E3S Web of Conferences **548**, 05004 (2024) https://doi.org/10.1051/e3sconf/202454805004
*AGRITECH-X 2024*

# Numerical solution of differential games with electric potential distributions in some domain

*Mashrabjon* Mamatov[1*], and *Nodir* Ibragimov[2]

[1]National University of Uzbekistan, University Street 4, Tashkent, 100174, Uzbekistan
[2]Termez State University, "Barkamol Avlod" Street 43, Termez, 190111, Uzbekistan

**Abstract.** Various applications of the problem of electrostatics are considered. It is shown that the Poisson equation describes the electrostatic field at the points where free charges are located. Assuming that the value of the free charge is a function controlled by two opposite sides, it is shown that for any control of the evader, the pursuer, having some advantage, can construct his control so that the potential distribution remains within certain predetermined limits. The problem of continuous pursuit by finite difference methods has been transformed into a discrete game. Using the formula for the obtained solution of a discrete equation with boundary conditions, theorems are proved on the possibility of completing the pursuit in the sense of getting into a small neighborhood of the terminal set of a discrete game. The method of normalization of the potential distribution is indicated.

## 1 Introduction

The calculation of electric and magnetic fields of various systems, the elements of which are conductors and dielectrics, is of practical interest for various fields of science: electro physics, radio electronics, and radio physics. The development of these areas of science imposes strict requirements on the accuracy of calculation methods. These methods should be economical and accessible to a wide range of engineers and other users. In addition, it is equally important to assess the accuracy of the results obtained, which is sometimes a more difficult task than obtaining the result itself.

For electrostatics on a plane, the most convenient mathematical apparatus is complex analysis. The function variables on the basis of physical concepts makes it possible to create new methods for solving the most important practical methods functions variables make it possible to find exact analytical solutions to electrostatic problems. And numerical methods give an approximate solution. The joint application of variation methods, complex and numerical analysis to the problems of electrostatics makes it possible to develop quite effective these.

Mathematically, a solution electrostatic problem with known potentials on conductors is reduced to finding a function that satisfies the Poisson equation and takes on a given value is called the Dirichlet problem. In most cases, the solution of electrostatic problems encounters significant difficulties, and it is possible to find an analytical solution of the Poisson equations

* Corresponding author: mamatovmsh@mail.ru

only in some cases with relatively simple geometric configurations of conductors. There is no single method for solving electrostatic problems, and different techniques are used for different types. The solution of the problem of electrostatics, from a mathematical point of view, consists in solving two Maxwell equations, which can be combined into one equation by simple transformations.

For a homogeneous and isotropic medium, the system of two Maxwell equations takes the form

$$\begin{cases} rot\bar{E} = 0; \\ div\bar{D} = \rho, \end{cases} \begin{cases} rot\bar{E} = 0; \\ div\bar{E} = \frac{\rho}{\varepsilon_a}. \end{cases} \tag{1}$$

Where $\bar{E} -$ the vector of the electromagnetic field strength, $\bar{D}$ —vector of electrical displacement or electrical induction. For homogeneous and isotropic media $\bar{D}$ end $\bar{E}$ is related by the following relation [1] - [3]:

$$\bar{D} = \varepsilon_a \bar{E} = \varepsilon_0 \varepsilon_r \bar{E}.$$

Here $\mathcal{E}_a = \mathcal{E}_0 \mathcal{E}_r -$ absolute permittivity of the medium, $\mathcal{E}_r -$ its relative permittivity, $\varepsilon_0 = \frac{10^7}{4\pi c^2} \approx 8.856 \cdot 10^{-12} \frac{\Phi}{M} -$ electrical constant. The electric displacement vector does not depend on the properties of the medium. Substitute $\bar{E} = -grad\varphi$ into the second equation (1), taking into account $divgrad\varphi = \nabla grad\varphi = \nabla\nabla\varphi = \nabla^2\varphi = \Delta\varphi$ we get that $div\bar{E} = -divgrad\varphi = -\nabla^2\varphi$ and therefore

$$\nabla^2\varphi = -\frac{\rho}{\varepsilon_a}. \tag{2}$$

Equation (2), as already noted, is called the Poisson equation, where $\nabla^2 = \Delta - Laplacian$. Taking into account $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ in the Cartesian coordinate system, equality (2) can be represented in the following form

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = -\frac{\rho}{\varepsilon_a}. \tag{3}$$

The Poisson equation describes the electrostatic field at the points where free charges are located, that is, at the points where$\rho \neq 0$. Here $\varphi$ —potential distribution. Consider the Dirichlet problem. The following task is set: For any measurable changes $\upsilon = \upsilon(x, y), \upsilon \in Q$ build such a measurable change $u = u(x, y), u \in P$ what, when$\rho = \rho(x, y) = \rho(u(x, y), \upsilon(x, y))$, $u \in P, \upsilon \in Q$, that (3) satisfies the condition $\varphi = \varphi(x, y) \in M$. Where $M$ —some predetermined set of $R. P, Q -$ Compact sets of $R$.

## 2 Materials and methods

A method for the analysis of electrostatic fields based on the organic combination of the variation methods is proposed. More precisely, the problem of the electric potential is studied by methods of the theory of differential games. The well-known methods of R. Isaacs, L.S. Pontryagin related to and discrete are used. Most cases, the results of calculating the distribution of the field are obtained in the form of analytical functions that allow you to find the potential or field strength at any point in space, with the possible exception of some singular points. However, with a more complex configuration of the electrodes, the analytical calculation becomes practically impossible and one has to use the so-called numerical methods of integration - the method of finite differences of the Laplace equations.

In order to study the problem posed by the methods of finite differences, we pass to the discrete analogue of equation (3). Considering that there are two variables and the notation $\varphi(x, y)|_{(x_i, y_j)} = \varphi(x_i, y_j) = \varphi_{i,j}$, for the partial second derivative

$$\left.\frac{\partial^2\varphi}{\partial x^2}\right|_{(x_i,y_j)} = \varphi_{i+1,j} - 2\varphi_{i,j} + \varphi_{i-1,j}, \left.\frac{\partial^2\varphi}{\partial y^2}\right|_{(x_i,y_j)} = \varphi_{i,j+1} - 2\varphi_{i,j} + \varphi_{i,j-1}.$$

Is obtained combining these two components

$$\nabla^2\varphi|_{(x_i,y_j)} = -4\varphi(x_i,y_j) + [\varphi(x_{i+1},y_j) + \varphi(x_{i-1},y_j) + \varphi(x_i,y_{j+1}) + \varphi(x_i,y_{j-1})] = -4\varphi_{i,j} + \varphi_{i-1,j} + \varphi_{i+1,j} + \varphi_{i,j-1} + \varphi_{i,j+1}.$$

From this it is clear that in order to $\varphi$ −the potential distribution was on some set we have to change i.e. control the Laplacian $\nabla^2\varphi(x_i,y_j)$ the same, taking into account presence of opposing sides, we obtain discrete games, the process of pursuit described by the equations [4]- [6]:

$$\begin{gathered}-4\varphi_{i,j} + \varphi_{i-1,j} + \varphi_{i+1,j} + \varphi_{i,j-1} + \varphi_{i,j+1} = -\frac{\rho(u_{i,j},v_{i,j})}{\varepsilon_a}, u_{i,j} \in P, v_{i,j} \in Q,\\ \varphi_{0,j} = 0, \varphi_{m+1,j} = 0, \varphi_{i,0} = 0, \varphi_{i,\theta} = 0,\\ i = 1,2,\dots,m, j = 1,2,\dots,\theta-1.\end{gathered} \tag{4}$$

The $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}$ potential distribution functions $\varphi = \varphi(x,y)$, a $\varphi_{i,j}$ −potential distribution at a point, $(x_i, y_j)$, convenient pursuit considered completed if $\varphi_{i,j}$: $\delta \le \varphi_{i,j} \le \delta + \varepsilon, i_0 \le i \le i_1, j_0 \le j \le j_1$ where $1 \le i_0, i_1 \le m, 1 \le j_0, j_1 \le \theta - 1$ for some predetermined $\delta > 0$, $\varepsilon > 0$, $\varphi_{i,j}$ potential distribution at predetermined points $(x_i, y_j)$ was at some point. Using the, at $i = 1,2,...,m$ (4) we obtain

$$\varphi_{0,j} + \varphi_{1,j-1} - 4\varphi_{1,j} + \varphi_{1,j+1} + \varphi_{2,j} = -\frac{\rho(u_{1,j},v_{1,j})}{\varepsilon_a},$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\varphi_{m-1,j} + \varphi_{m+1,j} - 4\varphi_{m,j} + \varphi_{m,j-1} + \varphi_{m,j+1} = -\frac{\rho(u_{m,j},v_{m,j})}{\varepsilon_a}.$$

Now we introduce and obtain a matrix discrete game of the second order

$$\begin{gathered}-\varphi_{j-1} + B\varphi_j - \varphi_{j+1} = -\frac{\rho(u_{i,j},v_{i,j})}{\varepsilon_a}, 1 \le j \le \theta - 1,\\ \varphi_0 = 0, \varphi_\theta = 0.\end{gathered} \tag{5}$$

Where $\varphi_j \in R^m$ end $u_j$ −pursuer control parameter; $v_j$ − evading player control parameter: $u_j \in R^m, v_j \in R^m$components that satisfy, $|u_{i,j}| \le \rho, |v_{i,j}| \le \sigma, \sigma < \rho$, $G-$ [6] of the form

$$G = \begin{pmatrix} 4 & 1 & \dots & \dots & 0\\ 1 & 4 & 1 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}.$$

The papers [7], [8] studied differential games with distributed parameters, and the papers [9]-[11] studied the connection between differential games with distributed parameters and a discrete one.

## 3 Results and discussion

Where (5) the movement of a point $\varphi$ $m$ dimensional Euclidean space $R^m$ described by the equations

$$-\varphi_{j-1} + B\varphi_j - \varphi_{j+1} = -\frac{\rho(u_{i,j}, v_{i,j})}{\varepsilon_a}, 1 \le j \le \theta - 1, \tag{6}$$

$$\varphi_0 = \xi_0, \varphi_\theta = \xi_\theta, \tag{7}$$

$B - m \times m$ - Matrix, $u$ - pursuit parameter, $\upsilon$ - escape parameter, $u_j \in P \subset R^m, v_j \in Q \subset R^m$, a sequence $u = u(.) = (u_1, u_2, \dots)$, $v$ - selected $v = v(.) = (v_1, v_2, \dots, v_{\theta-1}), v_j \in Q, j = 1,2, \dots, \theta - 1$. Besides, in $R^m$ is withdraw $\varphi_j$ escaping aims place this.

Definition. (6), (7) "boundary" position $(\xi_0, \xi_\theta)$ evasion, one $\varphi = \varphi(0) = (\varphi_0, \bar{\varphi}_1, \bar{\varphi}_2, \dots, \bar{\varphi}_{N-1}, \varphi_N)$ equations

$$-\varphi_{j-1} + B\varphi_j - \varphi_{j+1} = -\frac{\rho(u_{i,j}, v_{i,j})}{\varepsilon_a}, 1 \le j \le N - 1, \varphi_0 = \xi_0, \varphi_\theta = \xi_\theta,$$

at falls $: \overline{\varphi}_d \in M$ .

In (6), (7) - $U_n(x)$

$$U_n(x) = \begin{cases} \frac{\sin(n+1) \arccos x}{\sin \arccos x}, |x| \le 1 \\ \frac{1}{2\sqrt{x^2-1}} \left[ \left(x + \sqrt{x^2 - 1}\right)^{n+1} - \left(x + \sqrt{x^2 - 1}\right)^{-(n+1)} \right], |x| > 1. \end{cases} \tag{8}$$

[5], [6] contain the following recurrent relations

$$U_{n+2}(x) - 2xU_{n+1}(x) - U_n(x) = 0, n \ge 0, U_0(x) = 1, U_1(x) = 2x. \tag{9}$$

$U_n(X)$ -Matrix in (9). Hence from (8), (9) we obtain: $U_{-2}(X) = E$, $U_{-1}(X) = \bar{0}$, $U_0(X) = E$, $U_1(X) = 2X$. Where $E$ – single, $\overline{0}$ – null.

(6) Conditions (7) $\varphi_0 = \overline{\xi}_0$, $\varphi_\theta = \overline{\xi}_\theta$ are [6], chapter, equations

$$\varphi_n = U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n-1}\left(\frac{1}{2}B\right) \left[\xi_0 + \sum_{k=1}^{n-1} U_{k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(u_k, v_k)}{\varepsilon_a}\right)\right] +$$
$$+U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{n-1}\left(\frac{1}{2}B\right) \left[\xi_\theta + \sum_{k=n}^{\theta-1} U_{\theta-k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(u_k, v_k)}{\varepsilon_a}\right)\right]. \tag{10}$$

Assumption 1. $M = M_0 + M_1$, $M_0$ – linear subspace,$M_1$ – subset of subspace $L$, $M_0 \times L = R^n$

$\Pi$ - Operation of orthogonal projection $R^m$ on $L$. Let $M_{1,1} + M_{1,2} = M_1$ end

$$W_{1,1}(n) = \sum_{k=1}^{n-1} \bigcap_{\upsilon(k) \in Q} \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n-1}\left(\frac{1}{2}B\right) U_{k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(P, \upsilon(k))}{\varepsilon_a}\right) - M_{1,1},$$

$$W_{1,2}(n) = \sum_{k=n}^{\theta-1} \bigcap_{\upsilon(k) \in Q} \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{n-1}\left(\frac{1}{2}B\right) U_{\theta-k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(P, \upsilon(k))}{\varepsilon_a}\right) - M_{1,2}. \tag{11}$$

Assumption 2. Let there be such $n = n_0 \le \theta - 1$, where

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n_0-1}\left(\frac{1}{2}B\right) \xi_0\right] \in W_{1,1}(n_0),$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{n_0-1}\left(\tfrac{1}{2}B\right)\xi_\theta\right]\in W_{1,2}(n_0). \tag{12}$$

Theorem 1. If Assumptions 1 and 2 are satisfied, then in the game (6),(7) from the "boundary" position $(z_0, z_\theta)$ possible completion of the pursuit $N(z_0, z_\theta)\le n_0$ steps.

Proof of Theorem. It follows from (11) and (12) that there are such

$$a(k)\in\begin{cases}\bigcap\limits_{v(k)\in Q}\Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{\theta-n_0-1}\left(\tfrac{1}{2}B\right)U_{k-1}\left(\tfrac{1}{2}B\right)\left(-\frac{\rho(P,v(k))}{\varepsilon_a}\right), 1\le k\le n_0-1,\\ \bigcap\limits_{v(k)\in Q}\Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{n_0-1}\left(\tfrac{1}{2}B\right)U_{\theta-k-1}\left(\tfrac{1}{2}B\right)\left(-\frac{\rho(P,v(k))}{\varepsilon_a}\right), n_0\le k\le\theta-1,\end{cases} \tag{13}$$

$b_1\in M_{1,1}$, $b_2\in M_{1,2}$, where

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0\right]=\sum_{k=1}^{n_0-1}a(k)-b_1,$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{n_0-1}\left(\tfrac{1}{2}B\right)\xi_\theta\right]=\textstyle\sum_{k=n_0}^{\theta-1}a(k)-b_2. \tag{14}$$

Escaping, chasing player controls construct

$$a(k)\in\begin{cases}\Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{\theta-n_0-1}\left(\tfrac{1}{2}B\right)U_{k-1}\left(\tfrac{1}{2}B\right)\left(-\frac{\rho(\bar u(k),\bar v(k))}{\varepsilon_a}\right), 1\le k\le n_0-1,\\ \Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{n_0-1}\left(\tfrac{1}{2}B\right)U_{\theta-k-1}\left(\tfrac{1}{2}B\right)\left(-\frac{\rho(\bar u(k),\bar v(k))}{\varepsilon_a}\right), n_0\le k\le\theta-1.\end{cases} \tag{15}$$

$\bar v(k)$, $\bar u(k)$ in (6) and (10)

$$\varphi(n_0)=U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\left[\xi_0+\sum_{k=1}^{n_0-1}U_{k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(\bar u_k,\bar v_k)}{\varepsilon_a}\right)\right]+$$

$$+U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{n_0-1}\left(\frac{1}{2}B\right)\left[\xi_\theta+\sum_{k=n_0}^{\theta-1}U_{\theta-k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(\bar u_k,\bar v_k)}{\varepsilon_a}\right)\right].$$

Parts (14), (15)

$$\Pi\varphi(n_0)=\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0+\sum_{k=1}^{n_0-1}U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\cdot\right.$$

$$\left.\cdot U_{k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(\bar u_k,\bar v_k)}{\varepsilon_a}\right)\right]+\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{n_0-1}\left(\frac{1}{2}B\right)\xi_\theta+\right.$$

$$\left.+\sum_{k=n_0}^{\theta-1}U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{n_0-1}\left(\frac{1}{2}B\right)U_{\theta-k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(\bar u_k,\bar v_k)}{\varepsilon_a}\right)\right]=$$

$$=\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0\right]+\sum_{k=1}^{n_0-1}a(k)+$$

$$+\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{n_0-1}\left(\frac{1}{2}B\right)\xi_\theta\right]+\sum_{k=n_0}^{\theta-1}a(k)=\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0\right]-$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0\right]+b_1+\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right)U_{n_0-1}\left(\frac{1}{2}B\right)\xi_\theta\right]-$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{n_0-1}\left(\tfrac{1}{2}B\right)\xi_0\right]+b_2=b_1+b_2\in M_{1,1}+M_{1,2}.$$

It means that$\varphi(n_0)\in M$.

Now$n\le\theta-1$, $W_{2,1}(0)=-M_{1,1}$, $W_{2,2}=-M_{1,2}$,

$$W_{2,1}(n)=\bigcap_{v(k)\in Q}\left[W_{2,1}(n-1)+\Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right)U_{\theta-n-1}\left(\tfrac{1}{2}B\right)U_{k-1}\left(\tfrac{1}{2}B\right)\left(-\frac{\rho(P,v(k))}{\varepsilon_a}\right)\right],$$
$$1\le k\le n-1,$$

$$W_{2,2}(n) = \bigcap_{\upsilon(k)\in Q} \left[W_{2,2}(n-1) + \Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{n-1}\left(\tfrac{1}{2}B\right) U_{\theta-k-1}\left(\tfrac{1}{2}B\right)\left(-\tfrac{\rho(P,\upsilon(k))}{\varepsilon_a}\right)\right], \quad (16)$$
$$n \le k \le \theta - 1.$$

Assumption 3. Let there be such $n = n_0 \le \theta - 1$, what

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0\right] \in W_{2,1}(n_0),$$
$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{n_0-1}\left(\tfrac{1}{2}B\right)\xi_\theta\right] \in W_{2,2}(n_0). \quad (17)$$

Theorem 2. If Assumptions 3 are satisfied (6), (7) the $(z_0, z_\theta)$ possible completion of the pursuit $N(z_0, z_\theta) \le n_0$ steps.

Proof of Theorem. (16), (17)

$$a(k) \in \begin{cases} \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(P,\upsilon(k))}{\varepsilon_a}\right), 1 \le k \le n_0 - 1, \\ \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{n_0-1}\left(\frac{1}{2}B\right) U_{\theta-k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(P,\upsilon(k))}{\varepsilon_a}\right), n_0 \le k \le \theta - 1, \end{cases}$$

and $b_1 \in M_{1,1}$, $b_2 \in M_{1,2}$, what

$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{\theta-n_0-1}\left(\tfrac{1}{2}C\right)\xi_0\right] \in W_{2,1}(n) + a(n_0-1) + \cdots + a(n), 1 \le n \le n_0 - 1,$$
$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{n_0-1}\left(\tfrac{1}{2}B\right)\xi_\theta\right] \in W_{2,2}(n) + a(\theta-1) + a(\theta-2) + \cdots + a(n),$$
$$n_0 \le n \le \theta - 1.$$

Hence we get that

$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{\theta-n_0-1}\left(\tfrac{1}{2}B\right)\xi_0\right] = \textstyle\sum_{k=1}^{n_0-1} a(k) - b_1,$$
$$-\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{n_0-1}\left(\tfrac{1}{2}B\right)\xi_\theta\right] = \textstyle\sum_{k=n_0}^{\theta-1} a(k) - b_2. \quad (18)$$

Now $\upsilon = \bar{\upsilon}_k$, $k \le \theta - 1$, $u = \bar{u}_k$

$$a(k) = \begin{cases} \Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{\theta-n_0-1}\left(\tfrac{1}{2}B\right)\left(-\tfrac{\rho(\bar{u}_k,\bar{\upsilon}_k)}{\varepsilon_a}\right), 1 \le k \le n_0 - 1, \\ \Pi U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{n_0-1}\left(\tfrac{1}{2}B\right) U_{\theta-k-1}\left(\tfrac{1}{2}B\right)\left(-\tfrac{\rho(\bar{u}_k,\bar{\upsilon}_k)}{\varepsilon_a}\right), n_0 \le k \le \theta - 1. \end{cases} \quad (19)$$

$u = \bar{u}_k$, $\upsilon = \bar{\upsilon}_k$ and (6), (7), taking into account (18), (19), (10)

$$\Pi\varphi(n_0) = \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0 + \sum_{k=1}^{n_0-1} U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\cdot\right.$$
$$\left. U_{k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(\bar{u}_k,\bar{\upsilon}_k)}{\varepsilon_a}\right)\right] + \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{n_0-1}\left(\frac{1}{2}B\right)\xi_\theta + \right.$$
$$\left. + \sum_{k=n_0}^{\theta-1} U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{n_0-1}\left(\frac{1}{2}B\right) U_{\theta-k-1}\left(\frac{1}{2}B\right)\left(-\frac{\rho(\bar{u}_k,\bar{\upsilon}_k)}{\varepsilon_a}\right)\right] =$$
$$= \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}B\right) U_{\theta-n_0-1}\left(\frac{1}{2}B\right)\xi_0\right] + \sum_{k=1}^{n_0-1} a(k) +$$
$$+\Pi\left[U_{\theta-1}^{-1}\left(\tfrac{1}{2}B\right) U_{n_0-1}\left(\tfrac{1}{2}B\right)\xi_\theta\right] + \textstyle\sum_{k=n_0}^{\theta-1} a(k) = b_1 + b_2 \in M_{1,1} + M_{1,2} = M_1.$$

Hence the theorem is proven.

# 4 Conclusion

Differential to problems of electrostatics essential feature was that all attention was paid to the discretization of the Poisson equation described by the electrostatic field. Are discussed this that allow a posed. We study the case when the process of potential distribution elliptic of potential distribution, are obtained.

Assessing the achievements the development and application of methods, differential and discrete games of pursuit, we have to state that the pace of progress from theory to practice is still low. This is especially noticeable in relation to physical research. This approach still provides the broadest opportunities and, apparently, in the near future we can expect a very intensive development of this direction. Potential distribution developed theoretical methods for calculating the electric fields of various systems of conductors and dielectrics on a plane can be further applied in practical calculations in radio physics and radio electronics, some results of the presented scientific research are of interest for courses in mathematical physics and electrodynamics.

# References

1. K.A. Lurie, Optimal control in problems of mathematical physics (Moscow, Nauka, 1975)
2. R. Feynman, R. Layton, M. Sands, Electrodynamics **6,** 340 (1966)
3. T.E. Zima, E.A. Zima, *Theoretical foundations of electrical engineering. Fundamentals of the theory of the electromagnetic field:* Textbook (Novosibirsk: NGTU Publishing House, 2005)
4. A. Samarskii, *The Theory of Difference Schemes, Marcel Dekker Inc*. (New York, Basel, 2001)
5. A.A. Samarskii, E.S. Nikolaev, *Methods for solving grid equations* (M., Nauka, 1978)
6. O.A. Ladyzhenskaya, N.N. Uraltseva, *Linear and quasilinear elliptic equations* (New York-London, Academic Press, 1968)
7. N.Yu. Satimov, Math. Notes **29(3)**, 234-244 (1981)
8. M.Sh. Mamatov, Avtomat. Telemekh. **8**, 123-132 (2009)
9. M.Sh. Mamatov, Automation and Remote Control **70(8)**, 1376-1384 (2009)
10. N.Yu. Satimov, M. Tukhtasinov, Math. Notes **75(5)**, 669-675 (2004)
11. N.Yu. Satimov, M. Tukhtasinov, Russian Math. (Iz. VUZ) **51(1)**, 49-57 (2007)