar{g}_{m(d)} \times T'_{m \times 1} - \bar{g}_{m(d)} \times \hat{T}_{m \times 1}; \tag{8}$$

In (4)-(8) e denotes the total number of power steering couplings with a fixed nodal potential or hydraulic characteristic of the element $e = \left\{ J_{\pi(f)}^s \cup J_{\pi(f)}^s \cup J_{\pi(P)}^a \cup J_{\pi(P)}^s \right\}$; $n1, n2$ - accordingly the number of real SFS elements $n1 = \left\{ I^{sr} \right\}$, SS equivalents powered from the design subzone $n2 = \left\{ I^{ae} \right\}$; r - number of circuits in the BCS; $m = \left\{ J_{\eta(q)}^s \cup J_{\chi}^s \right\}$ - set of BCS nodes with non-fixed potential; the superscript “u” is a sign of the iterative process of adjusting the resistances of the control elements; the indices “sr” and “ae” denote, respectively, the elements of the SFS, equivalent structures of the SS; p - number of independent circuits ($p = e - 1$); the symbol “~” indicates elements that are newly included in the SFS (disturbing influence); all other designations are identical as for the model of the undisturbed state of the thermal power hydraulic system [4].

The system of equations (4) -(8) is closed, since the sum $(e + m)$ is equal to the total number of BCS nodes, and the number of contours r is equal to the total number of cross sections $(n_1 + n_2 = n)$.

It is noted in [2, 3] that energy equivalent should be used only for HS with isothermal flow in viscous media. It is also used in systems with significant non-isothermal properties, such as HSS. There is no significant error when applying energy equivalence, since the equivalence affects the open HW subsystem. Taking into account the temperature standard of the hot water system, which is 55°C , the temperature difference of the internal pipeline should not exceed 5°C . Taking into account these restrictions, the temperature mode of operation of the HE is determined and set in advance.

4 Conclusion

Model (4) - (8) is a qualitatively new approach to formalizing the problem of analyzing flow distribution in a system with adjustable parameters. The model includes all currently known models of stationary flow distribution, including systems with fixed BC in the UCS and models for the BCS. This general flow distribution model has generalized properties and can be applied not only to systems with centralized parameters. This expands its applicability to various HS. This method of flow distribution modeling can be used to analyze and optimize various horizontal wells and provides a more flexible and universal solution to such problems.

Model (4) - (8) can be considered as a generalized representation of a specific flow distribution model when describing the object of study and as a hydraulic circuit with adjustable parameters for non-isothermal flow of viscous media. This shows that the model can be used to analyze and describe the flow and properties of viscous media in such systems, with the ability to tune parameters and take into account non-isothermal flow. To more accurately understand and use the model, additional information may be required about equations (4) - (8) themselves and the context in which they are applied. This allows us to more accurately understand its applicability and capabilities for use in specific tasks and research.

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